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ON S-METRIC SPACES WITH SOME TOPOLOGICAL ASPECTS

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Abstract. The notion of a metric space is an important tool in functional analysis, nonlinear analysis and especially in topology. New generalizations of metric spaces have been introduced in recent years. For instance, S-metric and b-metric spaces are among the recent generalizations of a metric space. Fixed point theory has been intensively studied and generalized using various approaches on these new spaces. In this paper we consider the relationships among a metric, an S-metric and a b-metric. In this context, we define the topological equivalence between a metric and an S-metric. Especially, we focus on the fact that every S-metric does not always generate a metric. This is the main motivation of the recent fixed point studies for self-mappings on an S-metric space. Also we revisit the notion of a metric generated by an S-metric. We support our theoretical findings by necessary illustrative examples. As a consequence, existing studies based on the metric generated by an S-metric can be updated using a general S-metric whether generate a metric or not.

1. Introduction and Preliminaries

Let (X, d) be a metric space. Many generalizations of a metric space have been appeared in the literature, for example, a quasi-metric space, a rectangular metric space, a G-metric space, an S-metric space, a b-metric space, and so on. Fixed point theory has been intensively studied and generalized using various approaches on these generalized metric spaces (see [4], [7], [8], [9], [10] and the references therein). In this paper, mainly, we consider the relationships among a metric, an S-metric and a b-metric.

The notion of an S-metric space has been introduced as follows:

Definition 1.1. [19] Let $X \neq \emptyset$. An S-metric on X is a function such that $\mathcal{S} : X \times X \times X \to [0, \infty)$ satisfying the following conditions for all $x, y, z, a \in X$:

$(S1)$ $\mathcal{S}(x, y, z) = 0$ if and only if $x = y = z$,

$(S2)$ $\mathcal{S}(x, y, z) \leq \mathcal{S}(x, x, a) + \mathcal{S}(y, y, a) + \mathcal{S}(z, z, a)$.

Then the pair $(X, \mathcal{S})$ is called an S-metric space.

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Recently, new fixed-point results have been proved using various approaches to find the existence and uniqueness conditions for a fixed point of a self-mapping on an S-metric space (see [4], [5], [6], [13], [14], [15], [19], [20] and [21]). By a geometric viewpoint, some geometric properties of the fixed point set of a self-mapping on an S-metric space have been studied in the non unique fixed point case (see [1], [3], [11], [12], [16], [17], [18], [22] and the references therein).

Some properties and relationships between a metric and an S-metric were studied by several authors (see [5], [6], [14], [19], [20] and [21] for more details). Topological equivalence is an important issue both for studies on fixed point theory and on topology (for example, see [23] and the references therein). Hence, we present the topological equivalence between a metric and an S-metric along with the relationships among a metric, an S-metric and a b-metric.

On the other hand, we focus on the fact that there exist some examples of an S-metric which does not always generate a metric (for example, see [14] for more details). We revisit the notion of a metric generated by an S-metric. Hence, the existing studies based on the metric generated by an S-metric can be updated using a general S-metric whether generate a metric or not.

At first, we recall the following definitions and lemmas which will be needed in the sequel.

Definition 1.2. [2] Let $X \neq \emptyset$. A b-metric on X is a function $d : X \times X \to [0, \infty)$ if there exists a real number $b \geq 1$ such that the following conditions are satisfied for all $x, y, z \in X$:

$(B1)$ $d(x, y) = 0$ if and only if $x = y$,
$(B2)$ $d(x, y) = d(y, x)$,
$(B3)$ $d(x, z) \leq b\,[d(x, y) + d(y, z)]$.
Then the pair (X, d) is called a b-metric space.

For more details on the recent fixed point results on b-metric spaces see the recent survey [9] and the references therein.

Definition 1.3. [19] Let $(X, \mathcal{S})$ be an S-metric space.

(1) A sequence $\{x_n\}$ in X converges to x if and only if $\mathcal{S}(x_n, x_n, x) \to 0$ as $n \to \infty$. That is, there exists $n_0 \in \mathbb{N}$ such that for all $n \geq n_0$, $\mathcal{S}(x_n, x_n, x) < \varepsilon$ for each $\varepsilon > 0$. We denote this by $\lim\limits_{n\to\infty} x_n = x$ or $\lim\limits_{n\to\infty} \mathcal{S}(x_n, x_n, x) = 0$.
(2) A sequence $\{x_n\}$ in X is called a Cauchy sequence if $\mathcal{S}(x_n, x_n, x_m) \to 0$ as $n, m \to \infty$. That is, there exists $n_0 \in \mathbb{N}$ such that for all $n, m \geq n_0$, $\mathcal{S}(x_n, x_n, x_m) < \varepsilon$ for each $\varepsilon > 0$.
(3) The S-metric space $(X, \mathcal{S})$ is called complete if every Cauchy sequence is convergent.

Lemma 1.4. *[19] Let $(X, \mathcal{S})$ be an S-metric space. Then we have*

$$\mathcal{S}(x, x, y) = \mathcal{S}(y, y, x).$$

Lemma 1.5. *[19] Let $(X, \mathcal{S})$ be an S-metric space. If $\lim\limits_{n\to\infty} x_n = x$ and $\lim\limits_{n\to\infty} y_n = y$ then $\lim\limits_{n\to\infty} \mathcal{S}(x_n, x_n, y_n) = \mathcal{S}(x, x, y)$.*

2. Comparisons between metric and some generalized metrics

Let $(X,\mathcal{S})$ be an S-metric space. In [5], it was shown that every S-metric on X defines a metric d_S on X as follows:

$$d_S(x,y)=\mathcal{S}(x,x,y)+\mathcal{S}(y,y,x)=2\mathcal{S}(x,x,y), \tag{1}$$

for all $x,y\in X$. However, in [14] it was noticed that the function d_S defined in (1) does not always define a metric because of the reason that the triangle inequality is not satisfied for all elements of X everywhen. Also, an example of an S-metric which does not generate a metric was given (see [14] for more details). We give another examples.

Example 2.1. Let $X=\{a,b,c\}$ and the function $\mathcal{S}:X\times X\times X\to[0,\infty)$ be defined as

$$\begin{aligned}
&\mathcal{S}(a,a,c)=\mathcal{S}(c,c,a)=12,\\
&\mathcal{S}(b,b,c)=\mathcal{S}(c,c,b)=\mathcal{S}(a,a,b)=\mathcal{S}(b,b,a)=5,\\
&\mathcal{S}(x,y,z)=0 \text{ if } x=y=z,\\
&\mathcal{S}(x,y,z)=1 \text{ if otherwise},
\end{aligned}$$

for all $x,y,z\in X$. Then the function $\mathcal{S}$ is an S-metric and the pair $(X,\mathcal{S})$ is an S-metric space. However, the function d_S defined in (1) is not a metric on X. Indeed, for $x=a$, $y=c$ and $z=b$, we get

$$d_S(a,c)=24\nleq d_S(a,b)+d_S(b,c)=20.$$

We note that the function d_S is called the metric generated by the S-metric $\mathcal{S}$ in the case that d_S is a metric.

Now, we give the relationship between an S-metric and a b-metric.

Proposition 2.2. *Let $(X,\mathcal{S})$ be an S-metric space and the function $d:X\times X\to[0,\infty)$ be defined as*

$$d(x,y)=k\mathcal{S}(x,x,y),$$

for all $x,y\in X$ and some $k>0$. Then the function d is a b-metric on X.

Proof. Using the condition $(S1)$ and Lemma 1.4, we can easily seen that the conditions $(B1)$ and $(B2)$ are satisfied. Now we show that the condition $(B3)$ is satisfied. From the condition $(S2)$ and Lemma 1.4, we get

$$\begin{aligned}
d(x,z) &= k\mathcal{S}(x,x,z)\leq 2k\mathcal{S}(x,x,y)+k\mathcal{S}(z,z,y)\\
&= 2k\mathcal{S}(x,x,y)+k\mathcal{S}(y,y,z)\\
&= 2d(x,y)+d(y,z)
\end{aligned} \tag{2}$$

and

$$\begin{aligned}
d(x,z) &= k\mathcal{S}(x,x,z)=k\mathcal{S}(z,z,x)\leq 2k\mathcal{S}(z,z,y)+k\mathcal{S}(x,x,y)\\
&= 2k\mathcal{S}(y,y,z)+k\mathcal{S}(x,x,y)\\
&= 2d(y,z)+d(x,y).
\end{aligned} \tag{3}$$

From the inequalities (2) and (3), we have

$$d(x,z)\leq\frac{3}{2}\left[d(x,y)+d(y,z)\right].$$

Consequently, the function d is a b-metric with $b=\frac{3}{2}$. $\square$

Remark 2.3. 1) If we take $k=1$ in Proposition 2.2 then we get Proposition 2.1 on page 116 in [20].

2) If we take $k=2$ in Proposition 2.2 then we get the equality (1).

3) From Proposition 2.2, we deduce that the function d_S defined in (1) is a b-metric on X, but it is not always a metric since every b-metric need not to be a metric.

The relation between a metric and an S-metric is given in [6] as follows:

Lemma 2.4. *[6] Let (X,d) be a metric space. Then the following properties are satisfied*:

(1) *$\mathcal{S}_d(x,y,z)=d(x,z)+d(y,z)$ for all $x,y,z\in X$ is an S-metric on X.*
(2) *$x_n\to x$ in (X,d) if and only if $x_n\to x$ in $(X,\mathcal{S}_d)$.*
(3) *$\{x_n\}$ is Cauchy in (X,d) if and only if $\{x_n\}$ is Cauchy in $(X,\mathcal{S}_d)$.*
(4) *(X,d) is complete if and only if $(X,\mathcal{S}_d)$ is complete.*

The metric $\mathcal{S}_d$ is called the S-metric generated by d.

Now we present the following properties with some illustrative examples.

Proposition 2.5. *Let X be a nonempty set. If an S-metric is generated by any metric then this S-metric generates a metric d_S.*

Proof. Let $\mathcal{S}_d$ be an S-metric on X generated by a metric d. Then by Lemma 2.4 we have

$$\mathcal{S}_d(x,y,z)=d(x,z)+d(y,z),$$

for all $x,y,z\in X$. Then we obtain

$$\begin{aligned} d_{\mathcal{S}_d}(x,y) &= \mathcal{S}_d(x,x,y)+\mathcal{S}_d(y,y,x)=2\mathcal{S}_d(x,x,y)\\ &= 2\left[d(x,y)+d(x,y)\right]=4d(x,y), \end{aligned}$$

for all $x,y\in X$. Since d is a metric on X, then the function $d_{\mathcal{S}_d}$ defines a metric on X. □

We give the following corollary as a result of Proposition 2.5.

Corollary 2.6. *Let X be a nonempty set. If an S-metric is generated by any metric d then we have*

$$d_S(x,y)=4d(x,y).$$

The converse of Proposition 2.5 is not always true as seen in the following example.

Example 2.7. Let $X=\mathbb{R}^2$ and define the function

$$\mathcal{S}(x,y,z)=\sum_{i=1}^{2}\left(\left|x^{13}-z^{13}\right|+\left|x^{13}+z^{13}-2y^{13}\right|\right),$$

for all $x=(x_1,x_2)$, $y=(y_1,y_2)$, $z=(z_1,z_2)\in\mathbb{R}^2$. Then the pair $(\mathbb{R}^2,\mathcal{S})$ is an S-metric space with the S-metric which is not generated by any metric d, that is, $\mathcal{S}\neq\mathcal{S}_d$. On the contrary, we assume that there exists a metric d such that

$$\mathcal{S}(x,y,z)=\mathcal{S}_d(x,y,z)=d(x,z)+d(y,z),$$

for all $x,y,z\in\mathbb{R}^2$. Therefore we have

$$\mathcal{S}(x,x,z)=2d(x,z) \text{ and } d(x,z)=\sum_{i=1}^{2}\left|x^{13}-z^{13}\right|$$

and

$$\mathcal{S}(y,y,z)=2d(y,z) \text{ and } d(y,z)=\sum_{i=1}^{2}\left|y^{13}-z^{13}\right|,$$

for all $x,y,z\in\mathbb{R}^2$. So we get

$$\sum_{i=1}^{2}\left(\left|x^{13}-z^{13}\right|+\left|x^{13}+z^{13}-2y^{13}\right|\right)=\sum_{i=1}^{2}\left(\left|x^{13}-z^{13}\right|+\left|y^{13}-z^{13}\right|\right),$$

which is a contradiction for $x=(1,1)$, $y=(2,2)$, $z=(0,0)\in\mathbb{R}^2$. Consequently, $\mathcal{S}\neq\mathcal{S}_d$, that is, the S-metric is not generated by any metric d. However, this S-metric generates a metric d_S such that

$$d_S(x,y)=\mathcal{S}(x,x,y)+\mathcal{S}(y,y,x)=2\mathcal{S}(x,x,y)=2\sum_{i=1}^{2}\left|x^{13}-y^{13}\right|,$$

for all $x,y\in\mathbb{R}^2$.

Remark 2.8. Let X be a nonempty set, $\mathcal{S}_1$ be an S-metric on X which is not generated by any metric d and $\mathcal{S}_2$ be an S-metric on X which is generated by some metric d. Then $d_{\mathcal{S}_1}$ and $d_{\mathcal{S}_2}$ may be the same. For example, let $X=\mathbb{R}$ and the functions $\mathcal{S}_1,\mathcal{S}_2:X\times X\times X\to[0,\infty)$ be defined as

$$\mathcal{S}_1(x,y,z)=|x-z|+|x+z-2y|$$

and

$$\mathcal{S}_2(x,y,z)=|x-z|+|y-z|,$$

for all $x,y,z\in\mathbb{R}$. Then $\mathcal{S}_1$ is an S-metric on $\mathbb{R}$ which is not generated by any metric d and $\mathcal{S}_2$ be an S-metric which is called usual S-metric on $\mathbb{R}$ generated by usual metric d (see [14] and [20] for more details, respectively). By Lemma 2.4 we get

$$d_{S_1}(x,y)=d_{S_2}(x,y)=4|x-y|,$$

for all $x,y\in\mathbb{R}$. Consequently, $\mathcal{S}_1$ and $\mathcal{S}_2$ generate the same metric $d_S=d_{S_1}=d_{S_2}$.

Now we investigate another relationships between a metric and an S-metric with topological aspects. At first, we recall the following definitions and lemma on an S-metric space.

Definition 2.9. [19] Let $(X,\mathcal{S})$ be an S-metric space. For $r>0$ and $x\in X$, the open ball $B_S(x,r)$ with a center x and radius r is defined as follows:

$$B_S(x,r)=\{y\in X:\mathcal{S}(x,x,y)<r\}.$$

Definition 2.10. [19] Let $(X,\mathcal{S})$ be an S-metric space and $A\subseteq X$. For every $x\in A$, if there exists a $r>0$ such that

$$B_S(x,r)\subseteq A,$$

then the subset A is called an open subset of X.

Lemma 2.11. *[19] Let $(X,\mathcal{S})$ be an S-metric space. If $r>0$ and $x\in X$, the open ball $B_S(x,r)$ is an open subset of X.*

In the following definition, we give the notion of topological equivalence of a metric and an S-metric.

Definition 2.12. Let (X, d) be a metric space and $(X, \mathcal{S})$ be an S-metric space. The metric d and the S-metric $\mathcal{S}$ are said to be topological equivalent (briefly, equivalent) if they generate the same topology on X, that is, A is an open subset on (X, d) if and only if it is an open subset on $(X, \mathcal{S})$.

Using this definition, we obtain the following proposition.

Proposition 2.13. *Let (X, d) be a metric space and $(X, \mathcal{S})$ be an S-metric space. Then the metric d and the S-metric $\mathcal{S}$ are equivalent if and only if there exist radii $r_1, r_2, \rho_1, \rho_2 > 0$ such that*

$$B(x, r_1) \subset B_S(x, r_2)$$

and

$$B_S(x, \rho_1) \subset B(x, \rho_2),$$

for each $x \in X$.

Proof. Assume that the metric d and the S-metric $\mathcal{S}$ are equivalent. Let us consider an open ball $B_S(x, r_2)$ for each $x \in X$. Since the metric d and the S-metric S are equivalent then $B_S(x, r_2)$ is also open on (X, d). Therefore there exists an open ball such that

$$B(y, r_1) \subset B_S(x, r_2),$$

for each $y \in B_S(x, r_2)$. If we take $x = y$ then we get

$$B(x, r_1) \subset B_S(x, r_2).$$

Similarly we obtain

$$B_S(x, \rho_1) \subset B(x, \rho_2).$$

Conversely, let A be an open set on (X, d) and $x \in A$. Then there exists an open ball $B(x, \rho_2)$ such that

$$B(x, \rho_2) \subset A,$$

for each $x \in A$. By the hypothesis, there exists an open ball $B_S(x, \rho_1)$ on (X, S) such that

$$B_S(x, \rho_1) \subset B(x, \rho_2) \subset A.$$

Then A is an open set on $(X, \mathcal{S})$. Similarly, if A is an open set on $(X, \mathcal{S})$ then A is an open set on (X, d). $\square$

Using the idea of projection to reduce three dimensions to two dimensions, we give the following definition.

Definition 2.14. Let (X, d) be a metric space and $(X, \mathcal{S})$ be an S-metric space. If there exist numbers $k_1, k_2 > 0$ such that

$$k_1 \mathcal{S}(x, x, y) \leq d(x, y) \leq k_2 \mathcal{S}(x, x, y),$$

then the metric d and the S-metric $\mathcal{S}$ are said to $(\mathcal{S}, d)$-Lipschitz equivalent.

In the following proposition we see the relationships between topological equivalence and $(\mathcal{S}, d)$-Lipschitz equivalence.

Proposition 2.15. *Let (X, d) be a metric space and $(X, \mathcal{S})$ be an S-metric space. If the metric d and the S-metric $\mathcal{S}$ are $(\mathcal{S}, d)$-Lipschitz equivalent then they are equivalent.*

Proof. We prove that the metric d and the S-metric $\mathcal{S}$ are equivalent. To do this, we show that $B(x,k_1r) \subset B_S(x,r)$ and $B_S\left(x,\frac{r}{k_2}\right) \subset B(x,r)$. Assume that $y \in B(x,k_1r)$. Then we get

$$d(x,y) \leq k_1 r \text{ and } \frac{d(x,y)}{k_1} \leq r.$$

Hence we obtain

$$S(x,x,y) \leq \frac{d(x,y)}{k_1} \leq r,$$

that is, $y \in B_S(x,r)$. Therefore we have

$$B(x,k_1r) \subset B_S(x,r).$$

Using the above arguments, we see that

$$B_S\left(x,\frac{r}{k_2}\right) \subset B(x,r).$$

Consequently, the metric d and the S-metric $\mathcal{S}$ are equivalent. □

Proposition 2.16. *Let (X,d) be a metric space and $(X,\mathcal{S}_d)$ be an S-metric space with the S-metric $\mathcal{S}_d$ generated by d. Then the metric d and the S-metric $\mathcal{S}_d$ are equivalent.*

Proof. Since $\mathcal{S}_d$ is an S-metric generated by d, then from Lemma 2.4 we get

$$\mathcal{S}_d(x,y,z) = d(x,z) + d(y,z)$$

and so

$$\mathcal{S}_d(x,x,y) = 2d(x,y).$$

Therefore we have

$$\frac{1}{2}\mathcal{S}_d(x,x,y) \leq d(x,y) \leq \mathcal{S}_d(x,x,y),$$

for $k_1 = \frac{1}{2}$, $k_2 = 1$. Consequently, from Proposition 2.15 the metric d and the S-metric $\mathcal{S}_d$ are equivalent. □

Remark 2.17. Notice that a metric and an S-metric which is not generated by any metric can be equivalent. For example, let us consider the S-metric $\mathcal{S}_1$ defined in Remark 2.8 and the usual metric on $\mathbb{R}$. Therefore the usual metric and the S-metric $\mathcal{S}_1$ are equivalent.

Finally, even if an S-metric space is topologically equivalent to a metric space, but they are isometrically distinct.

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