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ON APPROXIMATE SOLUTION OF HIGH-ORDER LINEAR FREDHOLM INTEGRO-DIFFERENTIAL-DIFFERENCE EQUATIONS WITH VARIABLE COEFFICIENTS USING LEGENDRE COLLOCATION METHOD

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Abstract. In this paper, we applied method based on Legendre polynomial collocation method to obtain numerical solutions to linear differential difference equations with mixed conditions. The given problem is converted to a system of algebraic equations, which is then solved using matrix inversion method. Numerical examples are presented to test the efficiency and accuracy of the method.

1. Introduction

Differential difference equations are differential equations in which the derivatives of the unknown function at a certain time is given in terms of the values of the function at a previous time [2]. It can also be described as an equations containing shifts of the unknown function and its derivatives [11]. Integro-differential-difference equations have applications in elasticity, heat and mass transfer, biomechanics, games theory, queuing thoery and other fields as highlighted [10, 18, 19, 20, 21, 22]. The solution of integro-differential equations based on polynomial methods such as Taylor-polynomial approach [4], Hybrid Euler-Taylor matrix method [3], Legendre polynomials[6, 7, 12, 13] are all operational matrix approach. Also, a semi-analytic method based on differential transform method as elucidated in [17] gives a reliable approximate solution of integro-differential difference equations.

In this study, we modified and extended the new collocation method developed in [1, 8] for the solution of linear integro-differential equations with initial conditions to linear Fredholm integro-differential-difference equations with mixed condition in the form

$$\sum_{k=0}^{K} P_k(t)\, u^{(k)}(t) = \sum_{r=0}^{R} R_r(t)\, u^{(r)}(t-\tau) + g(t) + \int_a^b k(t,s)\, u(s-\tau)\, ds \qquad (1)$$

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with the mixed condition

$$\sum_{k=0}^{K-1}\left[\alpha_{ik}u^{(k)}(a)+\rho_{ik}u^{(k)}(b)+\theta_{ik}u^{(k)}(c)\right]=\lambda_i \tag{2}$$

where $P_k(t)$, $R_r(t)$,and $g(t)$ are analytic function defined on $a \le x \le b$. $k(t,s)$, the kernel of integrations defined on $a \le t, s \le b$ is continuous. The coefficients α_{ik}, ρ_{ik}, θ_{ik}, ξ_r, τ_r and λ_i are real, $R \le P$, $u(t)$ is the solution to (1) and (2) to be determined
The aim of the study is to solve (1) and (2) using approximate solution in the form

$$u_N(t)=\sum_{n=0}^{N} L_n(t)\,\beta_n,\ \ N\in\mathbb{R}^+ \tag{3}$$

L_n is the Legendre interpolating polynomial defined within the interval $[-1,1]$ given as

$$L_n(t)=\sum_{m=0}^{M} K(n,m)\,t^{n-2m} \tag{4}$$

where $K(n,m)=\dfrac{(-1)^m(2n-2m)!}{2^n m!\,(n-m)!\,(n-2m)!}$
The recursive formula is given as

$$L_{n+1}(t)=\frac{(2n+1)\,tL_n(t)-nL_{n-1}(t)}{n+1},\ \ n=1,2,\cdots,N \tag{5}$$

$L_0(t)=1$, $L_1(t)=t$, and β_n are constants to be determined.

2. Method of Solution

In this section, we discuss the algorithms involved in development of the new collocation method. The solution to difference equation can be expressed in the form

$$u_N(t-\tau)=\sum_{n=0}^{N} L_n(t-\tau)\,\beta_n, \tag{6}$$

therefore

$$u_N(t-\tau)=\sum_{n=0}^{N}\sum_{m=0}^{M} K(n,m)\,(t-\tau)^{n-2m}\,\beta_n, \tag{7}$$

Using Binomial theorem, we have

$$u_N(t-\tau)=\sum_{n=0}^{N}\sum_{m=0}^{M}\sum_{i=0}^{n-2m} K(n,m)\,\frac{(n-2m)!}{i!\,(n-2m-1)!}t^{n-2m-i}\tau^i\beta_n. \tag{8}$$

Substituting (4) and (8) into (1), we obtain

$$\begin{aligned}\sum_{n=0}^{N}\sum_{k=0}^{K}\sum_{m=0}^{M} P_k(t)\,K(n,m)\,\frac{d^k}{dt^k}t^{n-2m}\beta_n \quad = \quad & \sum_{n=0}^{N}\sum_{m=0}^{M}\sum_{i=0}^{n-2m}\sum_{r=0}^{R} R_r(t)\,\frac{Z(n,m)}{i!}\frac{d^r}{dt^r}t^{n-2m-i}\tau^i\beta_n \\ & +g(t)+\sum_{n=0}^{N}\sum_{m=0}^{M}\sum_{i=0}^{n-2m}\frac{Z(n,m)}{i!}\tau^i\int_a^b k(t,s)\,s^{n-2m-1}ds\beta_n,\end{aligned} \tag{9}$$

where $Z(n,m)=\dfrac{K(n,m)\,(n-2m)!}{(n-2m-1)!}$

Using $\frac{d^k}{dt^k}t^{n-2m} = \dfrac{(n-2m)!}{(n-2m-k)!}t^{n-2m-k}$ and $\frac{d^r}{dt^r}t^{n-2m-i} = \dfrac{(n-2m-i)!}{(n-2m-i-r)!}t^{n-2m-i-r}$, in (9) and collocating at $t_f = -\cos\left(\frac{\pi i}{N}\right)$, we obtained

$$\sum_{n=0}^{N} W_n(t_f)\,\beta_n = g(t_f), \tag{10}$$

where

$$\begin{aligned} W_n(t_f) \quad &= \quad \sum_{k=0}^{K}\sum_{m=0}^{M} P_k(t)\,Z(n,m)\,t_f^{n-2m-k} - \sum_{r=0}^{R}\sum_{m=0}^{M}\sum_{i=0}^{n-2m} R_r(t)\,B(n,m,i)\,t_f^{n-2m-r-1} \\ &\quad -\sum_{m=0}^{M}\sum_{i=0}^{n-2n} C(n,m,i)\int_a^b k(t,s)\,s^{n-2m-i}ds \end{aligned}$$

$$,\ B(n,m,i) = \frac{K(n,m)(n-2m)!}{(n-2m-r-1)!i!}\tau^i, C(n,m,i) = \frac{K(n,m)(n-2m)!}{(n-2m-i)!i!}\tau^i$$

$$C(n,m,i) = \frac{K(n,m)(n-2m)!}{(n-2m-i)!i!}\tau^i$$

$W_n(t_f)$ is of $[(N+1)\times(N+1)]$, $g(t_f)$ is $[(N+1)\times 1]$, and β_n is $[(N+1)\times 1]$ dimensions respectively.

On substituting (4) into (2), we obtained

$$Z(n,m,j)\,\beta_n = \lambda_i, \tag{11}$$

where

$$Z(n,m,j) = \sum_{k=0}^{K-1}\sum_{n=0}^{N}\sum_{m=0}^{M}\sum_{j=0}^{k} A(n,m,j)\left[\alpha_{i,k}a^{n-2m-j} + \rho_{i,j}b^{n-2m-j} + \theta_{i,j}c^{n-2m-j}\right]$$

$Z(n,m,j)$ is of $[1\times(N+1)]$, λ_i is $[1\times(N+1)]$ dimesnsion.

We replace the rows in (10) by (11), and then solve for the unknowns β_n.

3. Numerical Example

In this section, numerical examples are used to illustrate the new concept, efficiency, accuracy and simplicity of the new method. Let $U_N(t)$ and $U(t)$ be the approximate and numerical solution respectively, then $abs - e_N = |U_N(t) - U(t)|$ is the absolute error of N. All numerical solution are given in a tablular form except where the absolute error=0. All computations in this section are done with the aid of program written using MATLAB (2015a) and implemented using a PC.

Example 1 We consider linear third order Fredholm integro-differential difference equation with variable coefficient

$$\begin{aligned} u'''(t) - tu'(t) + u''(t-1) - tu(t-1) \quad &= \quad -(t+1)(\sin(t-1) + \cos t) \\ &\quad -\cos(2) + 1 + \int_{-1}^{1} u(y-1)\,dy \end{aligned} \tag{12}$$

with the condition
$u(0) = 0$, $u'(0) = 1$, $u''(0) = 0$,
comparing equation (12) with (1) then, we have
$P_3(t) = 1$, $R_2(t) = 1$, $P_1(t) = -t$, $R_0(t) = -t$, $g(t) = -(t+1)(\sin(t-1) + \cos(t)) -$

$\cos(2)+1,\ a=-1,\ b=1,\ k(t,y)=1.$
We solve this problem using $N=6,7,8$ and 9, the numerical solutions give

$$\begin{aligned} u_6(t) &= -\left(9.9377\times10^{-4}\right)t^6+\left(7.9621\times10^{-3}\right)t^5+\left(1.5091\times10^{-2}\right)t^4-0.18446t^3 \\ &\quad+\left(3.2102\times10^{-4}\right)t^2+0.99791t+1.4469\times10^{-3} \end{aligned}$$

$$\begin{aligned} u_7(t) &= -\left(1.4898\times10^{-4}\right)t^7-\left(1.5546\times10^{-4}\right)t^6+\left(8.2349\times10^{-3}\right)t^5 \\ &\quad+\left(2.285\times10^{-3}\right)t^4-0.16949t^3+4.6758\times10^{-8}t^2+t+1.653\times10^{-7} \end{aligned}$$

$$\begin{aligned} u_8(t) &= \left(1.5621\times10^{-6}\right)t^8-\left(1.4060\times10^{-4}\right)t^7-\left(1.9153\times10^{-4}\right)t^6 \\ &\quad+\left(8.2164\times10^{-3}\right)t^5+\left(2.7346\times10^{-3}\right)t^4-0.17004t^3 \\ &\quad-4.2902\times10^{-8}t^2+t-1.0479\times10^{-7} \end{aligned}$$

The numerical solutions and the exact solution are given in Table 1, we compare the absolute error of the new method with operational matrix method using Taylor and Legendre collocation respectively, the numerical solution clearly shows that the proposed method is accurate with good stability.

Table 1
Numerical Results for (12)

	exact	present method		
t_i	$\sin(t_i)$	$N=6$	$N=7$	$N=8$
-1	-0.84147098480	-0.80554831119	-0.83646306330	-0.83549471873
-0.6	-0.56464247339	-0.55604956476	-0.56373595111	-0.56356174242
-0.2	-0.19866933079	-0.19662476781	-0.19864254633	-0.19863809867
0.2	0.198669330795	0.199592347971	0.198650172931	0.198646612021
0.6	0.564642473395	0.562993309139	0.564314093477	0.564252502366
1.0	0.841470984807	0.837278684053	0.840722665699	0.840583781683

Table 2
Comparison of absolute error for (12)

	Taylor [7]		Legendre [11]		Fibonacci [9]	Present method	
t_i	abs-e_6	abs-e_7	abs-e_6	abs-e_7	abs-e_8	abs-e_6	abs-e_7
-1	8.58e-02	6.03e-02	3.83e-02	5.05e-03	2.72e-01	3.5923e-02	5.0079e-03
-0.6	1.50e-02	6.63e-03	7.00e-03	9.14e-04	6.90e-02	8.5929e-03	9.0652e-04
-0.2	4.85e-04	6.90e-05	2.04e-04	2.65e-05	2.82e-03	2.0446e-03	2.6784e-05
0.2	4.59e-04	5.30e-05	1.48e-04	1.91e-05	2.98e-03	9.2302e-04	1.9158e-05
0.6	1.28e-02	3.82e-03	2.55e-03	3.30e-04	8.30e-02	1.6492e-03	3.2838e-04
1.0	6.57e-02	2.73e-02	5.76e-03	7.53e-04	3.39e-01	4.1923e-03	7.4832e-04

Example 2, [14, 5] We consider third order mixed linear integro-differential difference equation

$$u'''(t)=u'(t)-2t(\cos(1)-\sin(1))-2\cos t+\int_{-1}^{1}tyu(y)\,dy \tag{13}$$

with the initial condition
$u(0)=0,\ u'(0)=1,\ u''(0)-2u'(0)=-2.$
Comparing equation (13) with (1), then, we have
$P_3(t)=1,\ P_1(t)=1,\ k(t,y)=ty,\ g(t)=2t(\cos(1)-\sin(1))-2\cos t.$

We solve this problem using N=6, and 7, the approximate solution are

$$\begin{aligned} U_6(t) &= \left(641\,1\times 10^{-7}\right)t^6+\left(7.637\,2\times 10^{-3}\right)t^5+\left(2.143\,8\times 10^{-5}\right)t^4-0.165\,70t^3 \\ &\quad +\left(1.065\,0\times 10^{-4}\right)t^2+1.000\,1t \end{aligned}$$

$$\begin{aligned} U_7(t) &= -\left(1.884\,6\times 10^{-4}\right)t^7+\left(8.320\,1\times 10^{-3}\right)t^5+\left(4.080\,4\times 10^{-8}\right)t^4-0.166\,66t^3 \\ &\quad -\left(7.865\,6\times 10^{-9}\right)t^2+t+1.058\,8\times 10^{-22} \end{aligned}$$

Comparison of the exact solutions and absolute error of the new method with the operational matrix using Chebyshev and Boubaker collocation methods respectively are given. Numerical solution shows that the new method has the best accuracy and stability as shown in Table 3

Table 3
Comparison of exact solution with numerical solution for (13)

	exact	Chebyshev [14]	Boubakar [5]		present method			
t_i	$\sin(t_i)$	abs-e_8	abs-e_6	abs-e_7	$N=6$	abs-e_6	$N=7$	abs-e_7
-0.2	-0.198669330	1.00e-8	0.00e-0	1.00e-6	-0.19869386	2.453e-5	-0.19866939	5.597e-8
-0.4	-0.389418342	2.00e-6	1.00e-6	1.00e-6	-0.38949852	8.017e-5	-0.38941871	3.683e-7
-0.6	-0.564642473	1.54e-5	1.00e-6	2.00e-6	-0.56482577	1.833e-4	-0.56464334	8.725e-7
-0.8	-0.717356090	4.96e-5	1.00e-6	3.00e-6	-0.71767307	3.169e-4	-0.71735738	1.294e-6
-1.0	-0.841470984	1.09e-4	2.00e-6	1.40e-5	-0.84191668	4.457e-4	-0.84147260	1.619e-6

Example 3, [9, 5] We consider the third order linear Fredholm integro-differential difference equation

$$\begin{aligned} u'''(t)-(t-1)u''(t)+(t-1)u'(t)+u'(t-1) &= e^{(t-1)}+t\left(et-\frac{1}{e}t-2\frac{1}{e}\right) \quad (14) \\ &\quad +\int_{-1}^{1}\left(ty-t^2\right)u(y)\,dy \end{aligned}$$

with the initial condition
$u(0)=u'(0)=u''(0)=1.$
Comparing equation (14) with (1) then, we have
$P_3(t)=1,\ P_2(t)=-(t-1),\ P_1(t)=(t-1),\ P_0(t)=-1,\ R_1(t)=1,\ g(t)=e^{(t-1)}+t\left(et-\frac{1}{e}t-2\frac{1}{e}\right), k(t,y)=\left(ty-t^2\right),\ a=-1,\ b=1.$
We solve this problem using N=6, 8, 9, and 10, the numerical solutions are given as

$$\begin{aligned} u_6(t) &= \left(1.335\,7\times 10^{-3}\right)t^6+\left(9.154\,7\times 10^{-3}\right)t^5+\left(4.197\,7\times 10^{-2}\right)t^4+0.165\,62t^3 \\ &\quad +0.500\,49t^2+1.000\,2t+1.001\,3 \end{aligned}$$

$$\begin{aligned} u_9(t) &= \left(2.585\,6\times 10^{-6}\right)t^9+\left(2.618\,3\times 10^{-5}\right)t^8+\left(1.989\,9\times 10^{-4}\right)t^7+\left(1.386\,4\times 10^{-3}\right)t^6 \\ &\quad +\left(8.333\,1\times 10^{-3}\right)t^5+\left(4.166\,8\times 10^{-2}\right)t^4+0.166\,67t^3+0.5t^2+t+1 \end{aligned}$$

$$\begin{aligned} u_{10}(t) &= \left(2.574\,7\times 10^{-7}\right)t^{10}+\left(2.891\,3\times 10^{-6}\right)t^9+\left(2.486\,2\times 10^{-5}\right)t^8 \\ &\quad +\left(1.981\,4\times 10^{-4}\right)t^7+\left(1.388\,9\times 10^{-3}\right)t^6+\left(8.333\,5\times 10^{-3}\right)t^5 \\ &\quad +\left(4.166\,7\times 10^{-2}\right)t^4+0.166\,67t^3+0.5t^2+t+1 \end{aligned}$$

Table 4 shows the comparison of the exact solution, numerical solution and absolute errors of the new method and the methods developed by Fibonacci collocation method [[6]] using operational matrix method. Results show that the new method has a better accuracy

Table 4
Comparison of exact solution, numerical solution and erors for (14)

	exact	Fibonacci [9]	present method					
t_i	$\exp(t_i)$	abs-e_9	N=6	abs-e_6	N=9	abs-e_9	N=10	abs-e_{10}
-0.2	0.818730753	7.845e-9	0.819980	1.249e-3	0.8187307	2.321e-9	0.8187307	1.768e-10
-0.4	0.670320046	8.495e-8	0.671654	1.337e-3	0.6703200	3.986e-8	0.6703200	5.273e-10
-0.6	0.548811636	3.612e-7	0.550350	1.539e-3	0.5488117	1.469e-7	0.5488116	7.146e-09
-0.8	0.449328964	4.540e-7	0.451185	1.856e-3	0.4493292	2.732e-7	0.4493289	2.016e-08
-1.0	0.367879441	9.388e-7	0.370119	2.240e-3	0.3678797	2.696e-7	0.3678794	1.952e-08

Example 4, [2] We consider the linear second order Fredholm integro-differential equation with variable coefficient

$$2u''(t) + 2u'(t) - 4u(t) + u''(t-1) + u'(t-1) - 2u(t-1) = -6t^2 + 10t + 8 \quad (15)$$

with the initial condition
$u(0) = 1,\ u'(0) = 2.$
comparing equation(15) with (1) then, we have
$P_2(t) = 2,\ P_1(t) = 2,\ R_2(t) = 1,\ R_1(t) = 1,\ R_0(t) = -2,\ g(t) = -6t^2 + 10t + 8,$
we solve this problem using N=6, and 8. The approximate solutions are

$$\begin{aligned} u_6(t) &= \left(2.8204 \times 10^{-3}\right) t^6 + \left(1.8062 \times 10^{-2}\right) t^5 + \left(8.4676 \times 10^{-2}\right) t^4 \\ &\quad + 0.32792 t^3 + 2.0034 t^2 + 1.9988 t + 1.0012 \end{aligned}$$

$$\begin{aligned} u_8(t) &= \left(3.8917 \times 10^{-5}\right) t^8 + \left(4.3703 \times 10^{-4}\right) t^7 + \left(2.8284 \times 10^{-3}\right) t^6 \\ &\quad + \left(1.6522 \times 10^{-2}\right) t^5 + \left(8.3216 \times 10^{-2}\right) t^4 + 0.33377 t^3 + 1.9998 t^2 + 2t + 1 \end{aligned}$$

Table 5
Comparison of exact solution, numerical solutions and absolute errors for (15)

	exact	Boubakar [2]		present method			
t_i	$2e^t + t^2 - 1$	abs-e_6	abs-e_8	N=6	abs-e_6	N=8	abs-e_8
-0.2	0.6774615	6.11e-04	2.94e-05	0.6790868	1.625e-03	0.6774517	9.752e-06
-0.4	0.5006400	2.48e-03	1.11e-04	0.5032382	2.598e-03	0.5005848	5.526e-05
-0.6	0.4576232	5.76e-03	2.16e-04	0.4620240	4.400e-03	0.4574673	1.559e-04
-0.8	0.5386579	1.04e-02	2.98e-04	0.5459578	7.300e-03	0.5383330	3.248e-04
-1.0	0.7357588	1.64e-02	3.12e-04	0.7473305	1.157e-02	0.7351917	5.671e-04

Comparison of the exact solution, numerical solution and absolute errors of the new method with operational matrix method using Boubakar collocation method, results show that the new method gives a better accuracy as shown in Table 5.

4. Conclusion

We have developed and presented a new numerical method for the solution of linear Fredholm integro differential difference equation with mixed condition in this paper. The method adopted in this study has a lesser computational burden and faster time of convergence when compared with existing methods. Numerical solutions show that as N increases, the accuracy increases. Moreover, the results has good stability as shown in the tables.

Conflict of Interest

The authors declare that there are no conflicts of interest regarding the publication of this paper

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References

[1] O. A. Agbolade,T. A. Anake, Solution of first order Volterra type linear differential equations by collocation method, *J. Appl. Math.*, Article ID. 1510267, doi:10.1155/2017/1510267

[2] T. Akkaya, S. Yalcinbas, Boubaker polynomial approach for solving high order linear differential difference equations, *AIP Conf. Proc.***1493**(26), (2012), doi:10.1063/1.4765464.

[3] M. A. Balci, M. Sezer, Hybrid Euler-Taylor matrix method for solving the generalized linear Fredholm integro-differential difference equations, *Appl. Math. Comput.*, **273**(2016), 33-41, doi:10.1016/j.amc.2015.09.085

[4] S. Davaeifar, J. Rashidina, M. Amirfakhrian, Berstein polynomial approach for solution of higher order mixed linear Fredholm integro differential difference equations with variable coefficients, *WMS J. Pure Appl. Math,* **7**(1), (2016), 46-62.

[5] M. Gulsu, Y. Ozturk, M. Sezer, A new collocation method for solution of mixed linear integro-differntial difference equations, *Appl. Math. Comput.*, **216**, (2010), 2183-2198,doi:10.1016/j.amc.2010.03.054

[6] M. Gulsu, M. Sezer, A Taylor polynomial approach for solving differential difference equations, *J. Comput. Appl. Math.*, **186**, (2006), 349-364.

[7] M. Gulsu, M. Sezer, Approximations of the solution of linear Fredholm integro differential difference of high order, *J. Franklin Inst.*, **343**, (2006), 720-737.

[8] Y. Jafarzadeh, B. Keramati, Numerical method for a system of integro-differential equations and convergence analysis by Taylor collocation, *Ain Shams Engineering Journal*, **9**, (2018), 1433-1438, doi:10.1016/j.asej.2016.08.014

[9] A. Kurt, S. Yalcinbas, M. Sezer, Fibonacci collocation method for solving high order linear Fredholm integro differential difference equations, *Int. J. Math. Math. Sci.*, (2013), Article ID 486013, doi:10.1155/2013/486013.

[10] P. Rai, K. K. Sharma, Numerical study of singularly perturbed differential difference equations arising in the modeling of neuronal variability, *Comput. Math. Appl.*, **63**(1), (2012), 118-132.

[11] A. Saadatmandi, M. Dehghan, Numerical solution of the higher order linear Fredholm integro differential difference equations with variable coefficients, *Comput. Math. Appl.* **59**,(2010), 2996-3004. doi:10.1016/j.camwa.2010.02.018.

[12] M. Sezer, M. Gulsu, Polynomial solution of the most general linear Fredholm Volterra integro differential difference equations by means of Taylor collocation method, *Appl. Math. Comput*, **185**, (2007), 646-657.

[13] M. Sezer, M. Gulsu, A new polynomial approach for solving difference and Fredholm integro differential equations with mixed augument, *Appl. Math. Comput*, **171**, (2005), 332-344.

[14] S. Yalcinbas, T. Akkays, A numerical approach for solving integr-differential-difference equations with Boubakar polynomial bases, *Ain Shams Engineering Journal*, **3**, 2012, 253-161, doi:10.1016/j.asej.2012.02.004

[15] S. Yulcinbas, M. Sezer, H. H. Sorkun, Legendre polynomial solution of high-order linear Fredholm integro-differential equations, *Appl. Math. Comput.*, **210**, (2009), 334-349.

[16] S. Yuzbasi, M. Karacayir, A numerical approach for solving high order delay Volterra integro differential equations, *Int. J. Comput. Methods*, (2018), **15**(3), doi:10.1142/s0219876218500421.

[17] M.T. Yaiuz, I. Ozkol. Solutions of Integro-differential Difference Equations via Differential Transform methods. Journal of Science and Engineering CUJSE **18**(1)(2021), 33-46.

[18] R.R. Dhundel, G.L.Waghmare. Solving Partial Integro Differential Equations Using Double Laplace Transform Method. American Journal of Computational and applied Mathematics, **5**(1), (2015), 7-10. .
[19] M. Dehghan, A. Saadatmandi. Chebyshev Finite Difference Method for Fredholm integro-differential equation. Int. Comput. Math. **85**(1) (2008), 123-130.
[20] Z. Jackiewicz, M Rahman, B.D. Welfert. Numerical Solution of a Fredholm integro-differential equation Modelling Neural Networks. Appl. Numer. Math., **56** (2006) pp. 423-432.
[21] A.M. Wazwaz, Partial Differential Equations and Solitary Waves Theory, Higher education Press, Beijing and Sringer-Verlag, Berlin Heidelberg, 2009.
[22] M. Dehghan, M.F. Shakeri. Solution of Parabolic Integro-Differential Equations arising in heat conduction in material with memory via He's Variational Iteration Techniques. International Journal of Bio Medical Engineering **26**, (2010), 710-715.

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