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CONTROLLABILITY RESULTS FOR SECOND ORDER IMPULSIVE STOCHASTIC FUNCTIONAL DIFFERENTIAL SYSTEMS WITH STATE-DEPENDENT DELAY

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Abstract. In this paper, we study the controllability results of second-order impulsive stochastic differential and neutral differential systems with state-dependent delay. Sufficient conditions for controllability of a class of second-order stochastic differential systems are formulated then the results are obtained by using the theory of strongly continuous cosine families and Sadovskii fixed point theorem. An example is provided to illustrate the theory.

1. Introduction

Stochastic differential equations have been considered extensively through discussion in the finite and infinite dimensional spaces. As a matter of fact, there exist broad literature on the related to the topic and it has played an important role in many ways such as option pricing, forecast of the growth of population, etc., and as an applications which cover the generalizations of stochastic differential equations arising in the fields such as electromagnetic theory, population dynamics, and heat conduction in material with memory. Random differential and integral equations play an important role in characterizing numerous social, physical, biological and engineering problems. For more details reader may refer [9, 15, 19, 32, 37, 39] and reference therein.

Impulsive systems arise naturally in various fields, such as mechanical systems, economics, engineering, biological systems and population dynamics, undergo abrupt changes in their state at certain moments between intervals of continuous evolution. Since many evolution process, optimal control models in economics, stimulated neural networks, frequency- modulated systems and some motions of missiles or aircrafts are characterized by the impulsive dynamical behavior. Nowadays, there has been increasing interest in the analysis and synthesis of impulsive systems due to their significance both in theory and applications. Thus the theory of impulsive differential equations has seen considerable development. For more details,

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see the monographs of Lakshmikantham et al. [34], Bainov and Simeonov [3] and Somoilenko and Perestuk [42].

Controllability play an important role in the analysis and design of control systems. Any control system is said to be controllable if every state corresponding to this process can be affected or controlled in respective time by some control signals. If the system cannot be controlled completely then different types of controllability can be defined such as approximate, null, local null and local approximate null controllability. For more details reader may refer the papers [4, 5, 6, 7, 11, 12, 30, 33, 35, 38, 40] and reference therein. Functional differential equations with state-dependent delay appear frequently in applications as model equations and for this reason the study of such equations gave received much attention in last few years, see for an instance [1, 2, 10, 16, 22, 27, 31, 36] and reference therein. The partial differential with differential equations with state dependent delay have been examine recently, for more details reader may refer [23, 24, 25, 26, 29] and reference therein.

In [8], P. Balasubramaniam et al. have studied approximate controllability of second-order stochastic distributed implicit functional differential systems with infinite delay.by using Sadovskii's fixed point theorem, whereas Yong Ren et al. [47] have proved second-order neutral impulsive stochastic evolution equations with delay and Chang et al. [13] have established the existence results for a second order impulsive functional differential equations with state-dependent delay by using Sadovskii's fixed point theorem, then Ganesan Arthi et al. [18] have examine the controllability of second-order impulsive functional differential equations with state-dependent delay by using Sadovskii's fixed point theorem. Recently, Jing Cui et al. [14] have investigate existence results for impulsive neutral second-order stochastic evolution equations with nonlocal conditions by using Sadovskii's fixed point theorem.

Inspired by the above mentioned works [8, 13, 14, 18, 47], the main purpose of this paper is to establish the controllability results for the following second order impulsive stochastic differential equations with state-dependent delay of the form

$$d[x'(t)] = \Big[Ax(t) + Bu(t)\Big]dt + f(t, x_{\rho(t,x_t)})dw(t), \quad t \in J := [0, b], \tag{1.1}$$

$$x_0 = \varphi \in \mathcal{B}, \quad x'(0) = \psi \in H, \tag{1.2}$$

$$\Delta x(t_k) = I_k(x_{t_k}), \qquad k = 1, 2, \ldots, m, \tag{1.3}$$

$$\Delta x'(t_k) = J_k(x_{t_k}), \qquad k = 1, 2, \ldots, m, \tag{1.4}$$

where A is the infinitesimal generator of a strongly continuous cosine family of bonded linear operator $C(t)$ on H. The control function $u(\cdot)$ is given in $L_2^{\mathcal{F}}(J, U)$; $x(t) \in H$; the histroy $x_t : (-\infty, 0] \to H, x_t(\theta) = x(t + \theta)$, for $t \geq 0$, belongs to phase space $\mathcal{B}$, which will be defined axiomatically in preliminaries; $0 < t_1 < t_2 < \cdots < t_m < b$ are prefixed numbers. Let K be another separable Hilbert space with inner product $(\cdot, \cdot)_K$ and norm $\|\cdot\|_K$. Suppose $\{W(t)\}_{t\geq 0}$ is a given K- valued Brownian motion or Wiener process with a finite trace nuclear covariance operator $Q \geq 0$. Throughout this paper we are employing the inner product and norm denoted respectively by $(\cdot, \cdot)$ and $\|\cdot\|$ for H, U and $L(K, H)$ denotes the space of all bounded linear operator from K into H. The functions $f : J \times \mathcal{B} \to L_Q(K, H)$, $\rho : J \times \mathcal{B} \to (-\infty, b]$ are measurable mapping in $L_Q(K, H)$-norm. Here $L_Q(K, H)$ denotes the space of all Q-Hilbert Schmidt operators from K into H which will be defined in next section. Let $\varphi(t) \in L^2(\Omega, \mathcal{B})$ and $\psi(t)$ is a H-valued $\mathcal{F}_t$-measurable random variables independent of Brownian motion $\{W(t)\}$ with a finite second moment. I_k and $J_k : \mathcal{B} \to H$ are appropriate functions. Moreover, let $0 < t_1 < \ldots < t_m < b$, $x(t_k^+)$ and $x(t_k^-)$ denote the right and left limits of $x(t)$ at $t = t_k$, $\Delta x(t_k) = x(t_k^+) - x(t_k^-)$ represents the jump in the state x at time t_k. Similarly $x'(t_k^+)$ and $x'(t_k^-)$ denote, respectively, the right and left limits of x' at t_k.

The rest of this paper is organized as follows. In Section 2, we introduce some basic notations and necessary preliminaries. In Section 3, we establish the controllability of second-order impulsive stochastic differential systems . In Section 4, we derive the controllability of second-order neutral impulsive stochastic differential systems. Finally, Section 5, paper concludes with an example is to illustrate the obtained results.

2. Preliminaries

Let $(K, \|\cdot\|_K)$ and $(H, \|\cdot\|_H)$ be the two separable Hilbert space with inner product $\langle\cdot,\cdot\rangle_K$ and $\langle\cdot,\cdot\rangle_H$, respectively. We denote $\mathcal{L}(K, H)$ be the set of all linear bounded operator from K into H, equipped with the usual operator norm $\|\cdot\|$. In this article, we use the symbol $\|\cdot\|$ to denote norms of operator regardless of the space involved when no confusion possibly arises.

Let $(\Omega, \mathcal{F}, P, H)$ be the complete probability space furnished with a complete family of right continuous increasing σ- algebra $\{\mathcal{F}_t, t \in J\}$ satisfying $\mathcal{F}_t \subset \mathcal{F}$. An H-valued random variable is an $\mathcal{F}$- measurable function $x(t) : \Omega \to H$ and a collection of random variables $S = \{x(t, \omega) : \Omega \to H \setminus t \in J\}$ is called stochastic process. Usually we write $x(t)$ instead of $x(t, \omega)$ and $x(t) : J \to H$ in the space of S. Let $\{e_i\}_{i=1}^{\infty}$ be a complete orthonormal basis of K. Suppose that $\{w(t) : t \geq 0\}$ is a cylindrical K-valued wiener process with a finite trace nuclear covariance operator $Q \geq 0$, denote $\mathrm{Tr}(Q) = \sum_{i=1}^{\infty} \lambda_i = \lambda < \infty$, which satisfies that $Qe_i = \lambda_i e_i$. So, actually, $\omega(t) = \sum_{i=1}^{\infty} \sqrt{\lambda_i} \omega_i(t) e_i$, where $\{\omega_i(t)\}_{i=1}^{\infty}$ are mutually independent one-dimensional standard Wiener processes. We assume that $\mathcal{F}_t = \sigma\{\omega(s) : 0 \leq s \leq t\}$ is the σ-algebra generated by ω and $\mathcal{F}_t = \mathcal{F}$. Let $\Psi \in \mathcal{L}(K, H)$ and define

$$\|\Psi\|_Q^2 = \mathrm{Tr}(\Psi Q \Psi^*) = \sum_{n=1}^{\infty} \|\sqrt{\lambda_n} \Psi e_n\|^2.$$

If $\|\Psi\|_Q < \infty$, then Ψ is called a Q-Hilbert-Schmidt operator. Let $\mathcal{L}_Q(K, H)$ denote the space of all Q-Hilbert-Schmidt operators $\Psi : K \to H$. The completion $\mathcal{L}_Q(K, H)$ of $\mathcal{L}(K, H)$ with respect to the topology induced by the norm $\|\cdot\|_Q$ where $\|\Psi\|_Q^2 = \langle\Psi, \Psi\rangle$ is a Hilbert space with the above norm topology. For more details reader may refer the reference [15].

The theory of cosine functions of operator is well known from few concepts and properties related to the second-order abstract Cauchy problem are going to be mentioned for more details reader may refer [17, 21, 43, 44]. Now, we say that the family $\{C(t) : t \in R\}$ of operators in $L(H)$ is a strongly continuous cosine family if

(i) $C(0) = I$, I is the identity operators in H,
(ii) $C(t + s) + C(t - s) = 2C(t)C(s)$, for all $s, t \in R$,
(iii) the map $t \to C(t)x$ is strongly continuous for each $x \in H$.

The strongly continuous sine family $\{S(t) : t \in R\}$, associated to the given strongly continuous cosine family $\{C(t) : t \in R\}$, is defined by

$$S(t)x = \int_0^t C(s)xds, \quad x \in H, \quad t \in R.$$

We denote N and $\tilde{N}$ are a pair of positive constants such that $\|C(t)\|^2 \leq N$ and $\|S(t)\|^2 \leq \tilde{N}$ for every $t \in J$. The infinitesimal genertor $A : H \to H$ of the cosine family $\{C(t) : t \in R\}$ is defined by

$$Ax = \left(\frac{d^2}{dt^2}\right)C(t)x|_{t=0},$$

for all $x \in D(A) = \{x \in H : C(\cdot)x \in C^2(R, H)\}$.
It is well known that the infinitesimal generator A is a closed, densely defined operator on H. Such cosine and corresponding sine families and their generators satisfy the following properties hold(see [45]).

Proposition 2.1. Suppose that A is the infinitesimal generator of a cosine family of operators $\{C(t) : t \in R\}$. Then, it holds the following.

(i) There exists $M^* \geq 1$ and $\alpha \geq 0$ such that $\|C(t)\| \leq M^* e^{\alpha|t|}$ and hence $\|S(t)\| \leq M * e^{\alpha|t|}$.
(ii) $A \int_s^r S(u)xdu = [C(r) - C(s)]x$, for all $0 \leq s \leq r < \infty$.
(iii) There exists $N^* \geq 1$ such that $\|S(s) - S(r)\| \leq N^* |\int_s^r e^{\alpha|s|}ds|$, for all $0 \leq s \leq r < \infty$.

The uniform boundedness principle, together with Proposition 2.1, part(i), implies that both $\{C(t) : t \in [0, b]\}$ and $\{S(t) : t \in [0, b]\}$ are uniformly bounded by some positive constants $N, \tilde{N}$ respectively.
The existence of solutions of the second order linear abstract Cauchy problem

$$x''(t) = Ax(t) + g(t), \quad t \in J, \tag{1.5}$$

$$x(0) = u, x'(0) = v, \tag{1.6}$$

where $g : J \to H$ is an integrable function, has been discussed in [43]. Similarly, the existence of solutions for semilinear second-order abstract Cauchy problem has been treated in [44].

Definition 2.1. The function $x(\cdot)$ given by

$$x(t) = C(t)u + S(t)v + \int_0^t S(t-s)g(s)ds, \quad t \in J,$$

is called a mild solution of (1.5)-(1.6) and, if $u \in H$, the function $x(\cdot)$ is continuously differentiable and

$$x'(t) = AS(t)u + C(t)v + \int_0^t C(t-s)g(s)ds.$$

For more details on cosine function theory, reader may refer [43, 45].
To consider the impulsive conditions (1.3)-(1.4), it is convenient to introduce some additional concepts and notations.
A function $u : [\mu, \tau] \to H$ is said to be normalized piecewise continuous function on $[\mu, \tau]$ if u is piecewise continuous and left continuous on $(\mu, \tau]$. We denoted by $\mathcal{PC}([\mu, \tau], H)$ the space of normalized piecewise continuous functions from $[\mu, \tau]$ into

H. In particular, we introduce the space $\mathcal{PC}$ formed by all normalized piecewise continuous function $u : [0,b] \to H$ such that $u(\cdot)$ is continuous at $t \neq t_k, u(t_k^-) = u(t_k)$ and $u(t_k^+)$ exists, for $k = 1, 2, \ldots, m$. In this paper, we always assume that $\mathcal{PC}$ is endowed with the norm $\|u\|_{\mathcal{PC}} = \left(\sup_{s\in J} E\|u(s)\|^2\right)^{\frac{1}{2}}$. It is clear that $(\mathcal{PC}, \|\cdot\|_{\mathcal{PC}})$ is a Banach space.
To simplify the notations, we put $t_0 = 0, t_{m+1} = b$ and, for $u \in \mathcal{PC}$, we denote by $\tilde{u_k}$, for $k = 1, 2, \ldots, m$ the function $\tilde{u_k} \in C(t_k, t_{k+1}, L^2(\Omega, H))$ given by $\tilde{u_k}(t) = u(t)$ for $t \in (t_k, t_{k+1}]$ and $\tilde{u_k}(t_k) = \lim_{t\to t_k^+} u(t)$. Moreover, for a set $B \subseteq \mathcal{PC}$, we denote by $\tilde{B}_k$ for $k = 1, 2, \ldots, m$, the set $\tilde{B}_k = \{\tilde{u}_k : u \in B\}$.
In this work, we will employ an axiomatic definition for the phase space $(\mathcal{B}, \|\cdot\|_{\mathcal{B}})$ is a seminormed linear space of $\mathcal{F}_0$-measurable functions maping $(-\infty, 0]$ into H_α and satisfies the following conditions[20, 28]:

(A1) If $x : (-\infty, \sigma + b] \to H_\alpha$, $b > 0$, is such that $x|_{[\sigma,\sigma+b]} \in \mathcal{PC}([\sigma, \sigma + b] : H_\alpha)$ and $x_\sigma \in \mathcal{B}$, then for every $t \in [\sigma, \sigma + b]$ the following conditions hold:
  (i) x_t is in $\mathcal{B}$,
  (ii) $\|x(t)\| \leq H\|x_t\|_{\mathcal{B}}$,
  (iii) $\|x_t\|_{\mathcal{B}} \leq K(t-\sigma) \sup\{\|x(s)\| : \sigma \leq s \leq t\} + M(t-\sigma)\|x_\sigma\|_{\mathcal{B}}$,
  where $H \geq 0$ is a constant; $K, M : [0, \infty) \to [1, \infty)$, K is continuous, M is locally bounded, and H, K, M are independent of $x(\cdot)$.
(A2) For the function $x(\cdot)$ in $(A1)$, x_t is a $\mathcal{B}$-valued continuous function on $[0, b]$.
(A3) The space $\mathcal{B}$ is complete.

The collection of all strongly measurable, square integrable, H-valued random variables, denoted by $L_2(\Omega, \mathcal{F}, P; H) \equiv L_2(\Omega, H)$ is a Banach space equipped with norm $\|x(\cdot)\|_{L_2} = (E\|x(\cdot, w)\|^2)^{\frac{1}{2}}$, where E denotes expectation defined by $Ex = \int_\Omega x(w)dP$. Let $J_1 = (-\infty, b]$ and $C(J_1, L_2(\Omega, H))$ be the Banach space of all continuous maps from J_1 into $L_2(\Omega, H)$ satisfying the conditions $\sup_{0\leq t\leq b} E\|x(t)\|^2 < \infty$. An important subspace is given by $L_2^0(\Omega, H)$ which denote the family of all $\mathcal{F}_0$-measurable, H-valued random variable $x(0)$. The notation $B_r[x, H]$ stands for the closed ball with center at x and radius $r > 0$ in H.

Let $\mathcal{C}$ be the closed subspace of all continuously differentiable process $x \in C^1(J, L_2(\Omega, H))$ consisting of $\mathcal{F}_t$-adapted measurable process such that the $\mathcal{F}_0$-adapted process $\varphi, \psi \in L_2^0(\Omega, \mathcal{B})$. Let $\|\cdot\|_{\mathcal{C}}$ be a seminorm in $\mathcal{C}$ defined by

$$\|x\|_{\mathcal{C}} = \left(\sup_{t\in J} \|x_t\|_{\mathcal{B}}^2\right)^{\frac{1}{2}},$$

where

$$\|x_s\|_{\mathcal{B}} \leq M_b E\|\varphi\|_{\mathcal{B}} + K_b \sup_{0\leq s\leq b} E\|x(s)\|_\alpha,$$

$K_b = \sup_{t\in J}\{K(t) : 0 \leq t \leq b\}$, $M_b = \sup_{t\in J}\{M(t) : 0 \leq t \leq b\}$. It is easy to verify that $\mathcal{C}$, endowed with the norm topology as defined above, is a Banach space.

Lemma 2.1. (Sadosvskii's Fixed Point Theorem[41]). Let F be condensing operator on a Banach space X. If $F(S) \subset S$ for a convex, closed and bounded sets S of X, then F has a fixed point in S.

3. Controllability Results For Second Order Impulsive Stochastic Systems

In this section, we prove the controllability of impulsive stochastic differential systems with state-dependent delay. Let $J_1 = (-\infty, b]$, here we present by defining the mild solution for the impulsive stochastic differential systems (1.1)-(1.4).

Definition 3.2. An $\mathcal{F}_t$-adapted stochastic process $x : (-\infty, b] \to H$ is called mild solution of the system (1.1)-(1.4) if $x \in C^1(J, L_2(\Omega, H))$, the function $S(t-s)f(t, x_{\rho(s,x_s)})$ is integrable on J, $x_0 = \varphi$, $x'(0) = \psi$ and $\Delta x(t_k) = I_k(x(t_k))$, $\Delta x'(t_k) = J_k(x(t_k)), k = 1, 2, \ldots, m$, $x(t)$ satisfied the following integral equation:

$$x(t) = C(t)\varphi(0) + S(t)\psi + \int_0^t S(t-s)Bu(s)ds + \int_0^t S(t-s)f(t, x_{\rho(s,x_s)})dw(s) + \sum_{0<t_k<t} C(t-t_k)I_k(x_{t_k}) + \sum_{0<t_k<t} S(t-t_k)J_k(x_{t_k}), \quad t \in J.$$

Definition 3.3. The nonlinear stochastic differential equations (1.1)-(1.4) is said to be controllable on the interval J_1, if for every continuous initial stochastic process $x_0 = \varphi \in \mathcal{B}, x'(0) = \psi$ defined on J_0, there exists a stochastic control $u \in L_2(J, U)$ which is adapted to the filtration $\{\mathcal{F}_t\}_{t\geq 0}$ such that the solution $x(\cdot)$ of (1.1)-(1.4) satisfies $x(b) = x_1$ where x_1 and b are preassigned terminal state and time, respectively.

In order to prove the main theorem, we always assume that $\rho : J \times \mathcal{B} \to (-\infty, b]$ is continuous and that $\varphi \in \mathcal{B}$. we assume the following hypotheses:

(H_φ) The function $t \to \varphi_t$ is continuous from $\mathcal{R}(\rho^-) = \{\rho(s, \psi) \leq 0, (s, \psi) \in J \times \mathcal{B}\}$ into $\mathcal{B}$ and there exists a continuous and bounded function $J^\varphi : \mathcal{R}(\rho^-) \to (0, \infty)$ such that $\|\varphi_t\| \leq J^\varphi(t)\|\varphi\|_{\mathcal{B}}$ for each $t \in \mathcal{R}(\rho^-)$.

(H1) The linear operator $W : L^2(J, U) \to L^2(\omega; H)$, defined by

$$Wu = \int_0^b T(b-s)Bu(s)ds,$$

has an induced inverse W^{-1} which takes values in $L^2(J, U)/KerW$ and there exist two positive constants M_2 and M_3 such that

$$\|B\|^2 \leq M_1 \qquad \text{and} \|W^{-1}\|^2 \leq M_2.$$

(H2) The function $f : J \times \mathcal{B} \to L_Q(K, H)$ satisfies the following conditions:

(i) The function $f(\cdot, \psi) : J \to L_Q(K, H)$ is strongly measurable.

(ii) The function $f(t, \cdot) : \mathcal{B} \to L_Q(K, H)$ is continuous for each $t \in J$.

(iii) There exists integrable function $p(t) : J \to [0, \infty)$ such that

$$E\|f(t, \varphi)\|_Q^2 \leq p(t)\Omega(\|\varphi\|_{\mathcal{B}}^2), \quad (t, \varphi) \in J \times \mathcal{B}.$$

where $\Omega : [0, \infty) \times (0, \infty)$ is a continuous nondecreasing function.

(iv) For every positive constant r, there exists an $h_r \in L^1(J)$ such that

$$\sup_{\|\varphi\|^2 \leq r} \|f(t, \varphi)\|^2 \leq h_r(t).$$

(v) $f : J \times \mathcal{B} \to L(K, H)$ is completely continuous. Then the operator

$$\Psi x(t) = \int_0^t S(t-s)f(s, x(s))dw(s) + \int_0^t S(t-s)(Bu_x)(s)ds, \quad t \in [0, b]$$

is completely continuous.

(H3) There exists a positive constants M_{I_k}, M_{J_k} such that

$$\|I_k(\Psi_1) - I_k(\Psi_2)\|^2 \leq M_{I_k}\|\Psi_1 - \Psi_2\|^2_{\mathcal{B}},\ \Psi_j \in \mathcal{B},\ j = 1,2,\ k = 1,2,\dots,m,$$
$$\|J_k(\Psi_1) - J_k(\Psi_2)\|^2 \leq M_{J_k}\|\Psi_1 - \Psi_2\|^2_{\mathcal{B}},\ \Psi_j \in \mathcal{B},\ j = 1,2,\ k = 1,2,\dots,m.$$

(H4) The maps $I_k, J_k : \mathcal{B} \to H, k = 1,2,\dots,m$ are completely continuous and there exist continuous non-decreasing functions $\Phi_k, \Gamma_k : [0,\infty) \to (0,\infty), k = 1,2,\dots,m$, such that

$$\|I_k(\Psi)\|^2 \leq \Phi_k(\|\Psi\|^2_{\mathcal{B}}), \quad \liminf_{\gamma\to+\infty} \frac{\Phi_k(\gamma)}{\gamma} = \gamma_k < \infty,$$
$$\|J_k(\Psi)\|^2 \leq \Gamma_k(\|\Psi\|^2_{\mathcal{B}}), \quad \liminf_{\gamma\to+\infty} \frac{\Gamma_k(\gamma)}{\gamma} = \sigma_k < \infty.$$

Remark 3.1. In the rest of this paper, $y : (-\infty, b] \to H$ is the function defined by $y(t) = \varphi(t)$ on $(-\infty, 0]$ and $y(t) = C(t)\varphi(0) + S(t)\psi$ for $t \in J$. Also $\|y\|_b, M_b, K_b$ and J_0^φ are constants defined by $\|y\|_b = \sup_{s\in[0,b]} \|y(s)\|$, $M_b = \sup_{s\in[0,b]} M(s)$, $K_b = \sup_{s\in[0,b]} K(s)$, $J_0^\varphi = \sup_{t\in\mathcal{R}(\rho^-)} J^\varphi(t)$.

Lemma 3.2. If $x : (-\infty, b] \to H$ is a function such that $x_0 = \varphi$ and $x|_I \in \mathcal{PC}(I : H)$, then

$$\|x_s\|_{\mathcal{B}} \leq (M_b + J^\varphi)\|\varphi\|_{\mathcal{B}} + K_b \sup\{\|x(\theta)\|;\ \theta \in [0, \max\{0,s\}]\}, \quad s \in \mathcal{R}(\rho^-) \cup J,$$

where $J^\varphi = \sup_{t\in\mathcal{R}(\rho^-)} J^\varphi(t)$, $M_b = \sup_{t\in J} M(t)$ and $K_b = \sup_{t\in J} K(t)$.

Theorem 3.1. Assume that the assumptions (H_φ), (H1)-(H4) hold. Then the system (1.1)-(1.4) is controllable on J_1 provided that

$$\left(36 + 36^2 b^2 M_1 M_2 M_3\right)\left[K_b\left(M_1 Tr(Q) \liminf_{\xi\to\infty} \frac{\Omega(\xi)}{\xi}\int_0^b p(s)ds + M_1\sum_{k=1}^m M_{I_k}\right)\right] < 1.$$

Proof. Consider the space $Y = \{x \in \mathcal{PC} : u(0) = \varphi(0)\}$ endowed with the uniform convergence topology. Using the assumption (H1), for an arbitrary function $x(\cdot)$, we define the control

$$\begin{aligned} u(t) =& W^{-1}\Bigg[x_1 - C(b)\varphi(0) - S(b)\psi - \int_0^b S(b-s)f(s, x_{\rho(s,x_s)})dw(s) \\ &- \sum_{k=1}^m C(b-t_k)I_k(x_{t_k}) - \sum_{k=1}^m S(b-t_k)J_k(x_{t_k})\Bigg](t). \end{aligned}$$

Using this control, we shall show that the operator $\Psi : Y \to Y$ defined by

$$\begin{aligned}\Psi x(t) = {} & C(t)\varphi(0) + S(t)\psi + \int_0^t S(t-s) f(s, \bar{x}_{\rho(s,\bar{x}_s)}) dw(s) \\ & + \int_0^t S(t-\eta) B W^{-1} \Bigg\{ x_1 - C(b)\varphi(0) - \int_0^b S(b-s) f(s, \bar{x}_{\rho(s,\bar{x}_s)}) dw(s) \\ & - S(b)\psi - \sum_{k=1}^m C(b-t_k) I_k(\bar{x}_{t_k}) - \sum_{k=1}^m S(b-t_k) J_k(\bar{x}_{t_k}) \Bigg\}(\eta) d\eta \\ & + \sum_{k=1}^m C(t-t_k) I_k(\bar{x}_{t_k}) + \sum_{0<t_k<t} S(t-t_k) J_k(\bar{x}_{t_k}), t \in J,\end{aligned}$$

has a fixed point $x(\cdot)$. This fixed point $x(\cdot)$ is then a mild solution of the system (1.1)-(1.4). Clearly, $(\Psi x)(b) = x_1$, which means that the control u steers the systems from the initial state φ to x_1 in time b, provided we can obtain a fixed point of the operator Ψ which implies that the systems is controllable. Here $\bar{x} : (-\infty, b] \to H$ is such that $\bar{x}_0 = \varphi$ and $\bar{x} = x$ on J. From the axiom (A1) and our assumption on φ, it is easy to see that $\Psi x \in \mathcal{PC}$.

Next we claim that there exists $r > 0$ such that $\Psi(B_r(y_{|_J}, Y)) \subseteq (B_r(y_{|_J}, Y))$. If we assume this property is false, then for every $r > 0$ there exist $x^r \in (B_r(y_{|_J}, Y))$ and $t^r \in J$ such that $r < E\|\Psi x^r(t^r) - y(t^r)\|^2$. Then by using Lemma 3.2 we get

$$\begin{aligned} r < {} & E\|\Psi x^r(t^r) - y(t^r)\|^2 \\ \leq {} & 36NH\|\varphi\|^2_{\mathcal{B}} + 36\tilde{N}\|\psi\|^2 + 36\tilde{N}Tr(Q)\int_0^{t_r} p(s)\Omega(\|\overline{x^r}_{\rho(s,\bar{x}^r_s)}\|^2_{\mathcal{B}})ds \\ & + 36^2\tilde{N}M_1M_2\int_0^{t_r}\Bigg[\|x_1\|^2 + NH\|\varphi\|^2_{\mathcal{B}} + \tilde{N}Tr(Q)\int_0^b p(s)\Omega(\|\overline{x^r}_{\rho(s,\bar{x}^r_s)}\|^2_{\mathcal{B}})ds \\ & + \tilde{N}\|\psi\|^2 + \sum_{k=1}^m N(M_{I_k}\|(\bar{x}_{t_k} - y_{t_k}\|^2_{\mathcal{B}} + \|I_k(y_{t_k})\|^2) \\ & + \sum_{k=1}^m \tilde{N}(M_{J_k}\|(\bar{x}_{t_k} - y_{t_k}\|^2_{\mathcal{B}} + \|J_k(y_{t_k})\|^2)\Bigg]d\eta \\ & + 36\sum_{k=1}^m N(M_{I_k}\|(\bar{x}_{t_k} - y_{t_k}\|^2_{\mathcal{B}} + \|I_k(y_{t_k})\|^2) \\ & + 36\sum_{k=1}^m \tilde{N}(M_{J_k}\|(\bar{x}_{t_k} - y_{t_k}\|^2_{\mathcal{B}} + \|J_k(y_{t_k})\|^2) \\ \leq {} & 36NH\|\varphi\|^2_{\mathcal{B}} + 36\tilde{N}\|\psi\|^2 + 36\tilde{N}Tr(Q)\Omega((M_b + J_0^{\varphi})\|\varphi\|^2_{\mathcal{B}} + K_b r \\ & + K_b\|y\|^2_b)\int_0^{t^r} p(s)ds + 36^2b^2M_1M_2\Bigg[\|x_1\|^2 + NH\|\varphi\|^2_{\mathcal{B}} + \tilde{N}\|\psi\|^2 \\ & + \tilde{N}Tr(Q)\Omega((M_b + J_0^{\varphi})\|\varphi\|^2_{\mathcal{B}} + K_b r + K_b\|y\|^2_b)\int_0^b p(s)ds \\ & + \sum_{k=1}^m N(M_{I_k}K_b r + \|I_k(y_{t_k})\|^2) + \sum_{k=1}^m \tilde{N}(M_{J_k}K_b r + \|J_k(y_{t_k})\|^2)\Bigg]\end{aligned}$$

$$+36\sum_{k=1}^{m}N(M_{I_k}K_br+\|I_k(y_{t_k})\|^2)+36\sum_{k=1}^{m}\tilde{N}(M_{J_k}K_br+\|J_k(y_{t_k})\|^2),$$

and hence

$$\begin{aligned}1\leq\big(36+36^2b^2M_1M_2M_3\big)\Bigg[K_b\Bigg(&\tilde{N}Tr(Q)\liminf_{\xi\to\infty}\frac{\Omega(\xi)}{\xi}\int_0^b p(s)ds\\&+\sum_{k=1}^{m}(NM_{I_k}+\tilde{N}M_{J_k}\Bigg)\Bigg],\end{aligned}$$

which is the contrary to the our assumption.

Let $r>0$ be such that $\Psi(B_r(y_{|_J},Y))\subset(B_r(y_{|_J},Y))$. In order to prove that Ψ is a condensing map on $\Psi(B_r(y_{|_J},Y))$ into $(B_r(y_{|_J},Y))$. We decompose Ψ as Ψ_1 and Ψ_2 (i.e) $\Psi=\Psi_1+\Psi_2$ where

$$\begin{aligned}\Psi_1x(t)=&S(t)\psi+\sum_{0<t_k<t}C(t-t_k)I_k(\bar{x}_{t_k})+\sum_{0<t_k<t}S(t-t_k)J_k(\bar{x}_{t_k}), && t\in J,\\ \Psi_2x(t)=&C(t)\varphi(0)+\int_0^t S(t-s)f(t,\bar{x}_{\rho(s,\bar{x}_s)})dw(s)+\int_0^t S(t-s)Bu(s)ds, && t\in J.\end{aligned}$$

Now

$$\begin{aligned}E\|Bu(s)\|^2\leq{}&36M_1M_2\Bigg[\|x_1\|^2+NH\|\varphi\|^2_{\mathcal{B}}+\tilde{N}\|\psi\|^2+Tr(Q)\tilde{N}\int_0^b h_rds\\&+N\sum_{k=1}^{m}\Phi_k\|\bar{x}_{t_k}\|^2+\tilde{N}\sum_{k=1}^{m}\Gamma_k\|\bar{x}_{t_k}\|^2\Bigg]\\ \leq{}&36M_1M_2\Bigg[\|x_1\|^2+NH\|\varphi\|^2_{\mathcal{B}}+\tilde{N}\|\psi\|^2+Tr(Q)\tilde{N}\int_0^b h_rds\\&+\sum_{k=1}^{m}r(N\Phi_k+\tilde{N}\Gamma_k)\Bigg]=P_0.\end{aligned}$$

Step 1. The set $\Psi_2(B_r(y_{|_J},Y))(t)=\{\Psi_2x(t):x\in(B_r(y_{|_J},Y))\}$ is relatively compact in X for every $t\in J$. The case $t=0$ is obvious. Let $0<\epsilon<t\leq b$. If $x\in(B_r(y_{|_J},Y))$, from Lemma 3.2 it follows that,

$$\|\bar{x}_{\rho(s,\bar{x}_s)}\|^2_{\mathcal{B}}\leq r^*=(M_b+\tilde{J}^{\varphi})\|\varphi\|^2_{\mathcal{B}}+K_br,$$

and so

$$\|\int_0^\tau S(\tau-s)f(t,\bar{x}_{\rho(s,\bar{x}_s)})dw(s)\|^2\leq r^{**}=Tr(Q)\Omega(r^*)\tilde{N}\int_0^b p(s)ds,\quad t\in J,$$

and

$$\|\int_0^\tau S(\tau-s)Bu(s)ds\|^2\leq g^*=\tilde{N}\int_0^b P_0ds,\quad \tau\in J.$$

Consequently, for $x \in (B_r(y_{|_J}, Y))$, we define that

$$\begin{aligned} E\|\Psi_2 x(t)\|^2 &= E\|C(t)\varphi(0) + S(\epsilon)\int_0^{t-\epsilon} S(t-\epsilon-s)f(t,\bar{x}_{\rho(s,\bar{x}_s)})dw(s) + \\ &+ \int_{t-\epsilon}^{t} S(t-s)f(t,\bar{x}_{\rho(s,\bar{x}_s)})dw(s) + S(\epsilon)\int_0^{t-\epsilon} S(t-\epsilon-s)Bu(s)ds \\ &+ \int_{t-\epsilon}^{t} S(t-s)Bu(s)ds\|^2 \\ &\in 9\{C(t)\phi(0)\} + 9S(\epsilon)B_{r^{**}}(0,H) + 9M_\epsilon + 9S(\epsilon)B_{g^*}(0,H) + 9G_\epsilon, \end{aligned}$$

where $\mathrm{diam}(M_\epsilon) \le 2\tilde{N}Tr(Q)\Omega(r^*)\int_{t-\epsilon}^{t} p(s)ds$ and $\mathrm{diam}(G_\epsilon) \le \tilde{N}\int_{t-\epsilon}^{t} P_0 ds$ which proves that $\Psi_2(B_r(y_{|_J}, Y))(t)$ is relatively compact in H.

Step 2. The function $\Psi_2(B_r(y_{|_J}, Y))$ is equicontinuous on J. Let $0 < t < b$ and $\epsilon > 0$. Since the semigroup $(T(t))_{t\ge 0}$ is strongly continuous and $\Psi_2(B_r(y_{|_J}, Y))$ is relatively compact in H, there exists $0 < \delta \le b - t$ such that

$$E\|S(h)x - x\|^2 < \epsilon, \quad x \in \Psi_2(B_r(y_{|_J}, Y)), \quad 0 < h < \delta.$$

Under these conditions, for $x \in \Psi_2(B_r(y_{|_J}, Y))$ and $0 < h < \delta$, we get

$$\begin{aligned} E\|\Psi_2 x(t+h) - \Psi_2 x(t)\|^2 &\le E\|S(t+h)\varphi(0) - S(t)\varphi(0)\|^2 + E\|S(h)x - x\|^2 \\ &\quad + E\|\int_t^{t+h} S(t-s)f(t,\bar{x}_{\rho(s,\bar{x}_s)})dw(s)\|^2 \\ &\quad + E\|\int_t^{t+h} S(t-s)Bu(s)ds\|^2 \\ &\le 9\tilde{N}\|(S(t+h) - I)\varphi(0)\|^2 + 9\epsilon \\ &\quad + 9\tilde{N}Tr(Q)\Omega(r^*)\int_t^{t+h} p(s)ds + 9\tilde{N}\int_t^{t+h} P_0 ds, \end{aligned}$$

which proves that the set function $\Psi_2(B_r(y_{|_J}, Y))$ is right equicontinuous at $t \in (0, b)$. Similarly, we can prove the right equicontinuity at zero and left equicontinuity at $t \in (0, b]$. Thus $\Psi_2(B_r(y_{|_J}, Y))$ is equicontinuous on J.

Step 3. The map $\Psi_2(\cdot)$ is continuous on $(B_r(y_{|_J}, Y))$. Let $(x^n)_{n\in N}$ be a sequence in $(B_r(y_{|_J}, Y))$ and $x \in (B_r(y_{|_J}, Y))$ such that $x^n \to x$ in $\mathcal{PC}$. From the Axioms, it is easy to see that $(\overline{x^n})_s \to \bar{x}_s$ as $n \to \infty$ uniformly for $s \in (-\infty, b]$ as $n \to \infty$. By assumption, we have

$$f(t, \overline{x^n}_{\rho(s,\bar{x}_s)}) \to f(t, \bar{x}_{\rho(s,\bar{x}_s)}) \quad \text{as} \quad n \to \infty,$$

for each $s \in [0, t]$, and since

$$\|f(t, \overline{x^n}_{\rho(s,\bar{x}^n_s)}) - f(t, \bar{x}_{\rho(s,\bar{x}_s)})\|^2 \le 2p(t)\Omega(r^*) \quad \text{as} \quad n \to \infty.$$

Now, a standard application of Lebesgue dominated convergence theorem, we have

$$\begin{aligned}
&E\|\Psi_2 x^n - \Psi_2 x\|^2_{\mathcal{B}}\\
&\leq E\|\int_0^t S(t-s)[f(t,\overline{x^n}_{\rho(s,\bar{x}^n_s)}) - f(t,\bar{x}_{\rho(s,\bar{x}_s)})]dw(s)\\
&\quad + \int_0^t S(t-\eta)B\Big[W^{-1}\Big\{x_1 - C(b)\varphi(0) - S(b)\psi\\
&\quad - \int_0^b S(b-s)f(s,\bar{x}^n_{\rho(s,\bar{x}^n_s)})dw(s) - \sum_{k=1}^m C(b-t_k)I_k(\bar{x}^n_{t_k})\\
&\quad - \sum_{k=1}^m S(b-t_k)J_k(\bar{x}^n_{t_k})\Big\} - W^{-1}\Big\{x_1 - C(b)\varphi(0)\\
&\quad - S(b)\psi - \int_0^b S(b-s)f(s,\bar{x}_{\rho(s,\bar{x}_s)})dw(s)\\
&\quad - \sum_{k=1}^m C(b-t_k)I_k(\bar{x}_{t_k}) - \sum_{k=1}^m S(b-t_k)J_k(\bar{x}_{t_k})\Big\}\Big](\eta)d\eta\|^2\\
&\leq 4Tr(Q)\tilde{N}\int_0^t E\|f(t,\overline{x^n}_{\rho(s,\bar{x}_s)}) - f(t,\bar{x}_{\rho(s,\bar{x}_s)})\|^2 ds\\
&\quad + 4\tilde{N}M_1M_2\int_0^b \Big[\tilde{N}\int_0^b \|f(t,\overline{x^n}_{\rho(s,\bar{x}_s)}) - f(t,\bar{x}_{\rho(s,\bar{x}_s)})\|^2 ds\\
&\quad + N\sum_{k=1}^m \|I_k(\overline{x^n}_{t_k}) - I_k(\bar{x}_{t_k})\|^2 + \tilde{N}\sum_{k=1}^m \|J_k(\overline{x^n}_{t_k}) - J_k(\bar{x}_{t_k})\|^2\Big]d\eta\\
&\quad \to 0 \quad \text{as} \quad n\to\infty.
\end{aligned}$$

Thus, $\Psi_2(\cdot)$ is continuous.

Step 4. The map $\Psi_1(\cdot)$ is a contraction on $(B_r(y_{|_J}, Y))$

$$\|\Psi_1 x - \Psi_1 y\|^2 \leq 4K_b \sum_{k=1}^m \Big(NM_{I_k} + \tilde{N}M_{J_k}\Big)\|x-y\|^2.$$

It follows that Ψ_1 is a contraction on $(B_r(y_{|_J}, Y))$ which implies that Ψ is a condensing operator on $(B_r(y_{|_J}, Y))$ into $(B_r(y_{|_J}, Y))$.

Finally, from Lemma 2.1, Ψ has a fixed point in Y which implies that any fixed point $\Psi(\cdot)$ is a mild solution of the problem (1.1)-(1.4). This completes the proof.

4. Controllability Results for Second Order Neutral Impulsive Stochastic Systems

In this section, we prove the controllability result for nonlinear systems with state-dependent delay. Consider the impulsive neutral stochastic control systems of the form

$$d[x'(t) - g(t,x_t)] = \Big[Ax(t) + Bu(t)\Big]dt + f(t,x_{\rho(t,x_t)})dw(t),\ t\in J := [0,b], \tag{4.1}$$

$$x_0 = \varphi \in \mathcal{B}, \quad x'(0) = \psi \in H, \tag{4.2}$$

$$\Delta x(t_k) = I_k(x_{t_k}), \qquad k = 1,2,\ldots,m, \tag{4.3}$$

$$\Delta x'(t_k) = J_k(x_{t_k}), \qquad k = 1,2,\ldots,m, \tag{4.4}$$

where A, B, ρ, f, I_k and J_k are defined in equations (1.1)-(1.4). Here $g : J \times \mathcal{B} \to H$ is an appropriate function. Furthermore, we assume the following conditions:

(H5) The function $g : J \times \mathcal{B} \to H$ is completely continuous and there exists $M_g > 0$ such that

$$\|g(t, \psi_1) - g(t, \psi_2)\|^2 \leq M_g \|\psi_1 - \psi_2\|^2, \quad (t, \psi_l) \in J \times \mathcal{B}, l = 1, 2.$$

(H6) There exists positive constants θ_1, θ_2 such that $\|g(t, \psi)\|^2 \leq \theta_1 \|\psi\|^2 + \theta_2$, for every $(t, \psi) \in J \times \mathcal{B}$.

Definition 4.4. An $\mathcal{F}_t$-adapted stochastic process $x : (-\infty, b] \to H$ is called mild solution of the system (4.1)-(4.4) if $x \in C^1(J, L_2(\Omega, H))$, the function $S(t-s)f(t, x_{\rho(s,x_s)})$ and $C(t-s)g(s, x_s)$ is integrable on J, $x_0 = \varphi$, $x'(0) = \psi$ and $\Delta x(t_k) = I_k(x(t_k))$, $\Delta x'(t_k) = J_k(x(t_k)), k = 1, 2, \ldots, m$, $x(t)$ satisfied the following integral equation:

$$\begin{aligned}
x(t) = C(t)\varphi(0) + S(t)[\psi - g(0, \varphi)] + \int_0^t C(t-s)g(s, x_s)ds \\
+ \int_0^t S(t-s)Bu(s)ds + \int_0^t S(t-s)f(s, x_{\rho(s,x_s)})dw(s) \\
+ \sum_{0<t_k<t} C(t-t_k)I_k(x_{t_k}) + \sum_{0<t_k<t} S(t-t_k)J_k(x_{t_k}), \quad t \in J.
\end{aligned}$$

Theorem 4.2. Assume that the assumptions (H_φ), (H1)-(H6) hold. Then the system (4.1)-(4.4) is controllable on $(-\infty, b]$ provided that

$$\begin{aligned}
1 \leq \big(49 + 49^2 b^2 \tilde{N} M_1 M_2\big)\Bigg[K_b\Bigg(b^2 N M_g + \tilde{N} Tr(Q) \liminf_{\xi \to \infty} \frac{\Omega(\xi)}{\xi} \int_0^b p(s)ds \\
+ \sum_{k=1}^{m} (N M_{I_k} + \tilde{N} M_{J_k})\Bigg)\Bigg].
\end{aligned}$$

Proof. Consider the space $Y = \{x \in \mathcal{PC} : u(0) = \varphi(0)\}$ endowed with the uniform convergence topology. Using the assumption (H1), for an arbitrary function $x(\cdot)$, we define the control

$$\begin{aligned}
u(t) = W^{-1}\Bigg[x_1 - C(b)\varphi(0) - S(b)[\psi - g(0, \varphi)] - \int_0^b C(b-s)g(s, x_s)ds \\
- \int_0^b S(b-s)f(s, x_{\rho(s,x_s)})dw(s) - \sum_{k=1}^{m} C(b-t_k)I_k(x_{t_k}) \\
- \sum_{k=1}^{m} S(b-t_k)J_k(x_{t_k})\Bigg](t).
\end{aligned}$$

Using this control, we shall show that the operator $\Psi : Y \to Y$ defined by

$$\begin{aligned}
\Psi x(t) &= C(t)\varphi(0) + S(t)[\psi - g(0,\varphi)] + \int_0^t C(t-s)g(s,x_s)ds \\
&+ \int_0^t S(t-s)f(s,\bar{x}_{\rho(s,\bar{x}_s)})dw(s) + \int_0^t S(t-\eta)BW^{-1}\Bigg[x_1 - C(b)\varphi(0) \\
&- S(b)[\psi - g(0,\varphi)] - \int_0^b S(b-s)f(s,\bar{x}_{\rho(s,\bar{x}_s)})dw(s) \\
&- \int_0^t C(b-s)g(s,x_s)ds - \sum_{k=1}^m C(b-t_k)I_k(\bar{x}_{t_k}) \\
&- \sum_{k=1}^m S(b-t_k)J_k(\bar{x}_{t_k})\Bigg](\eta)d\eta + \sum_{k=1}^m C(t-t_k)I_k(\bar{x}_{t_k}) \\
&+ \sum_{0<t_k<t} S(t-t_k)J_k(\bar{x}_{t_k}),\ t \in J,
\end{aligned}$$

has a fixed point $x(\cdot)$. The fixed point $x(\cdot)$ is then a mild solution of the system (4.1)-(4.4). Clearly, $(\Psi x)(b) = x_1$, which means that the control u steers the systems from the initial state φ to x_1 in time b, provided we can obtain a fixed point of the operator Ψ which implies that the systems is controllable. Here $\bar{x} : (-\infty, b] \to H$ is such that $\bar{x}_0 = \varphi$ and $\bar{x} = x$ on J. From the axiom (A1) and our assumption on φ, it is easy to see that $\Psi x \in \mathcal{PC}$.

Next we claim that there exists $r > 0$ such that $\Psi(B_r(y_{|_J}, Y)) \subset (B_r(y_{|_J}, Y))$. If this assume this property is false, then for every $r > \|\varphi\|^2$ there exist $x^r \in (B_r(y_{|_J}, Y))$ and $t^r \in J$ such that $r < E\|\Psi x^r(t^r) - y(t^r)\|^2$. Then by using Lemma 3.2, we get

$$\begin{aligned}
r &< E\|\Psi x^r(t^r) - y(t^r)\|^2 \\
&\leq 49NH\|\varphi\|_{\mathcal{B}}^2 + 98\tilde{N}[\|\psi\|^2 + \|g(0,\varphi)\|^2] + 49N\int_0^{t^r} \|g(s,(\bar{x}^r)_s) - g(s,y_s)\|^2 ds \\
&+ 49N\int_0^{t^r} \|g(s,y_s)\|^2 ds + 64\tilde{N}Tr(Q)\int_0^{t_r} p(s)\Omega(\|\overline{x^r}_{\rho(s,\bar{x}_s^r)})\|_{\mathcal{B}}^2)ds \\
&+ 49^2\tilde{N}M_1M_2\int_0^{t_r}\Bigg[\|x_1\|^2 + NH\|\varphi\|_{\mathcal{B}}^2 + 2\tilde{N}[\|\psi\|^2 + \|g(0,\varphi)\|^2] \\
&+ N\int_0^b \|g(s,(\bar{x}^r)_s) - g(s,y_s)\|^2 ds + N\int_0^b \|g(s,y_s)\|^2 ds \\
&+ \tilde{N}Tr(Q)\int_0^b p(s)\Omega(\|\overline{x^r}_{\rho(s,\bar{x}_s^r)})\|_{\mathcal{B}}^2)ds + \sum_{k=1}^m N(M_{I_k}\|(\bar{x}_{t_k} - y_{t_k}\|_{\mathcal{B}}^2 + \|I_k(y_{t_k})\|^2) \\
&+ \sum_{k=1}^m \tilde{N}(M_{J_k}\|(\bar{x}_{t_k} - y_{t_k}\|_{\mathcal{B}}^2 + \|J_k(y_{t_k})\|^2)\Bigg]d\eta \\
&+ 49\sum_{k=1}^m N(M_{I_k}\|(\bar{x}_{t_k} - y_{t_k}\|_{\mathcal{B}}^2 + \|I_k(y_{t_k})\|^2)
\end{aligned}$$

$$
\begin{aligned}
&+ 49\sum_{k=1}^{m}\tilde{N}(M_{J_k}\|(\bar{x}_{t_k}-y_{t_k}\|^2_{\mathcal{B}}+\|J_k(y_{t_k})\|^2)\\
&\le 49NH\|\varphi\|^2_{\mathcal{B}}+98\tilde{N}[\|\psi\|^2+\|g(0,\varphi)\|^2]+49NM_gK_b\int_0^{t^r}\|\bar{x}^r-y\|^2_s ds\\
&+49N\int_0^{t^r}(\theta_1\|y_s\|^2_{\mathcal{B}}+\theta_2)ds+49\tilde{N}Tr(Q)\Omega((M_b+J_0^{\varphi})\|\varphi\|^2_{\mathcal{B}}+K_br\\
&+K_b\|y\|^2_b)\int_0^b p(s)ds+49^2b^2\tilde{N}M_1M_2\Big[\|x_1\|^2+NH\|\varphi\|^2_{\mathcal{B}}+2\tilde{N}[\|\psi\|^2\\
&+g(0,\varphi)\|^2]+NM_gK_b\int_0^b\|\bar{x}^r-y\|^2_s ds+N\int_0^b(\theta_1\|y_s\|^2_{\mathcal{B}}+\theta_2)ds\\
&+\tilde{N}Tr(Q)\Omega((M_b+J_0^{\varphi})\|\varphi\|^2_{\mathcal{B}}+K_br+K_b\|y\|^2_b)\int_0^b p(s)ds\\
&+\sum_{k=1}^{m}N(M_{I_k}K_br+\|I_k(y_{t_k})\|^2)+\sum_{k=1}^{m}\tilde{N}(M_{J_k}K_br+\|J_k(y_{t_k})\|^2)\Big]\\
&+49\sum_{k=1}^{m}N(M_{I_k}K_br+\|I_k(y_{t_k})\|^2)+49\sum_{k=1}^{m}\tilde{N}(M_{J_k}K_br+\|J_k(y_{t_k})\|^2),
\end{aligned}
$$

and hence

$$
\begin{aligned}
1\le\Big(49+49^2b^2\tilde{N}M_1M_2\Big)\Big[K_b\Big(&b^2NM_g+\tilde{N}Tr(Q)\liminf_{\xi\to\infty}\frac{\Omega(\xi)}{\xi}\int_0^b p(s)ds\\
&+\sum_{k=1}^{m}(NM_{I_k}+\tilde{N}M_{J_k}\Big)\Big],
\end{aligned}
$$

which is the contrary to the our assumption.

Let $r>0$ be such that $\Psi(B_r(y_{|_J},Y))\subset(B_r(y_{|_J},Y))$. In order to prove that Ψ is a condensing map on $\Psi(B_r(y_{|_J},Y))$ into $(B_r(y_{|_J},Y))$. We decompose Ψ as Ψ_1 and Ψ_2 (i.e) $\Psi=\Psi_1+\Psi_2$ where

$$
\begin{aligned}
\Psi_1x(t)&=S(t)[\psi-g(0,\varphi)]+\int_0^t C(t-s)g(s,\bar{x}_s)ds\\
&\quad+\sum_{0<t_k<t}C(t-t_k)I_k(\bar{x}_{t_k})+\sum_{0<t_k<t}S(t-t_k)J_k(\bar{x}_{t_k}), \qquad t\in J,\\
\Psi_2x(t)&=C(t)\varphi(0)+\int_0^t S(t-s)f(t,\bar{x}_{\rho(s,\bar{x}_s)})dw(s)+\int_0^t S(t-s)Bu(s)ds, \quad t\in J.
\end{aligned}
$$

Now

$$
\begin{aligned}
E\|Bu(s)\|^2\le 36M_1M_2\Big[&\|x_1\|^2+NH\|\varphi\|^2_{\mathcal{B}}+2\tilde{N}[\|\psi\|^2+\theta_1\|\varphi\|^2+\theta_2]\\
&+N\int_0^b(\theta_1\|\bar{x}_s\|^2+\theta_2)+Tr(Q)\tilde{N}\int_0^b h_r ds\\
&+N\sum_{k=1}^{m}\Phi_k\|\bar{x}_{t_k}\|^2+\tilde{N}\sum_{k=1}^{m}\Gamma_k\|\bar{x}_{t_k}\|^2\Big]
\end{aligned}
$$

$$\leq 36M_1M_2\bigg[\|x_1\|^2 + NH\|\varphi\|_{\mathcal{B}}^2 + 2\tilde{N}[\|\psi\|^2 + \theta_1\|\varphi\|^2 + \theta_2]$$
$$+ Tr(Q)\tilde{N}\int_0^b h_r ds + b^2N(\theta_1 r + \theta_2) + \sum_{k=1}^m r(N\Phi_k + \tilde{N}\Gamma_k)\bigg] = \tilde{P_0}.$$

Similarly, same as in the proof of Theorem 3.1. we can conclude that Ψ is continuous and that Ψ_2 is completely continuous. Moreover, from estimate

$$\|\Psi_1 u - \Psi_1 v\|_{\mathcal{PC}}^2 \leq 16K_b\left[b^2M_gN + \sum_{k=1}^m\Big(NM_{I_k} + \tilde{N}M_{J_k}\Big)\right]\|u-v\|_{\mathcal{PC}}^2,$$

it follows that Ψ_1 is a contraction on $(B_r(y_{|_J}, Y))$ which implies that Ψ is a condensing operator on $(B_r(y_{|_J}, Y))$ into $(B_r(y_{|_J}, Y))$.

Finally, from Lemma 2.1, Ψ has a fixed point in Y which implies that any fixed point $\Psi(\cdot)$ is a mild solution of the problem (4.1)-(4.4). This completes the proof.

Theorem 4.3. Assume that the assumptions (H_φ), (H1)-(H6) hold. Then the system (4.1)-(4.4) is controllable on $(-\infty, b]$ provided that

$$1 \leq \Big(49 + 49^2b^2\tilde{N}M_1M_2\Big)\bigg[K_b\bigg(b^2NM_g + \tilde{N}Tr(Q)\liminf_{\xi\to\infty}\frac{\Omega(\xi)}{\xi}\int_0^b p(s)ds$$
$$+\sum_{k=1}^m(NM_{I_k} + \tilde{N}M_{J_k})\bigg)\bigg].$$

Proof. Consider the space $Y = \{x \in \mathcal{PC} : u(0) = \varphi(0)\}$ endowed with the uniform convergence topology. Using the assumption (H1), for an arbitrary function $x(\cdot)$, we define the control

$$u(t) = W^{-1}\bigg[x_1 - C(b)\varphi(0) - S(b)[\psi - g(0,\varphi)] - \int_0^b C(b-s)g(s,x_s)ds$$
$$- \int_0^b S(b-s)f(s, x_{\rho(s,x_s)})dw(s) - \sum_{k=1}^m C(b-t_k)I_k(x_{t_k})$$
$$- \sum_{k=1}^m S(b-t_k)J_k(x_{t_k})\bigg](t).$$

Using this control, we shall show that the operator $\Psi : Y \to Y$ defined by

$$\Psi x(t) = C(t)\varphi(0) + S(t)[\psi - g(0,\varphi)] + \int_0^t C(t-s)g(s,x_s)ds$$
$$+ \int_0^t S(t-s)f(s, \bar{x}_{\rho(s,\bar{x}_s)})dw(s) + \int_0^t S(t-\eta)BW^{-1}\bigg[x_1 - C(b)\varphi(0)$$
$$- S(b)[\psi - g(0,\varphi)] - \int_0^b S(b-s)f(s, \bar{x}_{\rho(s,\bar{x}_s)})dw(s)$$

$$
\begin{aligned}
&-\int_0^t C(b-s)g(s,x_s)ds-\sum_{k=1}^m C(b-t_k)I_k(\bar{x}_{t_k})\\
&-\sum_{k=1}^m S(b-t_k)J_k(\bar{x}_{t_k})\Bigg](\eta)d\eta+\sum_{k=1}^m C(t-t_k)I_k(\bar{x}_{t_k})\\
&+\sum_{0<t_k<t} S(t-t_k)J_k(\bar{x}_{t_k}), t\in J,
\end{aligned}
$$

has a fixed point $x(\cdot)$. This fixed point $x(\cdot)$ is then a mild solution of the system (4.1)-(4.4). Clearly, $(\Psi x)(b) = x_1$, which means that the control u steers the systems from the initial state φ to x_1 in time b, provided we can obtain a fixed point of the operator Ψ which implies that the systems is controllable. Here $\bar{x}$: $(-\infty, b] \to H$ is such that $\bar{x}_0 = \varphi$ and $\bar{x} = x$ on J. From the axiom (A1) and our assumption on φ, it is easy to see that $\Psi x \in \mathcal{PC}$.

Next we claim that there exists $r > 0$ such that $\Psi(B_r(y_{|_J}, Y)) \subset (B_r(y_{|_J}, Y))$. If this assume this property is false, then for every $r > 0$ there exist $x^r \in (B_r(y_{|_J}, Y))$ and $t^r \in J$ such that $r < E\|\Psi x^r(t^r) - y(t^r)\|^2$. Then by using Lemma 3.2, we get

$$
\begin{aligned}
r &< E\|\Psi x^r(t^r)-y(t^r)\|^2\\
&\le 49NH\|\varphi\|^2_{\mathcal{B}}+98\tilde{N}[\|\psi\|^2+\|g(0,\varphi)\|^2]+49N\int_0^{t^r}\|g(s,(\bar{x}^r)_s)-g(s,y_s)\|^2ds\\
&\quad+49N\int_0^{t^r}\|g(s,y_s)\|^2ds+64\tilde{N}Tr(Q)\int_0^{t_r}p(s)\Omega(\|\overline{x^r}_{\rho(s,\bar{x}^r_s)}\|^2_{\mathcal{B}})ds\\
&\quad+49^2\tilde{N}M_1M_2\int_0^{t_r}\Bigg[\|x_1\|^2+NH\|\varphi\|^2_{\mathcal{B}}+2\tilde{N}[\|\psi\|^2+\|g(0,\varphi)\|^2]\\
&\quad+N\int_0^b\|g(s,(\bar{x}^r)_s)-g(s,y_s)\|^2ds+N\int_0^b\|g(s,y_s)\|^2ds\\
&\quad+\tilde{N}Tr(Q)\int_0^b p(s)\Omega(\|\overline{x^r}_{\rho(s,\bar{x}^r_s)}\|^2_{\mathcal{B}})ds+\sum_{k=1}^m N\Phi_k\|(\bar{x}_{t_k}\|^2)+\sum_{k=1}^m\tilde{N}\Gamma\|(\bar{x}_{t_k}\|^2)\Bigg]d\eta\\
&\quad+49\sum_{k=1}^m N\Phi_k(\|\bar{x}_{t_k}\|^2_{\mathcal{B}})+49\sum_{k=1}^m\tilde{N}\Gamma_k\|(\bar{x}_{t_k}\|^2_{\mathcal{B}}),
\end{aligned}
$$

Since Φ_k and Γ_k are non-decreasing operators, we have

$$
\begin{aligned}
&\le 49NH\|\varphi\|^2_{\mathcal{B}}+98\tilde{N}[\|\psi\|^2+\|g(0,\varphi)\|^2]+49NM_gK_b\int_0^{t^r}\|\bar{x}^r-y\|^2_s ds\\
&\quad+49N\int_0^{t^r}(\theta_1\|y_s\|^2_{\mathcal{B}}+\theta_2)ds+49\tilde{N}Tr(Q)\Omega((M_b+J_0^{\varphi})\|\varphi\|^2_{\mathcal{B}}+K_br\\
&\quad+K_b\|y\|^2_b)\int_0^b p(s)ds+49^2b^2\tilde{N}M_1M_2\Big[\|x_1\|^2+NH\|\varphi\|^2_{\mathcal{B}}+2\tilde{N}[\|\psi\|^2+g(0,\varphi)\|^2]\\
&\quad+NM_gK_b\int_0^b\|\bar{x}^r-y\|^2_s ds+N\int_0^b(\theta_1\|y_s\|^2_{\mathcal{B}}+\theta_2)ds\\
&\quad+\tilde{N}Tr(Q)\Omega((M_b+J_0^{\varphi})\|\varphi\|^2_{\mathcal{B}}+K_br+K_b\|y\|^2_b)\int_0^b p(s)ds
\end{aligned}
$$

$$+\sum_{k=1}^{m} N\Phi_k(r^*) + \sum_{k=1}^{m} \tilde{N}\Gamma_k(r^*)\Bigg] + 49\sum_{k=1}^{m} N\Phi_k(r^*) + 49\sum_{k=1}^{m} \tilde{N}\Gamma_k(r^*),$$

where $\|\bar{x}_{t_k}\|^2 \le r^* = (M_b + J_0^{\varphi})\|\varphi\|_{\mathcal{B}}^2 + K_b(r + \|y\|_b^2)$ and hence

$$1 \le \Big(49 + 49^2 b^2 \tilde{N} M_1 M_2\Big)\Bigg[K_b\Bigg(b^2 N M_g + \tilde{N} Tr(Q) \liminf_{\xi\to\infty} \frac{\Omega(\xi)}{\xi}\int_0^b p(s)ds + \sum_{k=1}^{m}(N\zeta_k + \tilde{N}\sigma_k\Bigg)\Bigg],$$

which is the contrary to the our assumption.

Proceeding as in the proof of Theorem 4.2, we can conclude that $\Psi(\cdot)$ is a condensing map on $B_r(y_{|_J}, Y)$ and from Lemma 2.1, we conclude that there exists a mild solution $x(\cdot)$ from (4.1)-(4.4). This complete the proof.

Corollary 4.1. If all conditions of Theorem 4.2 hold except that (H2)(iii)replaced by

(C1) : there exists an integrable function $p : J \to [0, +\infty)$ and a constant $\tau \in [0, 1)$ such that

$$\|f(t, \psi)\|^2 \le p(t)(1 + \|\psi\|_{\mathcal{B}}^{\tau}), \quad \text{for each}(t, \psi) \in J \times \mathcal{B},$$

then the problem (4.1)-(4.4) admits at least one mild solution on $(-\infty, b]$ provided that

$$(49 + 49^2 b^2 \tilde{N} M_1 M_2)\Bigg[K_b\Bigg(\sum_{k=1}^{m} N M_{I_k} + \tilde{N} M_{J_k}\Bigg) + b^2 N M_g\Bigg] < 1.$$

Corollary 4.2. If all the conditions of Theorem 4.3 hold except that (H4) replaced by the following one,

(C2) : there exists positive constants $c_k, d_k, e_k, l_k, k = 1, 2, \ldots, m$, and constants $\mu, \upsilon \in [0, 1)$ such that for each $\Psi \in \mathcal{B}$,

$$\|I_k(\Psi)\|^2 \le c_k + d_k(\|\Psi\|_{\mathcal{B}})^{\mu}, \quad k = 1, 2, \ldots, m,$$

and

$$\|J_k(\Psi)\|^2 \le e_k + l_k(\|\Psi\|_{\mathcal{B}})^{\upsilon}, \quad k = 1, 2, \ldots, m,$$

then the problem (4.1)-(4.4) has at least one mild solution on $(-\infty, b]$ provided that

$$(49 + 49^2 b^2 \tilde{N} M_1 M_2)\Bigg[K_b\Bigg(\tilde{N} Tr(Q) \liminf_{\xi\to\infty} \frac{\Omega(\xi)}{\xi}\int_0^b p(s)ds + b^2 N M_g\Bigg)\Bigg] < 1.$$

Corollary 4.3. If all conditions of Theorem 4.3 hold except that (H2)(iii) and (H4) replaced by (C1) and (C2), then the problem has at least one mild solution on $(-\infty, b]$ provided that

$$K_b b^2 N M_g < 1.$$

5. Example

Example 1. Consider the following impulsive stochastic partial differential equation with state-dependent delay of the form

$$d\Big[\frac{\partial}{\partial t}w(t,y)\Big] = \Big[\frac{\partial^2}{\partial y^2}w(t,y) + z(y)u(t)\Big]dt$$
$$+\int_{-\infty}^{t} b(s-t)w(s-\rho_1(t)\rho_2(\|w(t)\|),y)d\beta(s), \qquad (5.1)$$
$$w(t,0 = w(t,\pi) = 0, \quad t \in J = [0,b], \qquad (5.2)$$
$$w(\tau,y) = \varphi(\tau,y), \quad \tau \in (-\infty,0], \quad y \in [0,\pi], \qquad (5.3)$$
$$\Delta w(t_k,y) = \int_{-\infty}^{t_k} a_k(t_k-s)w(s,y)ds, \quad y \in [0,\pi], \quad k = 1,2,\dots,m, \qquad (5.4)$$
$$\Delta w'(t_k,y) = \int_{-\infty}^{t_k} \tilde{a_k}(t_k-s)w(s,y)ds, \quad y \in [0,\pi], \quad k = 1,2,\dots,m, \qquad (5.5)$$

where the space $H = L^2([0,\pi])$, $\varphi \in \mathcal{B} = \mathcal{PC}_0 \times L^2(g,H)$ and $0 < t_1 < \cdots < t_m < b$ are prefixed numbers, then $\rho_i : [0,\infty) \to (0,\infty]$ is continuous, and $\beta(s)$ is a one-dimensional standard Wiener process. Define $A : H \to H$ by $Az = z''$ with domain $D(A) = \{z(\cdot) \in H : z,' \text{, are absolutely continuous, } z'' \in H, z(0) = z(\pi) = 0\}$.
The spectrum of A consists of the eigenvalues $-n^2$ for $n \in N$, with associated eigen vectors $e_n(y) = (\frac{2}{\pi})^{\frac{1}{2}} sin(ny)$. Futhermore, the set $\{e_n : n \in N\}$ is an orthonormal basis of H. In particular,

$$Az = \sum_{n=1}^{\infty} n^2 (z,e_n), \quad z \in D(A).$$

Moreover, the operator $C(t)$ defined by

$$C(t)x = \sum_{n=1}^{\infty} cos(nt) < z, e_n > e_n, \quad t \in R,$$

form a cosine function on H, with associated sine function

$$S(t)x = \sum_{n=1}^{\infty} \frac{sin(nt)}{n} < z, e_n > e_n, \quad t \in R.$$

From Ref. [43], for all $x \in H$, $t \in R$, $\|S(t)\| \le 1$ and $\|C(t)\| \le 1$. Let $\alpha < 0$, define the phase space

$$\mathcal{B} = \Big\{\phi \in C((-\infty,0],H) : \lim_{s\to-\infty} e^{\alpha s}\phi(s) \quad \text{exist in} \quad H\Big\},$$

and let $\|\phi\|_{\mathcal{B}} = \sup_{-\infty<s<0}\{e^{\alpha s}\ \|\phi(s)\|_{L_2}\}$. Then, $(\mathcal{B}, \|\phi\|_{\mathcal{B}})$ is a Banach space which satisfies the Axioms from (i)-(iii) with $L = 1, K_b = \max\{1, e^{-\alpha t}\}, M_b = e^{-\alpha t}$. Hence for $(t,\phi) \in [0,b] \times \mathcal{B}$, where $\phi(s)(y) = \phi(\theta,y), (s,y) \in (-\infty,0] \times [0,\pi]$, let $z(t)(y) = z(t,y)$.

To study the above systems, we impose the following conditions hold:

(i) The function $b : R \to R, \rho_i : [0,\infty) \to (0,\infty], i = 1,2$ are continuous, bounded

$$M_f = \Big(\int_{-\infty}^{0} \frac{(b^2(s))}{g(s)} ds\Big)^{\frac{1}{2}} < \infty.$$

(ii) The function $a_k : R \to R$ are continuous such that

$$M_{I_k} = \left(\int_{-\infty}^{0} \frac{(a_k^2(s))}{g(s)} ds \right)^{\frac{1}{2}}, \quad k = 1, 2, \ldots, m,$$

and

$$M_{J_k} = \left(\int_{-\infty}^{0} \frac{(\tilde{a}_k^2(s))}{g(s)} ds \right)^{\frac{1}{2}}, \quad k = 1, 2, \ldots, m.$$

Assume that the bounded linear operator $B \in L(R, H)$ is defined by

$$Bu(t) = z(y)u, \quad 0 \le y \le \pi, \quad u \in R, \quad z(y) \in L^2([0, \pi]).$$

By defining the operator $\rho, f : J \times \mathcal{B} \to H$ and $I_k, J_k : \mathcal{B} \to H$ by

$$\begin{aligned} f(t, \phi)(y) &= \int_{-\infty}^{0} b(s)\phi(s, y)ds, \\ \rho(t, \phi) &= s - \rho_1(s)\rho_2(\|\phi(0)\|), \\ I_k(\phi)(y) &= \int_{-\infty}^{0} a_k(-s)\phi(s, y)ds, \quad k = 1, 2, \ldots, m, \\ J_k(\phi)(y) &= \int_{-\infty}^{0} \tilde{a}_k(-s)\phi(s, y)ds, \quad k = 1, 2, \ldots, m, \end{aligned}$$

we can transform the systems (5.1)-(5.5) into the abstract impulsive Cauchy problem (1.1)-(1.4). Now the linear operator W is given by

$$Wu = \sum_{n=1}^{\infty} \int_0^{\pi} \frac{1}{n} sinns(\mu(s, y), e_n)e_n ds, \quad y \in [0, \pi].$$

Assume that this operator has a bounded inverse W^{-1} in $L^2(J, U)$. Moreover the function $F, I_k, k = 1, 2, \ldots, m$ are bounded linear operators with $\|f(t, \cdot)\|^2_{\mathcal{L}(\mathcal{B},\mathcal{H})} \le M_f$, $\|I_k\|^2_{\mathcal{L}(\mathcal{B},\mathcal{H})} \le M_{I_k}$, $\|J_k\|^2_{\mathcal{L}(\mathcal{B},\mathcal{H})} \le M_{J_k}$. Hence all the conditions of Theorem 3.1 have been satisfied for the system (5.1)-(5.5), and so system is controllable on J_1.

Example 2. Consider the following impulsive neutral stochastic partial differential equation with state-dependent delay of the form

$$\begin{aligned} d\left[\frac{\partial}{\partial t} w(t, y) + \int_{-\infty}^{t} \int_0^{\pi} a(t - s, \eta, y)w(s, \eta)d\eta ds\right] &= \left[\frac{\partial^2}{\partial y^2} w(t, y)\right. \\ &\left. + z(y)u(t)\right]dt + \int_{-\infty}^{t} b(s - t)w(s - \rho_1(t)\rho_2(\|w(t)\|), y)d\beta(s), \qquad (5.6) \end{aligned}$$

$$w(t, 0) = w(t, \pi) = 0, \quad t \in J, \qquad (5.7)$$

$$w(\tau, y) = \varphi(\tau, y), \quad \tau \in (-\infty, 0], \quad y \in [0, \pi], \qquad (5.8)$$

$$\Delta w(t_k, y) = \int_{-\infty}^{t_k} a_k(t_k - s)w(s, y)ds, \quad y \in [0, \pi], \ k = 1, 2, \ldots, m, \qquad (5.9)$$

$$\Delta w'(t_k, y) = \int_{-\infty}^{t_k} \tilde{a}_k(t_k - s)w(s, y)ds, \quad y \in [0, \pi], \ k = 1, 2, \ldots, m, \qquad (5.10)$$

where $\varphi, B, b, \rho_i, i = 1, 2$ and M_f are defined in Example 1. Assume that the conditions (ii)of the previous example holds and

(ii) The function $a(s,\eta,y)$, $\frac{\partial a(s,\eta,y)}{\partial y}$ are continuous and measurable, $a(s,\eta,\pi) = a(s,\eta,0) = 0$ and

$$M_g = \max\left[\left(\int_0^\pi \int_{-\infty}^0 \int_0^\pi \frac{1}{g(s)}\left(\frac{\partial^j a(s,\eta,y)}{\partial y^j}\right) d\eta ds dy\right)^{\frac{1}{2}} : j = 0,1\right] < \infty.$$

Define the function A, B, f, ρ, I_k, J_k and W as in Example 1 and the operator $g : J \times \mathcal{B} \to H$ by

$$g(\phi)(y) = \int_{-\infty}^0 \int_0^\pi a(s,\upsilon,y)\phi(s,\upsilon)d\upsilon ds,$$

we can transform the systems (5.6)-(5.10) into the abstract Cauchy problem (4.1)-(4.4). Moreover, the function g is a bonded linear operator with $\|g(t,\cdot)\|_{\mathcal{L}(\mathcal{B},\mathcal{H})} \leq M_g$. Hence all the conditions of Theorem 4.3 have been satisfied for the system (5.6)-(5.10), and so system is controllable on J_1.

6. Conclusion

In this paper, we discussed controllability results for the second order impulsive stochastic differential and neutral differential systems with state-dependent delay by using phase space axioms. Through the theory of strongly continuous cosine families of operators and the Sadovskii's fixed point theorem can be successfully used in the control problems to obtain sufficient conditions. Finally, an example is illustrated for the effectiveness of the controllability results.

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