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INTUITIONISTIC FUZZY Ψ-Φ-CONTRACTIVE MAPPINGS AND FIXED POINT THEOREMS IN NON-ARCHIMEDEAN INTUITIONISTIC FUZZY METRIC SPACES

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Abstract. In this paper intuitionistic fuzzy Banach contraction theorem for M-complete non-Archimedean intuitionistic fuzzy metric spaces and intuitionistic fuzzy Elelstein contraction theorem for non-Archimedean intuitionistic fuzzy metric spaces by intuitionistic fuzzy ψ-ϕ contractive mappings are proved.

1. Introduction

Theory of intuitionistic fuzzy set as a generalization of fuzzy set [8] was introduce by Atansov [7]. Grabiec [9] initiated the study of fixed point theory in fuzzy metric spaces. George and Veeramani [1] have pointed out that the definition of Cauchy sequence for fuzzy metric spaces given by Grabiec [9] is weaker and they gave one stronger definition of Cauchy sequence and termed as M-Cauchy sequence. The definition of Cauchy sequence given by Grabiec [9] has been termed as G-Cauchy sequence. With the help of fuzzy ψ-contractive mappings defined by Dorel Mihet[5], we define intuitionistic fuzzy ψ-ϕ contractive mappings. Our definition of intuitionistic fuzzy ψ-ϕ contractive mapping is more general than the definitions of intuitionistic fuzzy contractive mapping given by Abdul Mohamad [2] and by this contraction we prove an intuitionistic fuzzy Banach contraction theorem for M-complete non-Archimedean intuitionistic fuzzy metric spaces. We also prove intuitionistic fuzzy Elelstein contraction theorem for non-Archimedean intuitionistic fuzzy metric spaces without the continuity condition.

2. Preliminaries

We quote some definitions and statements of a few theorems which will be needed in the sequel.

Definition 2.1. [4] A binary operation $* : [0\,,\,1] \times [0\,,\,1] \to [0\,,\,1]$ is continuous t - norm if $*$ satisfies the following conditions:

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(i) $*$ is commutative and associative,
(ii) $*$ is continuous,
(iii) $a * 1 = a, \quad \forall\ a \in [0, 1]$,
(iv) $a * b \leq c * d$ whenever $a \leq c$, $b \leq d$ and $a, b, c, d \in [0, 1]$.

A few examples of continuous t-norm are $a * b = ab$, $a * b = \min\{a, b\}$, $a * b = \max\{a + b - 1, 0\}$.

Definition 2.2. [4]. A binary operation $\diamond : [0, 1] \times [0, 1] \to [0, 1]$ is continuous t-conorm if $\diamond$ satisfies the following conditions :
(i) $\diamond$ is commutative and associative,
(ii) $\diamond$ is continuous,
(iii) $a \diamond 0 = a, \quad \forall\ a \in [0, 1]$,
(iv) $a \diamond b \leq c \diamond d$ whenever $a \leq c$, $b \leq d$ and $a, b, c, d \in [0, 1]$.

A few examples of continuous t-conorm are $a \diamond b = a + b - ab$, $a \diamond b = \max\{a, b\}$, $a \diamond b = \min\{a + b, 1\}$.

Definition 2.3. [6] A 5-tuple $(X, \mu, \nu, *, \diamond)$ is said to be an intuitionistic fuzzy metric space if X is an arbitrary set, $*$ is a continuous t-norm, $\diamond$ is a continuous t-conorm, μ and ν are fuzzy sets on $X^2 \times (0, \infty)$ and μ denotes the degree of nearness, ν denotes the degree of non-nearness between x and y relative to t satisfying the following conditions: for all $x, y, z \in X$, $s, t > 0$,
(i) $\mu(x, y, t) + \nu(x, y, t) \leq 1$;
(ii) $\mu(x, y, 0) = 0$;
(iii) $\mu(x, y, t) = 1$ if and only if $x = y$;
(iv) $\mu(x, y, t) = \mu(y, x, t)$;
(v) $\mu(x, z, t + s) \geq \mu(x, y, t) * \mu(y, z, s)$;
(vi) $\mu(x, y, \cdot) : [0, \infty) \to [0, 1]$ is left-continuous;
(vii) $\nu(x, y, 0) = 1$;
($viii$) $\nu(x, y, t) = 0$ if and only if $x = y$;
(ix) $\nu(x, y, t) = \nu(y, x, t)$;
(x) $\nu(x, z, t + s) \leq \nu(x, y, t) \diamond \nu(y, z, s)$;
(xi) $\nu(x, y, \cdot) : [0, \infty) \to [0, 1]$ is right-continuous.

Remark 2.4. If in the above definition the triangular inequalities (v) and (x) are replaced by
$\mu(x, z, \max\{t, s\}) \geq \mu(x, y, t) * \nu(y, z, s)$ and
$\nu(x, z, \max\{t, s\}) \leq \nu(x, y, t) \diamond \nu(y, z, s)$.
Or, equivalently,
$\mu(x, z, t) \geq \mu(x, y, t) * \mu(y, z, t)$ and
$\nu(x, z, t) \leq \nu(x, y, t) \diamond \nu(y, z, t)$.
Then $(X, \mu, \nu, *, \diamond)$ is called non-Archimedean intuitionistic fuzzy metric space.

Definition 2.5. [2] Let $(X, \mu, \nu, *, \diamond)$ be an intuitionistic fuzzy metric space. A mapping $f : X \to X$ is intuitionistic fuzzy contractive if there exists $k \in (0, 1)$ such that $\frac{1}{\mu(f(x), f(y), t)} - 1 \leq k \left(\frac{1}{\mu(x, y, t)} - 1 \right)$ and $\frac{1}{\nu(f(x), f(y), t)} - 1 \leq \frac{1}{k} \left(\frac{1}{\nu(x, y, t)} - 1 \right)$ for all $x, y \in X$ and $t > 0$. (k is called contractive constant of f.)

Definition 2.6. [2] Let $(X, \mu, \nu, *, \diamond)$ be an intuitionistic fuzzy metric space. We will say that the sequence $\{x_n\}_n$ in X is intuitionistic fuzzy contractive if there exists $k \in (0,1)$ such that $\frac{1}{\mu(x_{n+1}, x_{n+2}, t)} - 1 \leq k \left(\frac{1}{\mu(x_n, x_{n+1}, t)} - 1 \right)$ and $\frac{1}{\nu(x_{n+1}, x_{n+2}, t)} - 1 \leq \frac{1}{k} \left(\frac{1}{\nu(x_n, x_{n+1}, t)} - 1 \right)$ for all $t > 0$ and $n \in \mathbb{N}$.

3. Intuitionistic fuzzy Ψ-Φ-contractive mappings

Definition 3.1. Let $(X, \mu, \nu, *, \diamond)$ be an intuitionistic fuzzy metric space.
(i) A sequence $\{x_n\}_n$ in X is called M-Cauchy sequence, if for each $\epsilon \in (0,1)$ and $t > 0$ there exists $n_0 \in \mathbb{N}$ such that $\mu(x_n, x_m, t) > 1 - \epsilon$ and $\nu(x_n, x_m, t) < \epsilon$ for all $m, n \geq n_0$.

(ii) A sequence $\{x_n\}_n$ in X is called G-Cauchy sequence if $\lim_{n \to \infty} \mu(x_n, x_{n+m}, t) = 1$ and $\lim_{n \to \infty} \nu(x_n, x_{n+m}, t) = 0$ for each $m \in \mathbb{N}$ and $t > 0$.

Definition 3.2. A sequence $\{x_n\}_n$ in an intuitionistic fuzzy metric space $(X, \mu, \nu, *, \diamond)$ is said to converge to $x \in X$ if $\lim_{n \to \infty} \mu(x_n, x, t) = 1$ and $\lim_{n \to \infty} \nu(x_n, x, t) = 0$ for all $t > 0$.

Definition 3.3. Let Ψ be the class of all mappings $\psi : [0,1] \to [0,1]$ such that ψ is continuous, non-decreasing and $\psi(t) < t, \forall t \in (0,1)$. Let Φ be the class of all mappings $\phi : [0,1] \to [0,1]$ such that ϕ is continuous, non-decreasing and $\phi(t) > t, \forall t \in (0,1)$. Let $(X, \mu, \nu, *, \diamond)$ be an intuitionistic fuzzy metric space and $\psi \in \Psi$ and $\phi \in \Phi$. A mapping $f : X \to X$ is called an intuitionistic fuzzy ψ-ϕ-contractive mapping if the following implications hold:

$$\mu(x, y, t) > 0 \Rightarrow \psi(\mu(f(x), f(y), t)) \geq \mu(x, y, t)$$
$$\nu(x, y, t) < 1 \Rightarrow \phi(\nu(f(x), f(y), t)) \leq \nu(x, y, t).$$

Example 3.4. *Let $(X, \mu, \nu, *, \diamond)$ be an intuitionistic fuzzy metric space and $f : X \to X$ satisfies $\frac{1}{\mu(f(x), f(y), t)} - 1 \leq k \left(\frac{1}{\mu(x,y,t)} - 1 \right)$ and $\frac{1}{\nu(f(x), f(y), t)} - 1 \leq \frac{1}{k} \left(\frac{1}{\nu(x,y,t)} - 1 \right)$ for all $x, y \in X$ and $t > 0$. Then for some $k \in (0,1)$ with $k > 1 - t$, f is an intuitionistic fuzzy ψ-ϕ-contractive mapping, with*

$$\psi(t) = 1 - k, \quad \phi(t) = \frac{t}{(1-k)t + k}$$

Example 3.5. *Let X be a non-empty set with at least two elements. If we define the intuitionistic fuzzy set (X, μ, ν) by $\mu(x, x, t) = 1$ and $\nu(x, x, t) = 0$ for all $x \in X$ and $t > 0$; and*

$$\mu(x, y, t) = \begin{cases} 0, & if\ t \leq 1 \\ 1, & if\ t > 1 \end{cases} \qquad \nu(x, y, t) = \begin{cases} 1, & if\ t \leq 1 \\ 0, & if\ t > 1 \end{cases}$$

for all $x, y \in X$, $x \neq y$, then $(X, \mu, \nu, *, \diamond)$ is an M-complete non-Archimedean intuitionistic fuzzy metric space under any continuous t-norm $*$ and continuous t-conorm $\diamond$. Now,
$\mu(x, y, t) < 1$
$\Rightarrow \mu(x, y, t) = 0$
$\Rightarrow \psi(\mu(f(x), f(y), t)) \geq \mu(x, y, t) = 0$ *and*

$\nu(x, y, t) > 0$
$\Rightarrow \nu(x, y, t) = 1$
$\Rightarrow \phi\,(\nu(f(x), f(y), t)) \leq \nu(x, y, t) = 1$
Therefore every mapping $f : (X, \mu, \nu, *, \diamond) \to (X, \mu, \nu, *, \diamond)$ *is an intuitionistic fuzzy* ψ-ϕ-*contractive mapping.*

Definition 3.6. An intuitionistic fuzzy ψ-ϕ-contractive sequence in an intuitionistic fuzzy metric space $(X, \mu, \nu, *, \diamond)$ is any sequence $\{x_n\}_n$ in X such that

$$\psi\,(\mu\,(x_{n+1}, x_{n+2}, t)) \geq \mu(x_{n+1}, x_n, t)$$
$$\phi\,(\nu\,(x_{n+1}, x_{n+2}, t)) \leq \nu(x_{n+1}, x_n, t).$$

An intuitionistic fuzzy metric space $(X, \mu, \nu, *, \diamond)$ is called M-complete (or, G-complete) if every M-Cauchy (or, G-Cauchy) sequence is convergent in X.

4. Fixed point theorems

Theorem 4.1. *Let* $(X, \mu, \nu, *, \diamond)$ *be an M-complete non-Archimedean intuitionistic fuzzy metric space and* $f : X \to X$ *be an intuitionistic fuzzy* ψ-ϕ-*contractive mapping. If there exists* $x \in X$ *such that* $\mu\,(x, f(x), t) > 0$ *and* $\nu\,(x, f(x), t) < 1$ *for all* $t > 0$, *then* f *has a unique fixed point.*

Proof. Let $x \in X$ be such that $\mu(x, f(x), t) > 0$ and $\nu(x, f(x), t) < 1$, $t > 0$ and $x_n = f^n(x)$, $n \in \mathbb{N}$, we have for all $t > 0$

$$\mu(x_0, x_1, t) \leq \psi(\mu(x_1, x_2, t)) < \mu(x_1, x_2, t)$$
$$\nu(x_0, x_1, t) \geq \phi(\nu(x_1, x_2, t)) > \nu(x_1, x_2, t)$$

and

$$\mu(x_1, x_2, t) \leq \psi(\mu(x_2, x_3, t)) < \mu(x_2, x_3, t)$$
$$\nu(x_1, x_2, t) \geq \phi(\nu(x_2, x_3, t)) > \nu(x_2, x_3, t)$$

Hence by induction $\forall\, t > 0$, $\mu(x_n, x_{n+1}, t) < \mu(x_{n+1}, x_{n+2}, t)$ and $\nu(x_n, x_{n+1}, t) > \nu(x_{n+1}, x_{n+2}, t)$. Therefore, for every $t > 0$, $\{\mu(x_n, x_{n+1}, t)\}$ is a non-decreasing sequence of numbers in $(0, 1]$ and $\{\nu(x_n, x_{n+1}, t)\}$ is a non-increasing sequence of numbers in $[0, 1)$.
Fix $t > 0$. Denote $\lim_{n \to \infty} \mu(x_n, x_{n+1}, t)$ by l and $\lim_{n \to \infty} \nu(x_n, x_{n+1}, t)$ by m. Then we have $l \in [0, 1]$ and $m \in [0, 1]$.
Since $\psi\,(\mu\,(x_{n+1}, x_{n+2}, t)) \geq \mu(x_n, x_{n+1}, t)$ and ψ is continuous, $\psi(l) \geq l$. This implies $l = 1$. Also, since $\phi\,(\nu\,(x_{n+1}, x_{n+2}, t)) \leq \nu(x_{n+1}, x_n, t)$ and ϕ is continuous, $\phi(m) \leq m$. This implies $m = 0$. Therefore,
$\lim_{n \to \infty} \mu(x_n, x_{n+1}, t) = 1$ and $\lim_{n \to \infty} \nu(x_n, x_{n+1}, t) = 0$.
If $\{x_n\}_n$ is not a M-cauchy sequence then there are $\epsilon \in (0, 1)$ and $t > 0$ such that for each $k \in \mathbb{N}$ there exist $m(k), n(k) \in \mathbb{N}$ with $m(k) > n(k) \geq k$ and
$\mu(x_{m(k)}, x_{n(k)}, t) \leq 1 - \epsilon$ and $\nu(x_{m(k)}, x_{n(k)}, t) \geq \epsilon$
Let for each k, $m(k)$ be the least positive integer exceeding $n(k)$ satisfying the above property, that is,
$\mu(x_{m(k)-1}, x_{n(k)}, t) \geq 1 - \epsilon$ and $\mu(x_{m(k)}, x_{n(k)}, t) \leq 1 - \epsilon$. Also,
$\nu(x_{m(k)-1}, x_{n(k)}, t) \leq \epsilon$ and $\nu(x_{m(k)}, x_{n(k)}, t) \geq \epsilon$.
Then for each positive integer k,

$1 - \epsilon \geq \mu(x_{m(k)}, x_{n(k)}, t)$

$$\geq \mu(x_{m(k)-1}, x_{n(k)}, t) * \mu(x_{m(k)-1}, x_{m(k)}, t)$$
$$\geq (1-\epsilon) * \mu(x_{m(k)-1}, x_{m(k)}, t).$$

and

$$\epsilon \leq \nu(x_{m(k)}, x_{n(k)}, t)$$
$$\leq \nu(x_{m(k)-1}, x_{n(k)}, t) \diamond \nu(x_{m(k)-1}, x_{m(k)}, t)$$
$$\leq \epsilon \diamond \nu(x_{m(k)-1}, x_{m(k)}, t).$$

Taking limit as $k \to \infty$ we have,
$\lim_{k \to \infty} \{(1-\epsilon) * \mu(x_{m(k)-1}, x_{m(k)}, t)\}$
$= (1-\epsilon) * \lim_{k \to \infty} \mu(x_{m(k)-1}, x_{m(k)}, t)$
$= (1-\epsilon) * 1$
$= (1-\epsilon)$
and $\lim_{k \to \infty} \{\epsilon \diamond \nu(x_{m(k)-1}, x_{m(k)}, t)\}$
$= \epsilon \diamond \lim_{k \to \infty} \nu(x_{m(k)-1}, x_{m(k)}, t)$
$= \epsilon \diamond 0 = \epsilon$.
It follows that $\lim_{k \to \infty} \mu(x_{m(k)}, x_{n(k)}, t) = 1-\epsilon$
and $\lim_{k \to \infty} \nu(x_{m(k)}, x_{n(k)}, t) = \epsilon$.
Now, $\mu(x_{m(k)}, x_{n(k)}, t) \leq \psi\left(\mu(x_{m(k)+1}, x_{n(k)+1}, t)\right)$
and $\nu(x_{m(k)}, x_{n(k)}, t) \geq \phi\left(\nu(x_{m(k)+1}, x_{n(k)+1}, t)\right)$.
Since ψ and ϕ are continuous taking limit as $k \to \infty$ we have,
$1-\epsilon \leq \psi(1-\epsilon) < 1-\epsilon$ and $\epsilon \geq \phi(\epsilon) > \epsilon$, which are contradictions. Thus $\{x_n\}_n$ is a M-cauchy sequence.
If $\lim_{n \to \infty} x_n = y$ then from $\psi(\mu(f(y), f(x_n), t)) \geq \mu(y, x_n, t)$ and
$\phi(\nu(f(y), f(x_n), t)) \leq \nu(y, x_n, t)$ it follows that $x_{n+1} \to f(y)$.
Therefore we have
$\mu(y, f(y), t) \geq \mu(y, x_n, t) * \mu(x_n, x_{n+1}, t) * \mu(x_{n+1}, f(y), t) \to 1$ as $n \to \infty$. This implies $\mu(y, f(y), t) = 1$.
$\nu(y, f(y), t) \leq \nu(y, x_n, t) \diamond \nu(x_n, x_{n+1}, t) \diamond \nu(x_{n+1}, f(y), t) \to 0$ as $n \to \infty$. This implies $\nu(y, f(y), t) = 0$. Hence, $f(y) = y$.
If x, y are fixed points of f then
$\mu(f(x), f(y), t) = \mu(x, y, t) \leq \psi(\mu(f(x), f(y), t))$
and $\nu(f(x), f(y), t) = \nu(x, y, t) \geq \phi(\nu(f(x), f(y), t))$, $\forall\, t > 0$.
If $x \neq y$ then $\mu(x, y, s) < 1$ and $\nu(x, y, s) > 0$ for some $s > 0$ i.e., $0 < \mu(x, y, s) < 1$ and $0 < \nu(x, y, s) < 1$ hold, implying
$\mu(f(x), f(y), s) \leq \psi(\mu(f(x), f(y), s)) < \mu(f(x), f(y), s)$
and $\nu(f(x), f(y), s) \geq \phi(\nu(f(x), f(y), s)) > \nu(f(x), f(y), s)$, which are contradictions.
Thus $x = y$.
This completes the proof.

Lemma 4.2. *Let $(X, \mu, \nu, *, \diamond)$ be a non-Archimedean intuitionistic fuzzy metric space. If $\{x_n\}_n$ and $\{y_n\}_n$ be two sequences in X converges to x and y respectively then $\lim_{n \to \infty} \mu(x_n, y_n, t) = \mu(x, y, t)$ and $\lim_{n \to \infty} \nu(x_n, y_n, t) = \nu(x, y, t)$.*

Proof. Since $(X, \mu, \nu, *, \diamond)$ be a non-Archimedean intuitionistic fuzzy metric space, therefore

$\mu\ (x_n, y_n, t\ ) \ \geq\ \mu(x_n, x, t) \ast \mu(x, y, t) \ast \mu(y, y_n, t)$
$\Rightarrow \lim_{n \to \infty} \mu\ (x_n, y_n, t\ ) \ \geq\ 1 \ast \mu(x, y, t) \ast 1 \ =\ \mu(x, y, t)\,.$
and $\nu\ (x_n, y_n, t\ ) \ \leq\ \nu(x_n, x, t) \diamond \nu(x, y, t) \diamond \nu(y, y_n, t)$
$\Rightarrow \lim_{n \to \infty} \nu\ (x_n, y_n, t\ ) \ \leq\ 0 \diamond \nu(x, y, t) \diamond 0 \ =\ \nu(x, y, t)\,.$

Also, $\mu(x, y, t) \ \geq\ \mu(x, x_n, t) \ast \mu(x_n, y_n, t) \ast \mu(y, y_n, t)$
$\Rightarrow \mu(x, y, t) \ \geq\ 1 \ast \lim_{n \to \infty} \mu(x_n, y_n, t) \ast 1 = \lim_{n \to \infty} \mu(x_n, y_n, t)$
and $\nu(x, y, t) \ \geq\ \nu(x, x_n, t) \ast \nu(x_n, y_n, t) \ast \nu(y, y_n, t)$
$\Rightarrow \nu(x, y, t) \ \geq\ 0 \diamond \lim_{n \to \infty} \nu(x_n, y_n, t) \diamond 0 = \lim_{n \to \infty} \nu(x_n, y_n, t).$
Hence the proof.
We prove the following theorem without the continuity condition.

Theorem 4.3. *Let $(X, \mu, \nu, \ast, \diamond)$ be a compact non-Archimedean intuitionistic fuzzy metric space. Let $f : X \to X$ be an intuitionistic fuzzy ψ-ϕ-contractive mapping. Then f has a unique fixed point.*

Proof. Let $x \in X$ and $x_n = f^n(x),\ n \in \mathbb{N}$. Assume $x_n \neq x_{n+1}$ for each n (if not $f(x_n) = x_n$).
Now assume $x_n \neq x_m$ $(n \neq m)$, otherwise for $m < n$ we get
$\mu(x_n, x_{n+1}, t) = \mu(x_m, x_{m+1}, t) \leq \psi\left(\mu(x_{m+1}, x_{m+2}, t)\right)$
$< \mu(x_{m+1}, x_{m+2}, t) < \cdots < \mu(x_n, x_{n+1}, t)$ and
$\nu(x_n, x_{n+1}, t) = \nu(x_m, x_{m+1}, t) \geq \phi\left(\nu(x_{m+1}, x_{m+2}, t)\right)$
$> \nu(x_{m+1}, x_{m+2}, t) > \cdots > \nu(x_n, x_{n+1}, t)$, a contradiction.
Since X is compact, $\{x_n\}_n$ in X has a convergent subsequence $\{x_{n_i}\}_{i \in \mathbb{N}}$ (say). Let $\{x_{n_i}\}_{i \in \mathbb{N}}$ converges to y. We also assume that $y, f(y) \notin \{x_n : n \in \mathbb{N}\}$ (if not, choose a subsequence with such a property). According to the above assumptions we may now write for all $i \in \mathbb{N}$ and $t > 0$

$$\mu(x_{n_i}, y, t) \leq \psi(\mu\left(f(x_{n_i}), f(y), t\right)) < \mu\left(f(x_{n_i}), f(y), t\right)$$
$$\nu(x_{n_i}, y, t) \geq \phi(\nu\left(f(x_{n_i}), f(y), t\right)) > \nu\left(f(x_{n_i}), f(y), t\right)$$

Since ψ and ϕ are continuous for all $x, y \in X$. From lemma 4.2 we obtain
$\lim_{i \to \infty} \mu(x_{n_i}, y, t) \leq \lim_{i \to \infty} \mu\left(f(x_{n_i}), f(y), t\right)$
$\Rightarrow 1 \leq \lim_{i \to \infty} \mu\left(f(x_{n_i}), f(y), t\right)$
$\Rightarrow \lim_{i \to \infty} \mu\left(f(x_{n_i}), f(y), t\right) = 1.$
and $\lim_{i \to \infty} \nu(x_{n_i}, y, t) \geq \lim_{i \to \infty} \nu\left(f(x_{n_i}), f(y), t\right)$
$\Rightarrow 0 \geq \lim_{i \to \infty} \nu\left(f(x_{n_i}), f(y), t\right)$
$\Rightarrow \lim_{i \to \infty} \nu\left(f(x_{n_i}), f(y), t\right) = 0.$ i.e.,

$$f(x_{n_i}) \ \to\ f(y) \tag{4.1}$$

Similarly, we obtain

$$f^2(x_{n_i}) \ \to\ f^2(y) \tag{4.2}$$

Now, we see that
$\mu(x_{n_1}, f(x_{n_1}), t) \leq \psi(\mu\left(f(x_{n_1}), f^2(x_{n_1}), t\right)) < \mu\left(f(x_{n_1}), f^2(x_{n_1}), t\right)$
$< \cdots < \mu(x_{n_i}, f(x_{n_i}), t) < \mu\left(f(x_{n_i}), f^2(x_{n_i}), t\right) < \cdots < 1.$ and
$\nu(x_{n_1}, f(x_{n_1}), t) \geq \phi(\nu\left(f(x_{n_1}), f^2(x_{n_1}), t\right)) > \nu\left(f(x_{n_1}), f^2(x_{n_1}), t\right)$
$> \cdots > \nu(x_{n_i}, f(x_{n_i}), t) > \nu\left(f(x_{n_i}), f^2(x_{n_i}), t\right) > \cdots > 0.$

Thus $\{\mu(x_{n_i}, f(x_{n_i}), t)\}_{i\in\mathbb{N}}$ and $\{\mu\left(f(x_{n_i}), f^2(x_{n_i}), t\right)\}_{i\in\mathbb{N}}$ converges to a common limit. Also, $\{\nu(x_{n_i}, f(x_{n_i}), t)\}_{i\in\mathbb{N}}$ and
$\{\nu\left(f(x_{n_i}), f^2(x_{n_i}), t\right)\}_{i\in\mathbb{N}}$ converges to a common limit.
So, by (4.1), (4.2) and lemma 4.2 we get

$$\mu(y, f(y), t) = \mu\left(\lim_{i\to\infty} x_{n_i},\, f(\lim_{i\to\infty} x_{n_i}),\, t\right) = \lim_{i\to\infty} \mu\left(x_{n_i},\, f(x_{n_i}),\, t\right)$$
$$= \lim_{i\to\infty} \mu\left(f(x_{n_i}),\, f^2(x_{n_i}),\, t\right) = \mu\left(f(\lim_{i\to\infty} x_{n_i}),\, f^2(\lim_{i\to\infty} x_{n_i}),\, t\right) = \mu(f(y), f^2(y), t)$$

and

$$\nu(y, f(y), t) = \nu\left(\lim_{i\to\infty} x_{n_i},\, f(\lim_{i\to\infty} x_{n_i}),\, t\right) = \lim_{i\to\infty} \nu\left(x_{n_i},\, f(x_{n_i}),\, t\right)$$
$$= \lim_{i\to\infty} \nu\left(f(x_{n_i}),\, f^2(x_{n_i}),\, t\right) = \nu\left(f(\lim_{i\to\infty} x_{n_i}),\, f^2(\lim_{i\to\infty} x_{n_i}),\, t\right) = \nu(f(y), f^2(y), t)$$

for all $t > 0$.
Suppose $f(y) \neq y$, then we have
$\mu(y, f(y), t) \leq \psi(\mu\left(f(y), f^2(y), t\right)) < \mu\left(f(y), f^2(y), t\right)$ and
$\nu(y, f(y), t) \geq \phi(\nu\left(f(y), f^2(y), t\right)) > \nu\left(f(y), f^2(y), t\right)$, $t > 0$, a contradiction.
Hence $y = f(y)$ is a fixed point.
If x, y are fixed points of f then
$\mu(f(x), f(y), t) = \mu(x, y, t) \leq \psi(\mu(f(x), f(y), t))$ and
$\nu(f(x), f(y), t) = \nu(x, y, t) \geq \phi(\nu(f(x), f(y), t))$, $\forall\, t > 0$.
Suppose that $x \neq y$, then $\mu(x, y, s) < 1$ and $\nu(x, y, s) > 0$ for some $s > 0$ i.e., $0 < \mu(x, y, s) < 1$ and $0 < \nu(x, y, s) < 1$ hold, impllying
$\mu\left(f(x), f(y), s\right) \leq \psi(\mu\left(f(x), f(y), s\right)) < \mu(f(x), f(y), s)$ and
$\nu\left(f(x), f(y), s\right) \geq \phi(\nu\left(f(x), f(y), s\right)) > \nu(f(x), f(y), s)$, which are contradictions.
Therefore it must be the case that $x = y$.
Hence the proof.

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