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COLLOCATION COMPUTATIONAL ALGORITHM FOR VOLTERRA-FREDHOLM INTEGRO-DIFFERENTIAL EQUATIONS

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Abstract. In this study, we present a collocation computational technique for solving Volterra-Fredholm Integro-Differential Equations (VFIDEs) via fourth kind Chebyshev polynomials as basis functions. The method assumed an approximate solution by means of the fourth kind Chebyshev polynomials, which were then substituted into the Volterra-Fredholm Integro-Differential Equations (VFIDEs) under consideration. Thereafter, the resulting equation is collocated at equally spaced points, which results in a system of linear algebraic equations with the unknown Chebyshev coefficients. The system of equations is then solved using the matrix inversion approach to obtain the unknown constants. The unknown constants are then substituted into the assumed approximate solution to obtain the required approximate solution. To test for the accuracy and efficiency of the scheme, six numerical examples were solved, and the results obtained show the method performs excellently compared to the results in the literature. Also, tables are used to outline the methods accuracy and efficiency.

1. Introduction

$$\varphi^r(s) + \sum_{i=0}^{r-1} \mu_i(s)\varphi^i(s) = f(s) + \lambda_1 \int_a^s K_1(s,t)\varphi(t)dt + \lambda_2 \int_a^b K_2(s,t)\varphi(t)dt, \quad (1)$$

with the initial conditions

$$\varphi^i(0) = \varphi_i, i = 0, 1, 2, \dots r-1. \quad (2)$$

Where $K_1(s,t)$ and $K_2(s,t)$ and $\mu_i(s), i = 0, 1, 2, \dots r$ are known functions on the interval $a \le s \le t \le b$. a, b, λ_1 , λ_2 are constant values, $f(s)$ is a known function and $\varphi(s)$ is the unknown function to be determined. Integro-differential equations are

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extensively used as mathematical models across a range of subjects. The study of integral and integro-differential equations has its roots in the work of Abel, Lotka, Fredholm, Malthus, and Verhulst, for more information, see [1] and references cited therein. Integro-Differential Equations (IDEs) have drawn a great deal of attention lately. Getting accurate approximations using numerical techniques will be very helpful because many IDEs cannot be solved analytically. The following are just a few of the authors who have offered numerical approaches to solve IDEs: Rationalized Haar function approach is used by [2] to solve a system of linear IDEs, Adomian decomposition method is implemented in [3] to solve BVPs for fourth-order IDEs, utilizing a variational iteration approach, [4] presented the solution of fourth order IDEs, Applying the differential transform method to solve high-order nonlinear Volterra-Fredholm IDEs is implemented by [5], For the solving linear FVIDE, [6] applied a fixed-point iterative algorithm, For solving Fredholm-Volterra Integro-Differential Equations (FVIDEs), [7], [8], and [9] used Chebyshev polynomials as basis functions, while [10] employed the Chebyshev wavelet approximation analytical solution for high-order IDEs. [11] presented a novel numerical method using the Chebyshev third-kind polynomials. The numerical solution of a system of linear fractional IDEs using the least-squares collocation Chebyshev technique is investigated by [12], In [13] work, two proposed approaches for rational Chebyshev functions are used to study the numerical solution of high-order linear IDEs with variable coefficients, A class of linear IDEs with weakly singular kernels were solved using the Bernstein series by [14], and For the VFIDEs, the collocation approach is used by [15], [16], [17], [18], [19], and [20]. Also, [21], [22], [23] and [24] contain a number of numerical techniques for solving the FIDEs. In this study, we provide a fourth kind of Chebyshev collocation technique for the class of problems in the earlier work that is motivated and inspired by the earlier work, with improved accuracy and requires less work.

2. Material and Method

Definition 1

Chebyshev Polynomials of the Fourth Kind (CPFK): The CPFK are orthogonal polynomials with respect to the weight function $w(s) = \sqrt{\frac{1-s}{1+s}} \forall \in [-1,1]$, is defined by $\psi_r(s) = \frac{\sin(r+\frac{1}{2})\theta}{\sin\frac{\theta}{2}}$, where $s = \cos\theta$ and the recurrence relation is given as:

$$\psi_{(r+1)}(s) = 2s\psi_r(s) - \psi_{r-1}(s), r \geq 1,$$

starting with $\psi_{(0)}(s) = 1$ and $\psi_{(1)}(s) = 2s+1$.
Hence, the first few CPFK are given below:
$\psi_{(0)}(s) = 1, \psi_{(1)}(s) = 2s+1, \psi_{(2)}(s) = 4s^2+2s-1, \psi_{(3)}(s) = 8s^3+4s^2-3s-1,$
$\psi_{(4)}(s) = 16s^4+8s^3-12s^2-2s+1$, $\psi_{(5)}(s) = 32s^5+16s^4-32s^3-12s^2+6s+1$

Definition 2

Shifted Chebyshev Polynomials of the Fourth Kind (SCPFK): The SCPFK is orthogonal polynomials with respect to the weight function $w^*(s) = \sqrt{\frac{1-s}{s}} \forall \in [0,1]$, is defined by $\psi^*_r(s) = \psi_{(r)}(2s+1)$ where $\psi_{(r)}(s)$ is CPFK.The recurrence relation is given by

$$\psi^*_{(r+1)}(s) = 2(2s+1)\psi^*_r(s) - \psi^*_{r-1}(s), r \geq 1,$$

starting with $\psi^*_{(0)}(s) = 1$ and $\psi^*_{(1)}(s) = 4s - 1$.
Hence, the first few SCPFK are given below:

$\psi^*_{(0)}(s) = 1, \psi^*_{(1)}(s) = 4s - 1, \psi^{[}_{(2)}*(s) = 16s^2 - 12s + 1, \psi^*_{(3)}(s) = 64s^3 - 80s^2 + 24s - 1,$
$\psi^*_{(4)}(s) = 256s^4 - 448s^3 + 240s^2 - 40s + 1,\ \psi^*_{(5)}(s) = 1024s^5 - 2304s^4 + 1792s^3 - 560s^2 + 60s - 1$

Definition 3
Collocation method: A method of evaluating an approximate solution in a suitable collection of functions, sometimes referred to as a trial solution or basis function.

Definition 4
Approximate solution:An approximate solution is an inexact representation of the exact solution that is still close enough to be useful.

Definition 5
Exact solution: The solution of an equation is called an exact solution if it can be expressed in a closed form, such as a polynomial, exponential function, trigonometric function or the combination of two or more of these elementary functions.

Definition 6
We defined absolute error as follows in this study: Absolute Error $= |\varphi(\mathbf{s}) - \varphi(s)|$; $-1 \leq s \leq 1$, where $\varphi(\mathbf{s})$is the exact solution and $\varphi(s)$ is the approximate solution.

3. Demonstration of the method

Proposed method
The work assumed an approximate solution by means of the fourth kind Chebyshev polynomial in the form:

$$\varphi^i(s) = \sum_{i=0}^{r} \psi_i(s)c_i \tag{3}$$

The unknowable constants to be determined are $c_i, i = 0(1)r$. Differentiating Eq. (3) with respect to rth-times as functions of s, to obtain the following equations Thus, substituting Eq. (3) into Eq. (1) gives:

$$\sum_{i=0}^{r} \mu_i(s)\psi_i^i(s)c_i = f(\omega) + \lambda_1 \int_a^b K_1(s,t)\psi_i(t)c_i dt + \lambda_2 \int_a^s K_2(s,t)\psi_i(t)c_i dt, \tag{4}$$

Let $\zeta(s) = \sum_{i=0}^{n} \mu_i(s)\psi_i^i(s)c_i$, $\eta(s) = \lambda_1 \int_a^b K_1(s,t)\psi_i^i(t)c_i dt$ and $\tau(s) = \lambda_2 \int_a^s K_2(s,t)\psi_i^i(t)c_i dt$

where $\zeta(s)$ is the differential part, $\eta(s)$ is the Fredholm integral part and $\tau(s)$ is the Volterra integral part. Thus, equation (4) becomes

$$\zeta(s) - \eta(s) - \tau(s) = f(s) \tag{5}$$

The linear algebraic system of equations in $(n+1)$ unknown constants $c_i's$ is obtained by collocating Eq. (5) at the equally spaced point $s_i = a + \frac{(b-a)i}{n}$, $(i = 0(1)(n))$.

Additional equations are obtained from Eq. (2), which are represented in matrix form:

$$\begin{pmatrix} A_{11} & A_{12} & A_{13} & \cdots & \cdots & \cdots & A_{1r} \\ A_{21} & A_{22} & A_{23} & \cdots & \cdots & \cdots & A_{2r} \\ \vdots & \vdots & \vdots & & \vdots & & \\ \vdots & \vdots & \vdots & & \vdots & & \\ A_{m1} & A_{m2} & A_{m3} & \cdots & \cdots & \cdots & A_{mr} \\ A_{11}^0 & A_{12}^0 & A_{13}^0 & \cdots & \cdots & \cdots & A_{1r}^0 \\ A_{21}^1 & A_{22}^1 & A_{23}^1 & \cdots & \cdots & \cdots & A_{2r}^1 \\ \vdots & \vdots & \vdots & & \vdots & & \\ \vdots & \vdots & \vdots & & \vdots & & \\ A_{m1}^{r-1} & A_{m2}^{r-1} & A_{m3}^{r-1} & \cdots & \cdots & \cdots & A_{mr}^{r-1} \end{pmatrix} \begin{pmatrix} c_0 \\ c_1 \\ \vdots \\ \vdots \\ \vdots \\ \vdots \\ \vdots \\ \vdots \\ c_r \end{pmatrix} = \begin{pmatrix} B_{11} \\ B_{22} \\ \vdots \\ \vdots \\ Bmr \\ R_{11}^0 \\ B_{22}^1 \vdots \\ \vdots \\ \\ B_{mr}^{r-1} \end{pmatrix} \tag{6}$$

where $A_i's$ and $A_i^{0's}$ are the coefficients of $c_i's$ and $B_i's$ are values of $f(s_i)$. The matrix inversion approach is then used to solve the system of equations in order to obtain the unknown constants.

$$\begin{pmatrix} c_0 \\ c_1 \\ \vdots \\ \vdots \\ \vdots \\ \vdots \\ \vdots \\ \vdots \\ c_r \end{pmatrix} = \begin{pmatrix} A_{11} & A_{12} & A_{13} & \cdots & \cdots & \cdots & A_{1r} \\ A_{21} & A_{22} & A_{23} & \cdots & \cdots & \cdots & A_{2r} \\ \vdots & \vdots & \vdots & & \vdots & & \\ \vdots & \vdots & \vdots & & \vdots & & \\ A_{m1} & A_{m2} & A_{m3} & \cdots & \cdots & \cdots & A_{mr} \\ A_{11}^0 & A_{12}^0 & A_{13}^0 & \cdots & \cdots & \cdots & A_{1r}^0 \\ A_{21}^1 & A_{22}^1 & A_{23}^1 & \cdots & \cdots & \cdots & A_{2n}^1 \\ \vdots & \vdots & \vdots & & \vdots & & \\ \vdots & \vdots & \vdots & & \vdots & & \\ A_{m1}^{r-1} & A_{m2}^{n-1} & A_{m3}^{n-1} & \cdots & \cdots & \cdots & A_{mr}^{r-1} \end{pmatrix}^{-1} \begin{pmatrix} B_{11} \\ B_{22} \\ \vdots \\ \vdots \\ Bmr \\ B_{11}^0 \\ B_{22}^1 \vdots \\ \vdots \\ \\ B_{mr}^{r-1} \end{pmatrix} \tag{7}$$

The required approximate solution is obtained by solving Eq. (7) and then substituting the unknown constant values into the assumed approximate solution.

4. Numerical Applications

Example 1 [7], [9]: Consider the following fifth-order Fredholm integro- differential equation

$$\varphi^v(s) - s^2\varphi^{'''}(s) - \varphi^{'}(s) - \omega\varphi(s) = \omega^2 \cos s - s \sin s + \int_{-1}^{1} \varphi(t)dt$$

Subject to the conditions $\varphi(0) = 0$, $\varphi^{'}(0) = 1$, $\varphi^{''}(0) = 0$, $\varphi^{'''}(0) = -1$,$\varphi^{iv}(0) = -1$. The exact solution is $\varphi(\omega) = \sin\omega$.

Using the method outlined above, we obtained the following unknown constants:

$c_0 = -0.440064481396165, c_1 = 0.440064505468773, c_2 = 0.0195564449518142,$
$c_3 = -0.0195564016204016, c_4 = -0.0002511296319910, c_5 = 0.000251137036237896$

$c_6 = 0.00000151176309613037, c_7 = -0.00000150694848807835,$
$c_8 = -5.13558381953082 \times 10^{-9},$
$c_9 = 5.13558387195263 \times 10^{-9}$
Consequently, the approximate solution is given as:
$\varphi(s) = s+0.000002629418942s^9+2.8392052\times10^{-11}+1.3\times10^{-14}s^8-0.0001981893290s^7-1.43 \times 10^{-13}s^6 + 0.008333170309s^5 - 6.88 \times 10^{-13}s^4 - 0.16666666667s^3 - 5.4121 \times 10^{-12}s^2$

Example 2 [9] Consider the Volterra integro-differential equation of second order

$$\varphi^{'}(s) + s\varphi^{'}(s) - s\varphi(s) = e^s - (s+1)\sin s + \int_{-1}^{s} \sin se^{-t}\varphi(t)dt$$

Subject to the conditions $\varphi(0) = 1$, $\varphi^{'}(0) = 1$. The exact solution is $\varphi(s) = e^s$
Using the method outlined above, we obtained the following unknown constants:
$c_0 = 0.700918984547003, c_1 = 0.429398759584840, c_2 = 0.113584921573100,$
$c_3 = 0.0194247870239250, c_4 = 0.00246650242558680, c_5 = 0.000247661871666485,$
$c_6 = 0.0000207850298229589, c_7 = 0.00000151091646799495, c_8 = 9.9906758324608\times 10^{-8}$
Thus, the approximate solution is given as:
$\varphi(s) = 0.9999999997s + 0.0002061853730s^7 + 1.000000003 + 0.00002557613013s^8 + 0.001388176740s^6+0.008326350282s^5+0.04166693068s^4+0.1666688805s^3+0.4999999693s^2$

Example 3 [9],[24] Consider the FVIDE of first order

$$\varphi^{'}(s) - \int_{-1}^{1} \sin(s-t)\varphi(t)dt - \int_{-1}^{s} st\varphi(t)dt = \frac{27}{5} + \frac{41s}{20} + 3s^2 - \frac{1}{2}s^3 - s^4 - \frac{3s^5}{4} - \frac{1s^6}{5}$$

$\varphi(0) = 1$, $-1 \le s \le 1$.

$$\varphi(s) = (s+1)^3$$

$c_0 = 0.375, c_1 = 1.125, c_2 = 0.624999999981010, c_3 = 0.125$
As a result, the approximate solution was found to be $\varphi(s) = 1 + 3s + 3s^2 + s^3$, which agrees perfectly with the exact solution.

Example 4 Consider the VFIDE of the first order

$$\varphi^{'}(s) = 9 - 5s - s^2 - s^3 + \int_{0}^{1} (s-t)\varphi(t)dt - \int_{0}^{1} (s-t)\varphi(t)dt$$

$\varphi(0) = 2$
The analytic solution is given as $\varphi(s) = 2 + 6s$

$c_0 = 3.49999999998781, c_1 = 1.49999999999793, c_2 = 2.04753436428007 \times 10^{-11},$
$c_3 = 1.03527741934784 \times 10^{-11}$

$$\varphi(s) = 2 + 6s - 5.006164370 \times 10^{-10}s^2 + 6.625775482 \times 10^{-10}s^3$$

Example 5 Consider the FVIDE of second order

$$\varphi^{''}(s) = -8 + 6s - 3s^2 + s^3 + \int_{-1}^{1} (1-2st)\varphi(t)dt - \int_{0}^{1} \varphi(t)dt$$

$\varphi^{'}(0) = 6$, $\varphi(0) = 2$

The analytic solution is given as $\varphi(s) = 2 + 6s - 3s^2$

$c_0 = 3.12499999999702, c_1 = 0.937499999994634, c_2 = -0.187500000002982,$
$c_3 = -5.96371910188687 \times 10^{-11}$

$$\varphi(s) = 2 + 6s - 3s^2 - 3.816780225 \times 10^{-11}$$

5. Numerical Results

Table 1. Shows comparison of the Absolute Error (AE) for example 1

s_i	[7] r=9	[9] r=9	Our Method r=9
-1.0	$1.359E-5$	$9.0E-8$	$4.150E-08$
-0.8	$3.194E-6$	$3.9E-8$	$2.143E-08$
-0.6	$5.345E-7$	$1.4E-8$	$7.648E-09$
-0.4	$4.896E-8$	$4.0E-9$	$1.359E-09$
-0.2	$1.056E-9$	$1.0E-9$	$8.378E-11$
0.0	0.00000	0.0000	$0.000E+00$
0.2	$5.123E-10$	$1.0E-9$	$2.822E-11$
0.4	$1.1835E-8$	$1.6E-8$	$1.301E-09$
0.6	$6.7471E-8$	$1.79E-7$	$7.592E-09$
0.8	$2.2275E-7$	$1.15E-6$	$2.137E-08$
1.0	$5.5371E-7$	$5.29E-6$	$4.111E-08$

Table 2. Shows comparison of the AE for example 2

s_i	[9]n=9	Our Method n=8
-1.0	$2.0697374E-2$	$2.011E-7$
-0.8	$1.1470384E-2$	$9.790E-8$
-0.6	$4.420981E-3$	$1.207E-7$
-0.4	$9.73181E-4$	$7.994E-8$
-0.2	$5.9298E-5$	$1.370E-8$
0.0	$1.0E-8$	$3.000E-9$
0.2	$8.849E-5$	$1.824E-8$
0.4	$1.21776E-3$	$8.442E-8$
0.6	$5.26806E-3$	$1.297E-7$
0.8	$1.334832E-2$	$1.087E-07$
1.0	$2.336246E-2$	$2.436E-07$

Table 3. Shows comparison of the Exact Solution and the Aprroximate Solution for example 4

s_i	Exact	Aprroximate	AE
0.0	2.00000	0.0000	0.00000
0.2	3.20000	3.20000	0.00000
0.4	4.40000	4.40000	0.00000
0.6	5.60000	5.60000	0.00000
0.8	6.80000	6.80000	0.00000
1.0	8.00000	8.00000	0.00000

Table 4. Shows comparison of the Exact Solution and the Aprroximate Solution for example 5

s_i	Exact	Aprroximate	AE
0.0	2.00000	0.0000	0.00000
0.2	3.08000	3.08000	0.00000
0.4	3.92000	3.92000	0.00000
0.6	4.52000	4.52000	0.00000
0.8	4.88000	4.88000	0.00000
1.0	4.97000	4.97000	0.00000

6. Discussion of results

Using Maple 18, all examples in this study have been solved. Table 1 and Table 2 show that the suggested method outperforms [7] and [9]. According to Table 3 and Table 4, the difference between the exact solution and the approximation solution tends to be zero when the suggested method is used.

7. Conclusion

The proposed scheme has been effectively used in this research to arrive at numerical solutions to VFIDEs via fourth-kind Chebyshev polynomials utilizing. , With the help of tables and figures, six numerical examples are used to outline the technique's accuracy and efficiency. Tables 1–4 demonstrate that the approach utilized was more accurate since the error table discovered is smaller than those found in [7] and [9].To this end, researchers can apply this technique to various other VFIDEs on the basis of this work.

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