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CLASS MAPPINGS PROPERTIES OF CONVOLUTIONS INVOLVING CERTAIN UNIVALENT FUNCTIONS ASSOCIATED WITH HYPERGEOMETRIC FUNCTIONS

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Abstract. The purpose of the present paper is to establish certain conditions to ensure that linear operator defined here maps a certain subclass of close-to-convex R^τ(A,B) into subclasses of convex and starlike functions $\alpha - UCV(\beta)$ and $\alpha - ST(\beta)$, respectively by making use of hypergeometric inequalities.

1. Introduction

Let $\mathcal{A}$ be the class consisting of functions of the form

$$f(z) = z + \sum_{n=2}^{\infty} a_n z^n \tag{1}$$

that are analytic in the open unit disk $E = \{z : |z| < 1\}$. As usual, denote by S the subclass of $\mathcal{A}$ consisting of function which are also univalent in E. Let $S^*(\alpha)$ and $C(\alpha)$ denote the subclass of S consisting of starlike and convex functions of order $\alpha (0 \le \alpha < 1)$ respectively [6]. $S^*(0) = S^*$ and $C(0) = C$ are respectively the classes of starlike and convex functions in S. Recently Bharti et al. [1] introduced the following subclasses of starlike and convex functions.

Definition 1. A function f of the form (1) is in $\alpha - ST(\beta)$ if it satisfies the condition

$$Re\left\{\frac{zf(z)}{f(z)}\right\} \ge \alpha \left|\frac{zf(z)}{f(z)} - 1\right| + \beta, \qquad \alpha \ge 0,\ 0 \le \beta < 1, \tag{2}$$

and $f \in \alpha - UCV(\beta)$ if and only if $zf \in \alpha - ST(\beta)$.

It should be noted that $1 - ST(0)$ is class of starlike functions corresponding to uniformly convex functions and $1 - UCV(0)$ is the class of uniformly convex functions given by Goodman [8] (also see [18]). Furthermore Bharti et al. [1] showed that

$$\alpha - ST(\beta) = S^*\left(\frac{\alpha+\beta}{1+\alpha}\right),\ \alpha - UCV(\beta) = C\left(\frac{\alpha+\beta}{1+\beta}\right)$$

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Definition 2. A function $f \in \mathcal{A}$ in said to be in the class $R^{\tau}(A,B)$. If it satisfies the inequality

$$\left|\frac{f'(z)-1}{(A-B)\tau - B[f'(z)-1]}\right| < 1 \, (z \in E : \tau \in C/\{0\}, -1 \leq B < A \leq 1) \, . \quad (3)$$

The class $R^{\tau}(A,B)$ was introduced by Dixit and Pal [4]. By giving appropriate values to the parameters τ, A and B, two interesting subclasses studied by Ponnusamy and Ronning [17] and Padmanabhan [13] can be reduced.

Let $F(a,b;c;z)$ be the (Gaussian) hypergeometric function defined by

$$F(a,b;c;z) = \sum_{n=0}^{\infty} \frac{(a)_n (b)_n}{(c)_n (1)_n} z^n \quad (4)$$

where $c \neq 0, -1, -2, ...$ and $(\lambda)_n$ is the Pochhammer symbol defined by

$$(\lambda)_n = \begin{cases} 1 & \text{if } n = 0 \\ \lambda(\lambda+1).........(\lambda+n-1) & \text{if } n \in N. = \{1,2,3.....\}. \end{cases} \quad (5)$$

We note that $F(a,b;c:1)$ converges for $Re\ (c-a-b) > 0$ and is related to the Gamma function by

$$F(a,b;c;1) = \frac{\Gamma(c)\Gamma(c-a-b)}{\Gamma(c-a)\Gamma(c-b)}. \quad (6)$$

The hypergeometric function $F(a,b;c;z)$ has been studied extensively by various authors and plays an important role in geometric function theory , We refer to([2], [3], [12], [14], [15], [16], [17], [19], [20]) and references therein for some interesting results.

We now recall the Hohlov operator $I_c^{ab} : \mathcal{A} \to \mathcal{A}$ defined in term of the Hadamard product (or convolution) by (cf. [9])

$$(I_c^{ab}(f))(z) = zF(a,b;c;z) * f(z) \ (f \in \mathcal{A}, \ z \in E). \quad (7)$$

Thus from (4) we have

$$(I_c^{ab}(f))(z) = z + \sum_{n=2}^{\infty} \frac{(a)_{n-1}(b)_{n-1}}{(c)_{n-1}(1)_{n-1}} a_n z^n \ (z \in E). \quad (8)$$

It is well known that the class S and many of its subclasses are not closed under the ring operations of usual addition and multiplication of functions. Therefore study of class preserving and class transforming operators is an interesting problem in geometric function theory, The operator $I_c^{ab}(f)$ is the natural extension of the Alexander, Libera, Bernardi and Carlsan - Shaffer operator (denoted here by A, L, B and $L(a,c)$ respectively). Thus

$$A(f) = I_2^{1,1}(f), \ L(f) = I_3^{1,2}(f), \ B(f) = I_{\gamma+2}^{1,\gamma+1}(f), \ L(a,b)(f) = I_c^{a,1}(f).$$

Dixit and Pathak [5] studied the mapping properties of a function f_μ to be as given in

$$f_\mu(z) = (1-\mu)zF(a,b;c;z) + \mu z[zF(a,b;c;z)] \ (\mu \geq 0) \quad (9)$$

and investigated the geometric properties of an integral operator of the form

$$I(z) = \int_0^z \frac{f_\mu(t)}{t} dt.$$

Further, Kim and Shon [10] considered a liner operator $M_\mu : \mathcal{A} \to \mathcal{A}$ defined by

$$M_\mu(f) = f_\mu(z)^* f(z) = z + \sum_{n=2}^{\infty}(1-\mu+\mu n)\frac{(a)_{n-1}(b)_{n-1}a_n z^n}{(c)_{n-1}(1)_{n-1}}. \tag{10}$$

For $\mu = 0$ in (10), we have $M_\mu(f) = (I_c^{a,b}(f))(z)$.

The purpose of the present paper is to make use of linear operator defined by (10) in order to establish a number of connections between the classes $R^\tau(A,B), \alpha-UCV(\beta)$ and $\alpha - ST$ and various other subclasses of $\mathcal{A}$.

2. Main Results

To establish our main results, we need each of the following results in our investigation.

Lemma 1. (see [1]) let $f \in \mathcal{A}$ be of the form (1).

$$\text{If } \sum_{n=2}^{\infty}[n(1+\alpha)-(\alpha+\beta)]a_n \le 1-\beta, \text{ then } f \in \alpha - ST(\beta). \tag{11}$$

Lemma 2. (see [1]) A function f of the form (1) is in $\alpha - UCV(\beta)$, if it satisfies the inequality

$$\sum_{n=2}^{\infty} n[n(1+\alpha)-(\alpha-\beta)]a_n \le 1-\beta. \tag{12}$$

Lemma 3. (see [4]). If $f \in R^\tau(A,B)$ is of the form (1) then

$$|a_n| \le (A-B)\frac{|\tau|}{n}, \ (n \in N/\{0\};\ \tau \in C/\{0\} \text{ and } -1 \le B < A \le 1) \tag{13}$$

The estimate in (13) is sharp.

Theorem 1. If $a > 1, b > 1$ and $c > a+b+1$. If $f \in R^\tau(A,B)$ and the inequality

$$\frac{(A-B)|\tau|\Gamma(c)\Gamma(c-a-b)}{\Gamma(c-a)\Gamma(c-b)}\left[\frac{\mu(\alpha+1)ab}{(c-a-b-1)} + \{1+\alpha-\mu(\alpha+\beta)\} + \frac{(\alpha+\beta)(\mu-1)(c-a-b)}{(a-1)(b-1)}\right]$$
$$\le (A-B)|\tau|\left[\frac{(\alpha+\beta)(\mu-1)(c-1)}{(a-1)(b-1)} + (1-\beta)\right] + (1-\beta)$$

is satisfied, then $M_\mu(f) \in \alpha - ST(\beta)$.

Proof. By Lemma 1, it suffices to show that

$$T_1 \equiv \sum_{n=1}^{\infty}[n(1+\alpha)-(\alpha+\beta)]\left|(1-\mu+\mu n)\frac{(a)_{n-1}(b)_{n-1}}{(c)_{n-1}(1)_{n-1}}a_n\right| \le 1-\beta.$$

Since $f \in R^\tau(A,B)$, then by Lemma 3, we have

$$|a_n| \le \frac{(A-B)|\tau|}{n}.$$

Hence

$$T_1 \leq \sum_{n=2}^{\infty}[n(1+\alpha)-(\alpha+\beta)]\left[(1-\mu+\mu n)\frac{(a)_{n-1}(b)_{n-1}}{(c)_{n-1}(1)_{n-1}}\frac{(A-B)|\tau|}{n}\right]$$

$$=(A-B)|\tau|\sum_{n=2}^{\infty}\left[\mu(1+\alpha)n+\{(1-\mu)(1+\alpha)-\mu(\alpha+\beta)\}-\frac{(\alpha+\beta)(1-\mu)}{n}\right]\frac{(a)_{n-1}(b)_{n-1}}{(c)_{n-1}(1)_{n-1}}$$

$$=(A-B)|\tau|\sum_{n=1}^{\infty}\left[\mu(1+\alpha)(n+1)\frac{(a)_n(b)_n}{(c)_n(1)_n}+\{(1-\mu)(1+\alpha)-\mu(\alpha+\beta)\}\frac{(a)_n(b)_n}{(c)_n(1)_n}\right.$$
$$\left.-(\alpha+\beta)(1-\mu)\frac{(a)_n(b)_n}{(c)_n(1)_n(n+1)}\right]$$

$$=(A-B)|\tau|\sum_{n=1}^{\infty}\left[\mu(1+\alpha)\frac{(a)_n(b)_n}{(c)_n(1)_{n-1}}+\mu(1+\alpha)\frac{(a)_n(b)_n}{(c)_n(1)_n}+\{(1-\mu)(1+\alpha)-\mu(\alpha+\beta)\}\right.$$
$$\left.\frac{(a)_n(b)_n}{(c)_n(1)_n}-(\alpha+\beta)(1-\mu)\frac{(a)_n(b)_n}{(c)_n(1)_{n+1}}\right]$$

$$=(A-B)|\tau|\sum_{n=1}^{\infty}\left[\mu(1+\alpha)\frac{ab}{c}\frac{(a+1)_{n-1}(b+1)_{n-1}}{(c+1)_{n-1}(1)_{n-1}}+\{\mu(1+\alpha)+(1-\mu)(1+\alpha)-\mu(\alpha+\beta)\}\right.$$
$$\left.\times\frac{(a)_n(b)_n}{(c)_n(1)_n}+(\alpha+\beta)(\mu-1)\frac{(c-1)}{(a-1)(b-1)}\frac{(a-1)_{n+1}(b-1)_{n+1}}{(c-1)_{n+1}(1)_{n+1}}\right]$$

$$=(A-B)|\tau|\left[\mu(1+\alpha)\frac{ab}{c}\sum_{n=0}^{\infty}\frac{(a+1)_n(b+1)_n}{(c+1)_n(1)_n}+\{1+\alpha-\mu(\alpha+\beta)\}\sum_{n=0}^{\infty}\frac{(a)_n(b)_n}{(c)_n(1)_n}\right.$$
$$\left.-\{1+\alpha-\mu(\alpha+\beta)\}+(\alpha+\beta)(\mu-1)\frac{(c-1)}{(a-1)(b-1)}\left\{\sum_{n=0}^{\infty}\frac{(a-1)_n(b-1)_n}{(c-1)_n(1)_n}-1-\frac{(a-1)(b-1)}{(c-1)}\right\}\right]$$

$$=(A-B)|\tau|\left[\mu(1+\alpha)\frac{ab}{c}\sum_{n=0}^{\infty}\frac{(a+1)_n(b+1)_n}{(c+1)_n(1)_n}+\{1+\alpha-\mu(\alpha+\beta)\}\sum_{n=0}^{\infty}\frac{(a)_n(b)_n}{(c)_n(1)_n}\right.$$
$$\left.-\{1+\alpha-\mu(\alpha+\beta)\}+(\alpha+\beta)(\mu-1)\frac{(c-1)}{(a-1)(b-1)}\left\{\sum_{n=0}^{\infty}\frac{(a-1)_n(b-1)_n}{(c-1)_n(1)_n}-1-\frac{(a-1)(b-1)}{(c-1)}\right\}\right]$$

$$=(A-B)|\tau|\left[\mu(1+\alpha)\frac{ab}{c}F(a+1,b+1;c+1;1)+\{1+\alpha-\mu(\alpha+\beta)\}F(a,b;c;1)\right.$$
$$\left.-\{1+\alpha-\mu(\alpha+\beta)\}+(\alpha+\beta)(\mu-1)\frac{(c-1)}{(a-1)(b-1)}\{F(a-1,b-1;c-1;1)-1-\frac{(a-1)(b-1)}{(c-1)}\}\right]$$

$$=(A-B)|\tau|\left[\mu(1+\alpha)\frac{ab}{c}\frac{\Gamma(c+1))\Gamma(c-a-b-1)}{\Gamma(c-a)\Gamma(c-b)}+\{1+\alpha-\mu(\alpha+\beta)\}\frac{\Gamma c\Gamma(c-a-b)}{\Gamma(c-a)\Gamma(c-b)}\right.$$
$$-\{1+\alpha-\mu(\alpha+\beta)\}+(\alpha+\beta)(\mu-1)\frac{(c-1)}{(a-1)(b-1)}\frac{\Gamma(c-1)\Gamma(c-a-b+1)}{\Gamma(c-a)\Gamma(c-b)}$$
$$\left.-\frac{(\alpha+\beta)(\mu-1)(c-1)}{(a-1)(b-1)}-(\alpha+\beta)(\mu-1)\right]$$

$$=(A-B)|\tau|\frac{\Gamma c\Gamma(c-a-b)}{\Gamma(c-a)\Gamma(c-b)}\left[\frac{\mu(1+\alpha)ab}{(c-a-b-1)}+\{1+\alpha-\mu(\alpha+\beta)\}\right.$$
$$\left.+\frac{(\alpha+\beta)(\mu-1)(c-a-b)}{(a-1)(b-1)}\right]-\frac{(\alpha+\beta)(\mu-1)(c-1)(A-B)|\tau|}{(a-1)(b-1)}-(1-\beta)(A-B)|\tau|.$$

But, this last expression is bounded above by $1-\beta$, if (12) holds. This completes the proof.
If we take $\alpha = 0$ and $\beta = 0$ in Theorem 1, we have the following corollary.

Corollary 1. If $a > 0, b > 0$ and $c > a+b+1$. If $f \in R^{\tau}(A,B)$ and the inequality $(A-B)|\tau|\dfrac{\Gamma c\Gamma(c-a-b)}{\Gamma(c-a)\Gamma(c-b)}\left[1+\dfrac{\mu ab}{(c-a-b-1)}\right] \leq 1+(A-B)|\tau|$ is satisfied.

Then $M_{\mu}(f) \in S^*$, where S^* is the class of starlike functions in S.

If we take $\alpha = 1$ and $\beta = 0$ in Theorem 1, we have

Corollary 2. If $a > 0, b > 0$ and $c > a+b+1$. If $f \in R^{\tau}(A,B)$ and the inequality

$$= \frac{(A-B)|\tau|\Gamma c\Gamma(c-a-b)}{\Gamma(c-a)\Gamma(c-b)}\left[\frac{2\mu ab}{(c-a-b-1)}+(2-\mu)+\frac{(\mu-1)(c-a-b)}{(a-1)(b-1)}\right]$$
$$\leq (A-B)|\tau|\left[\frac{(\mu-1)(c-1)}{(a-1)(b-1)}+1\right]+1$$

is satisfied, then $M_{\mu}(f) \in 1-ST(0)$, a class of starlike functions corresponding to uniformly convex functions.

Theorem 2. Let $a > 0, b > 0$ and $c > a+b+2$. If $f \in R^{\tau}(A,B)$ and the inequality

$$(A-B)|\tau|\left[\mu\frac{(1+\alpha)(a)_2(b)_2}{(c-a-b-2)_2}+(2\mu+\alpha\mu+1+\alpha-\mu\beta)\frac{ab}{(c-a-b-1)}+(1-\beta)\right]$$
$$\frac{\Gamma(c)\Gamma(c-a-b)}{\Gamma(c-a)\Gamma(c-b)} \leq (1-\beta)[1+(A-B)|\tau|]$$

is satisfied, then $M_{\mu}(f) \in \alpha-UCV(\beta)$.

Proof. By Lemma 2, it suffices to show that

$$T_2 \equiv \sum_{n=2}^{\infty} n[n(1+\alpha)-(\alpha+\beta)]|(1-\mu+\mu n)\frac{(a)_{n-1}(b)_{n-1}}{(c)_{n-1}(1)_{n-1}}a_n| \leq 1-\beta.$$

Since $f \in R^{\tau}(A,B)$, then by Lemma 3

$$|a_n| \leq \frac{(A-B)|\tau|}{n}.$$

Hence

$$T_2 \le \sum_{n=2}^{\infty}\left[n+n\alpha-(\alpha+\beta)\right]\left[1-\mu+\mu n\right]\frac{(a)_{n-1}(b)_{n-1}}{(c)_{n-1}(1)_{n-1}}(A-B)|\tau|$$

$$=(A-B)|\tau|\sum_{n=1}^{\infty}\left[(1+\alpha)\mu(n+1)^2+\left[(1-\mu)(1+\alpha)-\mu(\alpha+\beta)\right](n+1)-(\alpha+\beta)(1-\mu)\right]\frac{(a)_n(b)_n}{(c)_n(1)_n}$$

$$=(A-B)|\tau|\sum_{n=1}^{\infty}(1+\alpha)\mu n^2+2(1+\alpha)\mu n+(1+\alpha)\mu+\{(1-\mu)(1+\alpha)-\mu(\alpha+\beta)\}n$$

$$+\{(1-\mu)(1+\alpha)-\mu(\alpha+\beta)-(\alpha+\beta)(1-\mu)\}\times\frac{(a)_n(b)_n}{(c)_n(1)_n}$$

$$=(A-B)|\tau|\sum_{n=1}^{\infty}\left[(\mu+\alpha\mu)n^2+(\mu+1+\alpha-\mu\beta)n+(1-\beta)\right]\times\frac{(a)_n(b)_n}{(c)_n(1)_n}$$

$$=(A-B)|\tau|\sum_{n=1}^{\infty}\left[(\mu+\alpha\mu)n\frac{(a)_n(b)_n}{(c)_n(1)_{n-1}}+(\mu+1+\alpha-\mu\beta)\frac{(a)_n(b)_n}{(c)_n(1)_{n-1}}+(1-\beta)\frac{(a)_n(b)_n}{(c)_n(1)_n}\right]$$

$$=(A-B)|\tau|\sum_{n=1}^{\infty}\left[(\mu+\alpha\mu)(n-1)\frac{(a)_n(b)_n}{(c)_n(1)_{n-1}}+(\mu+\alpha\mu)\frac{(a)_n(b)_n}{(c)_n(1)_{n-1}}+(\mu+1+\alpha-\mu\beta)\right.$$

$$\left.\frac{(a)_n(b)_n}{(c)_n(1)_{n-1}}\right]+(1-\beta)\frac{(a)_n(b)_n}{(c)_n(1)_n}\Bigg]$$

$$=(A-B)|\tau|\sum_{n=1}^{\infty}\left[(\mu+\alpha\mu)\frac{(a)_n(b)_n}{(c)_n(1)_{n-2}}+(2\mu+\alpha\mu+1+\alpha-\mu\beta)\frac{(a)_n(b)_n}{(c)_n(1)_{n-1}}\right.$$

$$\left.+(1-\beta)\frac{(a)_n(b)_n}{(c)_n(1)_n}\right]\times\sum_{n=1}^{\infty}\frac{(a+1)_{n-1}(b+1)_{n-1}}{(c+1)_{n-1}(1)_{n-1}}+(1-\beta)\{\sum_{n=0}^{\infty}\frac{(a)_n(b)_n}{(c)_n(1)_n}-1\}\Bigg]$$

$$=(A-B)|\tau|\left[\mu(1+\alpha)\frac{(a)_2(b)_2}{(c)_2}\sum_{n=0}^{\infty}\frac{(a+2)_n(b+2)_n}{(c+2)_n(1)_n}+(2\mu+\alpha\mu+1+\alpha-\mu\beta)\frac{ab}{c}\right.$$

$$\left.\times\sum_{n=0}^{\infty}\frac{(a+1)_n(b+1)_n}{(c+1)_n(1)_n}+(1-\beta)\sum_{n=0}^{\infty}\frac{(a)_n(b)_n}{(c)_n(1)_n}-(1-\beta)\right]$$

$$=(A-B)|\tau|\left[\mu(1+\alpha)\frac{(a)_2(b)_2}{(c)_2}F(a+2,b+2;c+2;1)+(2\mu+\alpha\mu+1+\alpha-\mu\beta)\frac{ab}{c}\right.$$

$$\left.\times F(a+1,b+1;c+1;1)+(1-\beta)F(a,b;c;1)-(1-\beta)\right]$$

$$=(A-B)|\tau|\left[\mu(1+\alpha)\frac{(a)_2(b)_2}{(c)_2}\frac{\Gamma(c+2)\Gamma(c-a-b-2)}{\Gamma(c-a)\Gamma(c-b)}+(2\mu+\alpha\mu+1+\alpha-\mu\beta)\frac{ab}{c}\right.$$

$$\left.\frac{\Gamma(c+1)\Gamma(c-a-b-1)}{\Gamma(c-a)\Gamma(c-b)}+(1-\beta)\frac{\Gamma(c)\Gamma(c-a-b)}{\Gamma(c-a)\Gamma(c-b)}-(1-\beta)\right]$$

$$=(A-B)|\tau|\left[\mu(1+\alpha)\frac{(a)_2(b)_2}{(c-a-b-2)_2}\frac{\Gamma(c)\Gamma(c-a-b)}{\Gamma(c-a)\Gamma(c-b)}+(2\mu+\alpha\mu+1+\alpha-\mu\beta)\right.$$

$$\left.\frac{ab\Gamma(c)\Gamma(c-a-b)}{(c-a-b-1)\Gamma(c-b)\Gamma c-b}+(1-\beta)\frac{\Gamma(c)\Gamma(c-a-b)}{\Gamma(c-a)\Gamma(c-b)}\right]-(1-\beta)(A-B)|\tau|$$

$$=(A-B)|\tau|\left[\mu(1+\alpha)\frac{(a)_2(b)_2}{(c-a-b-2)_2}+(2\mu+\alpha\mu+1+\alpha-\mu\beta)\right.$$

$$\left.\frac{ab}{(c-a-b-1)}+(1-\beta)\right]\times\frac{\Gamma(c)\Gamma(c-a-b)}{\Gamma(c-a)\Gamma(c-b)}-(1-\beta)(A-B)|\tau|$$

But, this last expression is bounded above by $(1-\beta)$ if (13) holds.
This completes the proof.

If we take $\alpha = 0, \beta = 0$ in Theorem 2, we have the following corollary.

Corollary 3. Let $a > 0, b > 0$ and $c > a + b + 2$. If $f \in R^{\tau}(A, B)$ and the inequality

$$(A-B)|\tau|\left[\frac{\mu(a)_2(b)_2}{(c-a-b-2)_2} + (2\mu+1)\frac{ab}{(c-a-b-1)} + 1\right] \frac{\Gamma(c)\Gamma(c-a-b)}{\Gamma(c-a)\Gamma(c-b)} \leq 1 + (A-B)|\tau|$$

is satisfied. Then $M_\mu(f) \in C$. Where C is the class of convex functions in S.
If we take $\alpha = 1$ and $\beta = 0$ in Theorem 2 we have the following corollary

Corollary 4. Let $a > 0, b > 0$ and $c > a+b+2$. If $f \in R^{\tau}(A, B)$ and the inequality

$$(A-B)|\tau|\left[\frac{\mu(a)_2(b)_2}{(c-a-b-2)_2} + (3\mu+1)\frac{ab}{(c-a-b-1)} + 1\right] \frac{\Gamma(c)\Gamma(c-a-b)}{\Gamma(c-a)\Gamma(c-b)} \leq 1 + (A-B)|\tau|$$

is satisfied. Then $M_\mu(f) \in 1 - UCV(0)$, a class of convex functions.

Theorem 3. Let $a > 1, b > 1$ and $c > a+b+1$. If $f \in R^{\tau}(A, B)$ and the inequality

$$\frac{(A-B)|\tau|\Gamma(c)\Gamma(c-a-b)}{\Gamma(c-a)\Gamma(c-b)}\left[\frac{\mu(\alpha+1)ab}{(c-a-b-1)} + \{(1+\alpha) - \mu(\alpha+\beta)\} + \frac{(\alpha+\beta)(\mu-1)(c-a-b)}{(a-1)(b-1)}\right] \leq (A-B)|\tau|\left[\frac{(\alpha+\beta)(\mu-1)(c-1)}{(a-1)(b-1)} + (1-\beta)\right] + (1-\beta)$$

is satisfied, then $I(z)^* f(z) \in \alpha - UCV(\beta)$.

Proof. Here

$$I(z)^* f(z) = z + \sum_{n=2}^{\infty}(1-\mu+\mu n)\frac{(a)_{n-1}(b)_{n-1}}{(c)_{n-1}(1)_{n-1}}\frac{a_n}{n}z^n$$

By Lemma 2, it suffices to show that

$$T_3 \equiv \sum_{n=2}^{\infty} n[n(1+\alpha) - (\alpha+\beta)]\left[(1-\mu+\mu n)\frac{(a)_{n-1}(b)_{n-1}}{(c)_{n-1}(1)_{n-1}}\frac{a_n}{n}\right] \leq 1-\beta.$$

Since $f \in R^{\tau}(A, B)$, then by Lemma 3, we have $|a_n| \leq \frac{(A-B)|\tau|}{n}$.

Now rest of the proof of Theorem 3 is exactly the same to that of Theorem 1, therefore, we omit the details involved.

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