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ALMOST PERIODIC SOLUTIONS FOR FOX PRODUCTION HARVESTING MODEL WITH DELAY AND IMPULSES

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Abstract. By employing the contraction mapping principle and applying the Gronwall–Bellman's inequality, sufficient conditions are established to prove the existence and exponential stability of positive almost periodic solution for Fox production harvesting model with delay and impulses.

1. Introduction

Consider the following equation of population dynamics [1, 2]

$$x'(t) = -xF(t,x) + xG(t,x), \;\; x'(t) = \frac{\mathrm{d}x}{\mathrm{d}t}, \tag{1}$$

where $x = x(t)$ is the size of population, $F(t,x)$ is the per–capita harvesting rate and $G(t,x)$ is the per–capita fecundity rate. Let $G(t,x)$ and $F(t,x)$ be defined in the form

$$F(t,x) = \alpha(t) \;\text{ and }\; G(t,x) = \beta(t)\ln^{\gamma}\Big(\frac{K(t)}{x(t)}\Big), \; \gamma > 0$$

then equation (1) becomes

$$x'(t) = -\alpha(t)x(t) + \beta(t)x(t)\ln^{\gamma}\Big(\frac{K(t)}{x(t)}\Big), \tag{2}$$

where $\alpha(t)$ is a variable harvesting rate, $\beta(t)$ is an intrinsic factor and $K(t)$ is a varying environmental carrying capacity. The positive parameter γ is referred to as an interaction parameter [1, 3, 4]. Indeed, if $\gamma > 1$ then intra–specific competition is high whereas if $0 < \gamma < 1$ then the competition is low. For $\gamma = 1$, equation (2) reduces to a classical Gompertzian model with harvesting [2, 5]. Equation (2) is called a Fox surplus production model that has been used to build up certain prediction models such as microbial growth model, demographic model and fisheries model. This equation is considered to be an efficient alternative to the well known γ–logistic model. Specifically, Fox model is more appropriate upon describing lower population density. We refer the reader to the papers [6, 7] where the existence of periodic solutions, stability, oscillation and the global attractivity of

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the solutions have been studied. In the recent paper [8], the authors have employed the continuation theorem to investigate the existence of positive almost periodic solution of equation (2). For more related topics, the papers [9, 10, 11, 12, 13] are recommended.

In the real world phenomena, the parameters can be nonlinear functions. The variation of the environment, however, plays an important role in many biological and ecological dynamical systems. In particular, the effects of a periodically varying environment are important for evolutionary theory as the selective forces on systems in a fluctuating environment differ from those in a stable environment. Thus, the assumption of periodicity of the parameters are a way of incorporating the periodicity of the environment. It has been suggested by Nicholson [14] that any periodical change of climate tends to impose its period upon oscillations of internal origin or to cause such oscillations to have a harmonic relation to periodic climatic changes.

On the other hand, some dynamical systems which describe real phenomena are characterized by the fact that at certain moments in their evolution they undergo rapid changes. Most notably this takes place due to certain seasonal effects such as weather, resource availability, food supplies, mating habits, etc. These phenomena are best described by the so called impulsive differential equations. Thus, it is more realistic to consider the case of combined effects: periodicity of the environment, time delays and impulse actions. Namely, an equation of the form

$$\left\{\begin{array}{rl} x'(t) & = -\alpha(t)x(t) + \beta(t)x(t-\tau)\ln^{\gamma}\left(\frac{K(t)}{x(t-\tau)}\right), \ \ t \neq \theta_k, \\ \Delta x(\theta_k) & := x(\theta_k^+) - x(\theta_k^-) = \eta_k x(\theta_k^-) + \delta_k, \ \ k \in \mathbb{N}, \end{array}\right. \tag{3}$$

where θ_k represent the instants at which the population suffers a sudden increment of δ_k units. For more information regarding the theory of impulsive delay differential equations; we refer the readers to the references [15, 16, 17, 18, 19, 20].

One can easily figure out that most of the equations in the above mentioned papers are considered under periodic assumptions. Nevertheless, the generalization to almost periodic functions which are functions that are periodic to some error has comparably less attention among researchers; we mention here the papers [21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34] in which the authors study the notion of almost periodicity of delay differential/difference equations with or without impulses. Motivated by the above said justifications, we shall study the almost periodicity of Fox model with delay and impulses (3). Indeed, sufficient conditions are established to prove the existence and exponential stability of positive almost periodic solution of this model. Our approach in this paper is different and is based on using the contraction mapping principle as well as applying Gronwall–Bellman's inequality.

2. Essential definitions and lemmas

Let $\{\theta_k\}_{k\in\mathbb{N}}$ be a fixed sequence such that $\sigma \leq \theta_1 < \theta_2 < \ldots < \theta_k < \theta_{k+1} < \ldots$ where $\lim_{k\to\infty}\theta_k = \infty$ and σ is a positive number.

Denote by $PLC([\sigma-\tau,\sigma],\mathbb{R}^+)$ the space of all piecewise left continuous functions $\varphi : [\sigma-\tau,\sigma] \to \mathbb{R}^+$ with points of discontinuity of the first kind at $t = \theta_k, \ k \in \mathbb{N}$. By a solution of (3), we mean a function $x(t)$ defined on $[\sigma-\tau,\infty)$ and satisfying equation (3) for $t \geq \sigma$. For a given initial function $\xi \in PLC([\sigma-\tau,\sigma],\mathbb{R}^+)$, it is well known that equation (3) has a unique solution $x(t) = x(t;\sigma,\xi)$ defined on

$[\sigma-\tau,\infty)$ and satisfying the initial condition

$$x(t;\sigma,\xi)=\xi(t),\ \ \sigma-\tau\le t\le\sigma. \tag{4}$$

Due to purpose of real applications, we will restrict our attention to positive solutions.

To say that impulsive delay differential equations have positive almost periodic solutions, one need to adopt the following definitions of almost periodicity for such type of equations.

The definitions are borrowed from the monograph [15].

Definition 1. *The set of sequences* $\{\theta_k^p\}$, $\theta_k^p=\theta_{k+p}-\theta_k,\ k,p\in\mathbb{N}$, *is said to be uniformly almost periodic if for arbitrary* $\varepsilon>0$ *there exists a relatively dense set of* $\varepsilon-$*almost periods common for any sequences.*

Definition 2. *A function* $\varphi\in PLC(\mathbb{R}^+,\mathbb{R}^+)$ *is said to be almost periodic if the following conditions hold:*

- *(a1) The set of sequences* $\{\theta_k^p\}$ *is uniformly almost periodic;*
- *(a2) For any* $\varepsilon>0$ *there exists a real number* $\delta=\delta(\varepsilon)>0$ *such that if the points* $t^{'}$ *and* $t^{''}$ *belong to the same interval of continuity of* $\varphi(t)$ *and satisfy the inequality* $|t^{'}-t^{''}|<\delta$, *then* $|\varphi(t^{'})-\varphi(t^{''})|<\varepsilon$.
- *(a3) For any* $\varepsilon>0$ *there exists a relatively dense set* T *of* $\varepsilon-$*almost periods such that if* $\omega\in T$ *then* $|\varphi(t+\omega)-\varphi(t)|<\varepsilon$ *for all* $t\in\mathbb{R}^+$ *satisfying the condition* $|t-\theta_k|>\varepsilon$, $k\in\mathbb{N}$. *The elements of* T *are called* $\varepsilon-$*almost periods.*

Related to equation (3), we consider the linear equation

$$\left\{\begin{array}{ll} x'(t) & =-\alpha(t)x(t),\ \ t\ne\theta_k,\\ \Delta x(\theta_k) & =\eta_k x(\theta_k),\ \ k\in\mathbb{N}.\end{array}\right. \tag{5}$$

It is well known [15] that equation (5) with an initial condition $x(t_0)=x_0$ has a unique solution represented by the form

$$x(t;t_0,x_0)=X(t,t_0)x_0,\quad t_0,x_0\in\mathbb{R}^+,$$

where X is the Cauchy matrix of (5) defined as follows:

$$X(t,s)=\left\{\begin{array}{ll} e^{-\int_s^t\alpha(r)dr}, & \theta_{k-1}<s\le t\le\theta_k,\\ \prod_{i=m}^k(1+\eta_i)e^{-\int_s^t\alpha(r)dr}, & \theta_{m-1}<s\le\theta_m\le\theta_k<t\le\theta_{k+1}.\end{array}\right. \tag{6}$$

Throughout this paper, we introduce the following conditions (C) for equation (3):

- (C1) The function $\alpha\in C[\mathbb{R}^+,\mathbb{R}^+]$ is almost periodic in the sense of Bohr and there exists a constant μ such that $\alpha(t)\ge\mu>0$;
- (C2) The sequence $\{\eta_k\}$ is almost periodic and $-1\le\eta_k\le0$, $k\in\mathbb{N}$;
- (C3) The set of sequences $\{\theta_k^p\}$ is uniformly almost periodic and there exists $\eta>0$ such that $\inf_{k\in\mathbb{N}}\theta_k^1=\eta>0$;
- (C4) The function $\beta(t)\in C[\mathbb{R}^+,\mathbb{R}^+]$ is almost periodic in the sense of Bohr and $\sup_{t\in\mathbb{R}^+}|\beta(t)|<\nu$ where $\nu>0$ and $\beta(0)=0$;
- (C5) The sequence $\{\delta_k\}$ is almost periodic and $\sup_{k\in\mathbb{N}}|\delta_k|<\kappa$, $k\in\mathbb{N}$;
- (C6) The function $K(t)\in C[\mathbb{R}^+,\mathbb{R}^+]$ is almost periodic in the sense of Bohr and $\sup_{t\in\mathbb{R}^+}|K(t)|<\rho$ where $\rho>0$.

The following results prove helpful.

Lemma 1. [15] *Let conditions* (C) *hold. Then for each* $\varepsilon > 0$ *there exists* ε_1, $0 < \varepsilon_1 < \varepsilon$, *relatively dense sets* T *of positive real numbers and* Q *of natural numbers such that the following relations are fulfilled:*

(b1) $|\alpha(t+\omega) - \alpha(t)| < \varepsilon$, $t \in \mathbb{R}^+$, $\omega \in T$;
(b2) $|\beta(t+\omega) - \beta(t)| < \varepsilon$, $t \in \mathbb{R}^+$, $\omega \in T$;
(b3) $|K(t+\omega) - K(t)| < \varepsilon$, $t \in \mathbb{R}^+$, $\omega \in T$;
(b4) $|\eta_{k+p} - \eta_k| < \varepsilon$, $p \in Q$, $k \in \mathbb{N}$;
(b5) $|\delta_{k+p} - \delta_k| < \varepsilon$, $p \in Q$, $k \in \mathbb{N}$;
(b6) $|\theta_k^p - \omega| < \varepsilon_1$, $\omega \in T$, $p \in Q$, $k \in \mathbb{N}$.

Lemma 2. [15] *Let condition* (C3) *be fulfilled. Then for each* $j > 0$ *there exists a positive integer* N *such that on each interval of length* j *there is no more than* N *elements of the sequence* $\{\theta_k\}$, *i.e.,*

$$i(s,t) \le N(t-s) + N,$$

where $i(s,t)$ *is the number of the points* θ_k *in the interval* (s,t).

The following lemmas provide a bound for the Cauchy matrix $X(t,s)$ of equation (5).

Lemma 3. *Let conditions* (C1)–(C3) *be satisfied. Then for the Cauchy matrix* $X(t,s)$ *of equation (5) there exists a positive constant* μ *such that*

$$X(t,s) \le e^{-\mu(t-s)},\ t \ge s,\ t,s \in \mathbb{R}^+. \tag{7}$$

Proof. In virtue of condition (C2), we deduce that the sequence $\{\eta_k\}$ is bounded. Further, it follows that $1 + \eta_k \le 1$. Thus, from formula (6) and condition (C1), we get

$$X(t,s) \le e^{-\mu(t-s)},\ t \ge s,\ t,s \in \mathbb{R}^+.$$

Lemma 4. *Let conditions* (C1)–(C3) *be satisfied. Then each* $\varepsilon > 0,\ t \in \mathbb{R}^+,\ s \in \mathbb{R}^+,\ t \ge s,\ |t-\theta_k| > \varepsilon,\ |s-\theta_k| > \varepsilon,\ k \in \mathbb{N}$ *there exists a relatively dense set* T *of* $\varepsilon-$*almost periods of the function* $\alpha(t)$ *and a positive constant* M *such that for* $\omega \in T$ *it follows*

$$\Big|X(t+\omega, s+\omega) - X(t,s)\Big| \le \varepsilon M e^{-\frac{\mu}{2}(t-s)}. \tag{8}$$

Proof. Consider the sets T and Q defined as in Lemma 1. Let $\omega \in T$. Since the matrix $H(t+\omega, s+\omega)$ is a solution of equation (5), we have the following

$$\begin{aligned}
&\tfrac{\partial}{\partial t}X = \alpha(t)X(t+\omega,s+\omega) + \Big[\alpha(t) - \alpha(t+\omega)\Big]X(t+\omega,s+\omega),\ t \ne \theta_k^{'},\\
&\Delta X(\theta_k^{'}, s) = \eta_k X(\theta_k+\omega, s+\omega) + (\eta_k - \eta_{k+p})X(\theta_k^{'}+\omega, s+\omega),
\end{aligned}$$

where $\theta_k^{'} = \theta_k - p,\ p \in Q,\ k \in \mathbb{N}$. Then

$$\begin{aligned}
X(t+\omega, s+\omega) &= X(t,s) + \int_s^t X(t,r)[\alpha(r) - \alpha(r+\omega)]X(r+\omega, s+\omega)\,\mathrm{d}r\\
&+ \sum_{s<\theta_k^{'}<t} X(t,\theta_k^{'}+0)\Big[\eta_{k+p} - \eta_k\Big]X(\theta_k^{'}+\omega, s+\omega).
\end{aligned} \tag{9}$$

In view of Lemma 1 it follows that if $|t-\theta_k^{'}| > \varepsilon$, then $\theta_{k+p}^{'} < t+\omega < \theta_{k+p+1}^{'}$. Further, we obtain

$$\Big|X(t+\omega,s+\omega) - X(t,s)\Big| \le \varepsilon(t-s)e^{-\mu(t-s)} + \varepsilon i(s,t)e^{-\mu(t-s)} \tag{10}$$

for $|t-\theta_k^{'}|>\varepsilon,\ |s-\theta_k^{'}|>\varepsilon$ where $i(s,t)$ is the number of the points $\theta_k^{'}$ in the interval (s,t). From Lemma 2, (10) becomes

$$\begin{aligned} &\Big|X(t+\omega,s+\omega)-X(t,s)\Big| \\ \leq\ & \varepsilon\Big[\frac{2}{\mu}\Big\{\frac{\mu}{2}(t-s)e^{\frac{-\mu}{2}(t-s)}\Big\}+N\frac{2}{\mu}\Big\{\frac{\mu}{2}(t-s)e^{\frac{-\mu}{2}(t-s)}\Big\}+Ne^{\frac{-\mu}{2}(t-s)}\Big]e^{\frac{-\mu}{2}(t-s)}. \end{aligned}$$

By using the inequalities $e^{-\frac{\mu}{2}(t-s)}<1$ and $\frac{\mu}{2}(t-s)e^{-\frac{\mu}{2}(t-s)}\leq 1$, we get

$$\Big|X(t+\omega,s+\omega)-X(t,s)\Big|\leq \varepsilon M,$$

where

$$M=\frac{2}{\mu}\Big(1+N+\frac{\mu}{2}N\Big).$$

3. The Main results

Throughout this section, it is assumed that

$$\nu\rho\gamma<\mu. \tag{11}$$

Theorem 1. *Let conditions (C) hold. Then there exists a unique positive almost periodic solution $x(t)$ of (3).*

Proof. Let $D\subset PLC(\mathbb{R}^+,\mathbb{R}^+)$ denote the set of all positive almost periodic functions $\varphi(t)$ with

$$\|\varphi\|\leq K,$$

where

$$\|\varphi\|=\sup_{t\in\mathbb{R}}|\varphi(t)| \ \text{ and } \ K:=\frac{\nu\rho\gamma}{\mu}+\frac{2}{1-e^{-\mu}}\kappa N.$$

Define an operator F in D by the formula

$$[F\varphi](t)=\int_{-\infty}^{t}X(t,s)\beta(s)\varphi(s-\tau)\ln^{\gamma}\Big(\frac{K(s)}{\varphi(s-\tau)}\Big)\,\mathrm{d}s+\sum_{\theta_k<t}X(t,\theta_k)\delta_k. \tag{12}$$

One can easily check that $F\varphi$ is a solution of equation (3). In the following, we first show that F maps the set D into itself. In view of relation (7) and conditions (C1),(C4)–(C6) we obtain

$$\begin{aligned} \|F\varphi\| &\leq \sup_{t\in\mathbb{R}^+}\Big\{\int_{-\infty}^{t}X(t,s)\big|\beta(s)\big|\Big|\varphi(s-\tau)\ln^{\gamma}\Big(\frac{K(s)}{\varphi(s-\tau)}\Big)\Big|\,\mathrm{d}s+\sum_{\theta_k<t}X(t,\theta_k)|\delta_k|\Big\} \\ &\leq \sup_{t\in\mathbb{R}^+}\Big\{\gamma\int_{-\infty}^{t}X(t,s)\big|\beta(s)\big||K(s)|\,\mathrm{d}s+\sum_{\theta_k<t}X(t,\theta_k)|\delta_k|\Big\} \\ &< \frac{\nu\rho\gamma}{\mu}+\frac{2}{1-e^{-\mu}}\kappa N=K \end{aligned} \tag{13}$$

for arbitrary $\varphi\in D$.

Now, we shall prove that $F\varphi$ is almost periodic. Indeed, let $\omega \in T,\ p \in Q$ where the sets T and Q are defined as in Lemma 1, it follows that

$$\begin{aligned}\Big\|F\varphi(t+\omega) &- F\varphi(t)\Big\| \\ &\le \sup_{t\in\mathbb{R}^+}\Big\{\int_{-\infty}^{t}\Big|X(t+\omega,s+\omega)-X(t,s)\Big| \\ &\times \big|\beta(s+\omega)\big|\varphi(s+\omega-\tau)\ln^{\gamma}\Big(\frac{K(s+\omega)}{\varphi(s+\omega-\tau)}\Big)\,\mathrm{d}s \\ &+ \int_{-\infty}^{t}X(t,s)\Big|\big|\beta(s+\omega)\big|\varphi(s+\omega-\tau)\ln^{\gamma}\Big(\frac{K(s+\omega)}{\varphi(s+\omega-\tau)}\Big) \\ &- \big|\beta(s)\big|\varphi(s-\tau)\ln^{\gamma}\Big(\frac{K(s)}{\varphi(s-\tau)}\Big)\Big|\,\mathrm{d}s \\ &+ \sum_{\theta_k<t}\Big|X(t+\omega,\theta_{k+p})-X(t,\theta_k)\Big||\delta_{k+p}| \\ &+ \sum_{\theta_k<t}X(t,\theta_k)\Big|\delta_{k+p}-\delta_k\Big|\Big\}\end{aligned}$$

or

$$\begin{aligned}&\Big\|F\varphi(t+\omega)-F\varphi(t)\Big\| \\ \le\ & \sup_{t\in\mathbb{R}^+}\Big\{\int_{-\infty}^{t}\Big|X(t+\omega,s+\omega)-X(t,s)\Big| \\ \times\ & \big|\beta(s+\omega)\big|\varphi(s+\omega-\tau)\ln^{\gamma}\Big(\frac{K(s+\omega)}{\varphi(s+\omega-\tau)}\Big)\,\mathrm{d}s \qquad (14) \\ +\ & \int_{-\infty}^{t}X(t,s)\Big\{\big|\beta(s+\omega)-\beta(s)\big|\varphi(s+\omega-\tau)\ln^{\gamma}\Big(\frac{K(s+\omega)}{\varphi(s+\omega-\tau)}\Big) \\ +\ & \big|\beta(s)\big|\Big|\varphi(s+\omega-\tau)\ln^{\gamma}\Big(\frac{K(s+\omega)}{\varphi(s+\omega-\tau)}\Big)-\varphi(s-\tau)\ln^{\gamma}\Big(\frac{K(s)}{\varphi(s-\tau)}\Big)\Big|\Big\}\,\mathrm{d}s \\ +\ & \sum_{\theta_k<t}\Big|X(t+\omega,\theta_{k+p})-X(t,\theta_k)\Big||\delta_{k+p}| \\ +\ & \sum_{\theta_k<t}X(t,\theta_k)|\delta_{k+p}-\delta_k|\Big\}\le\varepsilon C_1, \qquad (15)\end{aligned}$$

where

$$C_1=\frac{2}{\mu}\nu\rho\gamma M+\frac{1}{\mu}(\rho\gamma+2\nu\rho\gamma)+\kappa M\frac{2N}{1-e^{-\frac{\mu}{2}}}+\frac{2N}{1-e^{-\mu}}.$$

In virtue of (13) and (14), we deduce that $F\varphi\in D$. Therefore, F is a self–mapping from D to D.

Finally, we prove that F is a contraction mapping on D. Let $\varphi,\psi\in D$. From

(12), we have

$$\begin{aligned}\Big\|F\varphi - F\psi\Big\| &\leq \int_{-\infty}^{t} X(t,s)|\beta(s)| \\ &\times \Big|\varphi(s-\tau)\ln^{\gamma}\Big(\frac{K(s)}{\varphi(s-\tau)}\Big) - \psi(s-\tau)\ln^{\gamma}\Big(\frac{K(s)}{\psi(s-\tau)}\Big)\Big|\,\mathrm{d}s \\ &\leq \frac{\nu\rho\gamma}{\mu}\|\varphi-\psi\|. \end{aligned} \tag{16}$$

The assumption that $\nu\rho\gamma < \mu$ implies that F is a contraction mapping on D. Then there exists a unique fixed point $x \in D$ such that $Fx = x$. This implies that (3) has a unique positive almost periodic solution $x(t)$.

Theorem 2. *Let conditions (C) hold. Then the unique positive almost periodic solution $x(t)$ of (3) is exponentially stable.*

Proof. Let $y(t)$ be an arbitrary solution of (3) supplemented with the initial condition

$$y(t) = \zeta(t),\ \zeta \in PLC([\sigma-\tau,\sigma],\mathbb{R}^{+}).$$

Let $x(t)$ be the unique positive almost periodic solution of (3) with the initial condition (4). It follows that

$$\begin{aligned} x(t) - y(t) &= X(t,\sigma)(\xi-\zeta) \\ &+ \int_{\sigma}^{t} X(t,s)\beta(s) \\ &\times \Big(x(s-\tau)\ln^{\gamma}\Big(\frac{K(s)}{x(s-\tau)}\Big) - y(s-\tau)\ln^{\gamma}\Big(\frac{K(s)}{y(s-\tau)}\Big)\Big)\,\mathrm{d}s. \end{aligned}$$

Taking the norm of both sides, we get

$$\Big\|x(t)-y(t)\Big\| \leq e^{-\mu(t-\sigma)}\|\xi-\zeta\| + \int_{\sigma}^{t} e^{-\mu(t-s)}\nu\rho\gamma\|x(s)-y(s)\|\,\mathrm{d}s.$$

Setting $z(t) = \|x(t)-y(t)\|e^{\mu t}$ and applying Gronwall–Bellman's inequality we end up with the expression

$$\Big\|x(t)-y(t)\Big\| \leq \|\xi-\zeta\|e^{-(\mu-\nu\rho\gamma)(t-\sigma)}.$$

The assumption that $\nu\rho\gamma < \mu$ implies that the unique positive almost periodic solution of equation (3) is exponentially stable.

Example 1. *Let conditions (C) hold. If $\sup_{t\in\mathbb{R}^{+}}\{\gamma\beta(t)K(t)\} < \sup_{t\in\mathbb{R}^{+}}\alpha(t)$ then there exists a unique positive almost periodic exponential stable solution $x(t)$ of*

$$\begin{cases} x'(t) &= -\alpha(t)x(t) + \beta(t)x(t-\tau)\ln^{\gamma}\Big(\frac{K(t)}{x(t-\tau)}\Big),\ t \neq \theta_k, \\ \Delta x(\theta_k) &= \eta_k x(\theta_k),\ k \in \mathbb{N}. \end{cases} \tag{17}$$

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