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CUBIC B-SPLINE COLLOCATION ALGORITHM FOR THE NUMERICAL SOLUTION OF NEWELL WHITEHEAD SEGEL TYPE EQUATIONS

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Abstract. In this paper, numerical solutions of the Newell Whitehead Segel Type equations (NWS) are obtained using different cubic B-spline basis. A linear Von-Neumann stability analysis shows that the numerical scheme is unconditionally stable. Accuracy of the method is discussed by computing the max numerical error. The numerical result shows that the presented method is a successful numerical technique for solving NWS equation.

1. Introduction

The general nonlinear parabolic partial differential equation is of the form

$$u_t = ku_{xx} + au + bu^q + \psi(x, t, u, u_x) \tag{1}$$

where k, a, b and q are real constants, then (1) gives rise to well-known models in fluid dynamics. For $a = -4, b = 4, k = 1$ and $q = 3$, (1) becomes the Allen-Cahn equation [1]. For $b = -a = 1, k = 1$ and $q = 2$, (1) reduces to the well-known Fisher's equation [2]. By setting the values as $b = -1, a = 1, k = 1, q = 2$ and the source term $\psi(x, t, u, u_x) = uu_x$ gives Burgers-fisher equation. Changing the values to $k = 1, a = -\beta, b = 1 + \beta, q = 2$ and $\psi(x, t, u, u_x) = -u^3$ (1) becomes Huxley equation. However, by letting the values to $k = 1, a = -\beta, b = 1 + \beta, q = 2$ and $\psi(x, t, u, u_x) = uu_x - u^3$ leads to Burgers-Huxley equation. Another form by letting $k = 1, a = \alpha, b = -1, q = 2$ and $\psi(x, t, u, u_x) = u^3 - u^2$ gives Nagumo reaction diffusion equation. Last, by replacing the coefficient b with $-b$ and letting $q = 2$ (1) becomes the famous Newell Whitehead Segel Equation. The Newell Whitehead Segel type Equations describes the dynamical behavior near the bifurcation point of the Rayleigh-Benard convection of binary fluid mixtures [3].

The Rayleigh -Benard convection is a type of natural convection, occurring in a plane horizontal layer of fluid heated from below, in which the fluid develops

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a regular pattern of convection cells known as Bernard cells. When the heating is sufficiently intensive, convective motion of the fluid is developed spontaneously then the hot fluid moves upward, and the cold fluid moves downward. Rayleigh Bernard convection is one of the most commonly studied convection phenomena because of its analytical and experimental accessibility. The convection patterns are the most carefully examined example of self-organizing nonlinear systems [4]. Buoyancy and gravity are responsible for the appearance of convection cells. The initial movement is the upwelling of warmer liquid from the heated bottom layer [5]. Fig. (1) and Fig. (2) Shows the Rayleigh -Benard convection phenomenon and the convection cells in a gravity field.

Fig. (1)

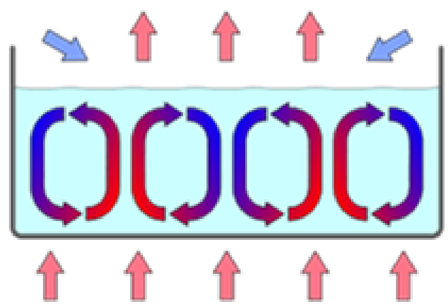


Fig. (2)

There are two kinds of patterns that are observed especially often. The first one is the roll pattern (or stripe pattern) in which the fluid streamlines form cylinders. These cylinders may be bent and they may form spirals or target-like patterns. Another typical pattern is the hexagonal one in which the liquid flow is divided into honeycomb cells. For some fluids, the motion is downward in the center of each cell and upward on the border between the cells; for other fluids, the motion is in the opposite direction. The same patterns, stripes and hexagons, appear in completely different physical systems and on different spatial scales. For instance, stripe patterns are observed in human fingerprints, on Zebra's skin and in the visual cortex. Hexagonal patterns result from the propagation of laser beams through a nonlinear medium and in systems with chemically reacting and diffusing species [6].

Recently a lot of research work has been presented for solving NWS type equations. Ezzati and Shakibi [7] applied the Adomian's Decomposition and multi-quadric quasi-interpolation methods for solving Newell-whitehead equation. Kheiri et. al., [8] Adapted the Homotopy analysis and Homotopy Pade methods to find the numerical solution of NWS equation. Aasaraai [9] considered the differential transformation method for solving this equation with both constant and variable coefficients finding the analytical solution of equation. Macias-Diaz and Ruiz-Ramirez [10], proposed a finite-difference scheme to approximate the solutions of a generalization of the classical NWS equation. Lastly, Nourazar et. al., applied the Homotopy perturbation method for five different cases of the equation [11].

In this paper, we adapt a collocation method based on cubic B-spline functions with different type basis functions to solve (1) such as the trigonometric B-spline that has been presented by I. G. Burova in [13], [14] and [15] that has the same form as in [17] that we will use as our basis functions in the later section. We solve (1) according to the following classes of initial and boundary conditions as follows

Class No (1)

initial condition $u(x,0) = g(x)$

Dirichlet boundary conditions $u(a,t) = g_1(t), u(b,t) = g_2(t)$

Class No (2)

initial condition $u(x,0) = g(x)$

Neumann boundary conditions $u_x(a,t) = g_3(t), u_x(b,t) = g_4(t)$

The method is tested for six different test problems with different boundary conditions. The most important feature of this method is that it is easy to implement to both linear and nonlinear problems with no computational or time effort.

In section 2, we apply the cubic B-spline collocation method to solve the NWS equation where we use uniform, trigonometric and extended cubic B-spline as basis functions. In section 3 we propose a stability analysis using Von-Neumann method showing to be unconditionally stable. Finally, section 4 is the closing stage where we present the numerical results of our method for six test problems.

2. Application of the collocation method

2.1. **Uniform Cubic B-spline (UCBS).** We introduce the cubic spline space and basis functions to construct an interpolant S(x) satisfying certain end conditions and then derive several asymptotic expansions to be used in the formulation of the cubic spline collocation method.

Let $\Delta \equiv \{a = x_0 < x_1 < ..... < x_{N-1} < x_N = b\}$ be a uniform partition of the interval $[a,b]$ with 6 additional knots outside the region, positioned at:

$$x_{-3} < x_{-2} < x_{-1} < x_0,$$
$$x_N < x_{N+1} < x_{N+2} < x_{N+3},$$

and $h = x_i - x_{i-1} = (b-a)/N$ as a step size. Consider a smooth quartic spline $S(x)$ that is an element of $S_3(\Delta) \equiv \{q(x)/q(x) \in C^2[a,b], q(x)$ is a polynomial of degree at most 3 on the partition $\Delta\}$. Consider the B-splines basis in $S_3(\Delta)$ is defined as follows in different ways.

The B-spline basis can be defined as

$$B_i(x) = \frac{1}{6\,h^3}\begin{cases} x^3 & 0 \le x < h \\ -3\,x^3 + 12\,h\,x^2 - 12\,h^2\,x + 4\,h^3, & h \le x < 2\,h \\ 3\,x^3 - 24\,h\,x^2 + 60\,h^2\,x - 44\,h^3, & 2\,h \le x < 3\,h \\ -x^3 + 12\,h\,x^2 - 48\,h^2\,x + 64\,h^3 & 3\,h \le x < 4\,h \end{cases} \tag{2}$$

where

$$B_{i-1}(x) = B_i(x - (i-1)h), i = 2, 3, ... \tag{3}$$

The $B_i(x)$ and their first two derivatives, evaluated at the nodal points are needed. Their coefficients are given in Table 1.

Table 1: coefficients of B_i and its derivatives

x	x_{i-1}	x_i	x_{i+1}
B_i	$\frac{1}{6}$	$\frac{4}{6}$	$\frac{1}{6}$
hB'_i	$\frac{-3}{6}$	0	$\frac{3}{6}$
$h^2B''_i$	1	-2	1

2.2. **Extended cubic uniform B-spline (ECBS).** We introduce the blending function of the extended cubic uniform B-spline with degree 4, $EB_i(x)$ is given by [16] as

$$EB_i(x) = \frac{1}{24\,h^4}\begin{cases} 4h(1-\lambda)z_i^3 + 3\lambda z_i^3, & x \in I_i \\ (4-\lambda)h^4 + 12h^3 z_{i+1} + 6h^2(2+\lambda)z_{i+1}^2 - 12hz_{i+1}^3 - 3\lambda z_{i+1}^4, & x \in I_{i+1} \\ (4-\lambda)h^4 - 12h^3 z_{i+1} + 6h^2(2+\lambda)z_{i+1}^2 + 12hz_{i+1}^3 - 3\lambda z_{i+1}^4, & x \in I_{i+2} \\ 4h(1-\lambda)z_i^3 + 3\lambda z_i^3, & x \in I_{i+3} \end{cases} \tag{4}$$

Where $z_i = x - x_i, I_i \equiv [x_i, x_{i+1}]$, and the parameter λ satisfies $-8 \leq \lambda \leq 1$.

The $EB_i(x)$ and their first two derivatives, evaluated at the nodal points are needed. Their coefficients are given in Table2.

Table 2: coefficients of EB_i and its derivatives

x	x_{i-2}	x_{i-1}	x_i	x_{i+1}	x_{i+2}
EB_i	0	$\frac{4-\lambda}{24}$	$\frac{8+\lambda}{12}$	$\frac{4+\lambda}{24}$	0
EB'_i	0	$\frac{1}{2h}$	0	$\frac{-1}{2h}$	0
EB''_i	0	$\frac{2+\lambda}{2h^2}$	$-\frac{2+\lambda}{h^2}$	$\frac{2+\lambda}{2h^2}$	0

2.3. **Trigonometric cubic B-spline (TCBS).** Let $TS_3(\Delta)$ be the space of cubic trigonometric spline functions over the partition Δ. We can define the cubic trigonometric B-spline functions $TB_i(x)$, for $i = -1, 0, 1, ..., n+1$ for $TS_3(\Delta)$ after including two more points on each side of the partition Δ. Thus the cubic trigonometric B-spline functions is defined as in [17] as

$$TB_i(x) = \frac{1}{\mu(h)}\begin{cases} \sin^3(\frac{x-x_{i-2}}{2}), & x \in [x_{i-2}, x_{i-1}] \\ \sin(\frac{x-x_{i-2}}{2})\left[\sin(\frac{x-x_{i-2}}{2})\sin(\frac{x_i-x}{2}) + \sin(\frac{x_{i+1}-x}{2})\sin(\frac{x-x_{i+1}}{2})\right] + & \\ \sin(\frac{x-x_{i-1}}{2})\sin^2(\frac{x_{i+1}-x}{2}), & x \in [x_{i-1}, x_i] \\ \sin(\frac{x_{i+2}-x}{2})\left[\sin(\frac{x-x_{i-1}}{2})\sin(\frac{x_{i+1}-x}{2}) + \sin(\frac{x_{i+2}-x}{2})\sin(\frac{x-x_i}{2})\right] + & \\ \sin(\frac{x-x_{i+2}}{2})\sin^2(\frac{x_{i+1}-x}{2}), & x \in [x_i, x_{i+1}] \\ \sin^3(\frac{x_{i+2}-x}{2}), & x \in [x_{i+1}, x_{i+2}] \end{cases} \tag{5}$$

Where

$$\mu(h) = sin(\tfrac{h}{2})sin(h)sin(\tfrac{3h}{2})$$

The $TB_i(x)$ and their first two derivatives, shown at the nodal points are needed. Their coefficients are given in Table 3.

Table 3: coefficients of TB_i and its derivatives

x	x_{i-2}	x_{i-1}	x_i	x_{i+1}	x_{i+2}
TB_i	0	$\sin^2(\frac{h}{2})\csc(h)\csc(\frac{3h}{2})$	$\frac{2}{1+2\cos(h)}$	$\sin^2(\frac{h}{2})\csc(h)\csc(\frac{3h}{2})$	0
TB_i'	0	$\frac{3}{4}\csc(\frac{3h}{2})$	0	$\frac{-3}{4}\csc(\frac{3h}{2})$	0
TB_i''	0	$\frac{3(1+3\cos(h))\csc^2(\frac{h}{2})}{13(2\cos\frac{h}{2}+\cos(\frac{3h}{2}))}$	$\frac{-3\cot^2(\frac{h}{2})}{2+4\cos(h)}$	$\frac{3(1+3\cos(h))\csc^2(\frac{h}{2})}{13(2\cos\frac{h}{2}+\cos(\frac{3h}{2}))}$	0

3. Implementation of the method

The approximate solution U to the exact solution $u(x,t)$ will be sought in form of an expansion of uniform B-splines

$$U_m(x) = \sum_{m=-1}^{N+1} \delta_m B_m(x), \tag{6}$$

where δ_m are unknown parameters to be determined using cubic B-spline collocation form of (6). The nodal values $U_m, U_m^{'}$ and $(U_m^{''})$ at the knots x_m are derived from expression (6) and Table 1 in the following form

$$U(x_m) = \frac{1}{6}(\delta_{m+1} + 4\delta_m + \delta_{m-1}), \tag{7}$$

$$U^{'}(x_m) = \frac{3}{6h}(\delta_{m+1} - \delta_{m-1}), \tag{8}$$

$$U^{''}(x_m) = \frac{1}{h^2}(\delta_{m+1} - 2\delta_m + \delta_{m-1}), \tag{9}$$

These knots are used as collocation points. We discretize the time derivative by a difference formula to the space derivative

$$U_t(x,t_n) = \frac{(U(x,t_{n+1}) - U(x,t_n))}{\tau}. \tag{10}$$

Substitute (10) into (1), yields

$$U(x,t_{n+1}) = U(x,t_n) + \tau k U_{xx}(x,t_n) + a\tau U(x,t_n) + b\tau(U(x,t_n))^q + \tau\psi(x,t_n,u^n,u_x^n) \tag{11}$$

where $\tau = \Delta t$ is the time step and the superscripts n and $n+1$ are successive time levels. Hence (11) can be rearranged as

$$U(x,t_{n+1}) = \tau k U_{xx}(x,t_n) + b\tau(U(x,t_n))^q + (1+a\tau)U(x,t_n) + \tau\psi(x,t_n,u^n,u_x^n) \tag{12}$$

substituting (6) into (12) for time step t_n

$$(\sum_{m=-1}^{N+1}(\delta_m^{n+1}B_m(x)) = \sum_{m=-1}^{N+1}(\delta_m^n B_m''(x))) + (1+a\tau)(\sum_{m=-1}^{N+1}(\delta_m^n B_m(x))$$
$$+ b\tau((\sum_{m=-1}^{N+1}(\delta_m^n B_m(x)))^q + \psi(x_m,t_n,u^n,u_x^n) \tag{13}$$

Then putting the values of the nodal values U and its derivatives using (7) to (9) at the knots in (13) yields the following difference equation with the variables δ.

$$\delta_{m+1}^{n+1}+4\delta_{m}^{n+1}+\delta_{m-1}^{n+1}=a^{'}\delta_{m+1}^{n}+b^{'}\delta_{m}^{n}+c^{'}\delta_{m-1}^{n}+\tau\psi(x_m,t_n,u^n,u_x^n) \tag{14}$$

Where $m=0,1,......,N$

$$a^{'}=c^{'}=X+Y+Z, \qquad b^{'}=4X-2Y+(4^q)Z$$

$$X=a\tau+1, \qquad Y=\frac{6\tau}{h^2}, \qquad Z=b\tau(1)^q=b\tau$$

The system (14) results in $(N+1)$ linear equations in $(N+3)$ unknowns. To solve this system two additional constrains are required. These constrains are obtained from the boundary conditions. Applying the boundary conditions enables us to add the parameters δ_{-1},δ_{N+1} in the system and then the system can be reduced to $(N+3)\times(N+3)$ solvable system diven by

$$\mathbf{A}\delta^{n+1}=\mathbf{B}\delta^{n}+\mathbf{C} \tag{15}$$

Where $\delta=(\delta_{-1},\delta_0,......,\delta_{N+1})^T$ and the matrix $\mathbf{A},\mathbf{B}$ and $\mathbf{C}$ is given by

$$\mathbf{B}=\begin{pmatrix} \frac{1}{6} & \frac{4}{6} & \frac{1}{6} & 0 & 0 & 0 & 0 & 0 & 0 \\ a^{'} & b^{'} & c^{'} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & a^{'} & b^{'} & c^{'} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & a^{'} & b^{'} & c^{'} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \ddots & \ddots & \ddots & \ddots & \ddots & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & a^{'} & b^{'} & c^{'} \\ 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{6} & \frac{4}{6} & \frac{1}{6} \end{pmatrix},$$

$$\mathbf{A}=\begin{pmatrix} 1 & 4 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 4 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 4 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 4 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & \ddots & \ddots & \ddots & \ddots & \ddots & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 4 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 4 & 1 \end{pmatrix},$$

And

$$\mathbf{C}=[g_1(t_n),\psi(x_0,t_n,u^n,u_x^n),\psi(x_1,t_n,u^n,u_x^n),\cdots\cdots,\psi(x_{m+1},t_n,u^n,u_x^n),g_2(t_n)]^T$$

where $\psi(x,t,u,u_x)$ denotes the right hand side of the equation after calculating the initial state from the boundary conditions. All calculations were carried out using Matlab 7 (R 2008 a). The same procedure is carried out using the trigonometric B-spline and the extended B-spline in the same sense of the uniform B-spline as stated in the previous section by changing the basis used.

4. Stability analysis

In this section we will investigate the stability analysis by applying Von-Neumann stability analysis. For sake of simplicity we let $\psi(x,t,u,u_x)=0$. We substitute

$\delta_m^n = \epsilon_n e^{i\beta h}$ into linearized form of (14)where β is the mode number, h is size element and i=$\sqrt{-1}$ we obtain,

$$\epsilon^n(a^{'}e^{i(m+1)\phi} + b^{'}e^{i(m)\phi} + c^{'}e^{i(m-1)\phi}) = \epsilon^{n+1}(e^{i(m+1)\phi} + 4e^{i(m)\phi} + e^{i(m-1)\phi})$$

Here we have taken, $\beta h = \phi$, dividing both sides by $e^{i(m)\phi}$ we get

$$\epsilon^n(a^{'}e^{i\phi} + b^{'} + c^{'}e^{-i\phi}) = \epsilon^{n+1}(e^{i\phi} + 4 + e^{-i\phi})$$

$$\epsilon = \frac{(a^{'}e^{i\phi} + b^{'} + c^{'}e^{-i\phi})}{(e^{i\phi} + 4 + e^{-i\phi})} \tag{16}$$

and by simplifying (16), we get

$$\epsilon = \frac{\hat{A}}{\hat{C}}$$

Where $\hat{A}, \hat{C}$ can be put in the form

$$\hat{A} = (a' + c')\cos\phi + b', \quad \hat{C} = 2\cos\phi + 4 \tag{17}$$

after simplifying and substitution we see that

$$\hat{A}^2 - \hat{C}^2 = -24a^2\tau^2 - 72a\tau - 24ab\tau^2 - 12ab\tau^2 4^q - 24b\tau - 12b\tau 4^q - 4b^2\tau^2$$
$$-2^{(2+2q)}b^2\tau^2 - 16^q b^2\tau^2 \leq 1$$

Which implies that the scheme is unconditionally stable.

5. Test problems

Numerical method described in the previous section is tested on six test problems from [9], [11] and [12] for different values of coefficients for getting solutions of the NWS equation in order to demonstrate the robustness and numerical accuracy. The L_∞ error norm

$$L_\infty = |U - U_N|_\infty = max_j |U_j - (U_N)_j^n|$$

is used to measure error between the exact and numerical solutions.

5.1. **Problem (1).** In (1) $a = 2, b = -3, q = 2, k = 1$ and $\psi(x, t, u, u_x) = 0$ then the NWS equation is written with class no (1) of boundary conditions as in [9]

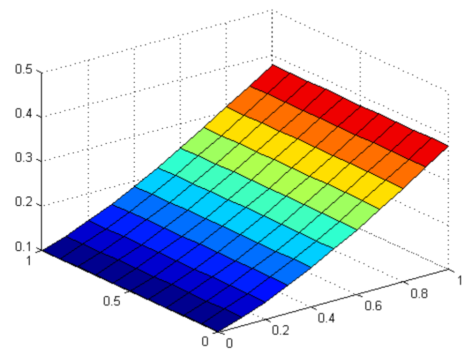
$$u_t - u_{xx} - 2u + 3u^2 = 0,$$

$$u(x, 0) = \lambda, \quad u(0, t) = \frac{(-2\lambda e^{2t})}{(-2 + 3\lambda(1 - e^{2t}))},$$

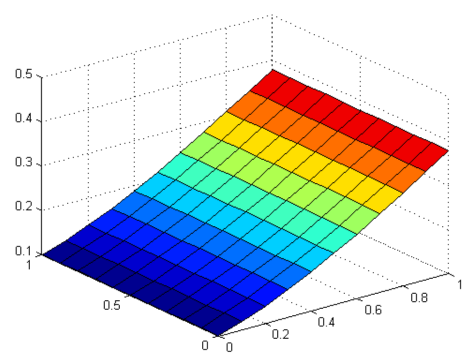
$$u(1, t) = \frac{(-2\lambda e^{2t})}{(-2 + 3\lambda(1 - e^{2t}))}, \qquad u(x, t) = \frac{(-2\lambda e^{2t})}{(-2 + 3\lambda(1 - e^{2t}))}$$

Table 4: Absolute maximum error for $0 \leq x \leq 1$,$0 \leq t \leq 1$ and λ=0.1

x/t	Method	0.2	0.4	0.6	0.8	1.0
0.2	UCBS	8.323E-04	1.011E-03	8.581E-04	4.745E-04	1.991E-05
	TCBS	8.295E-04	1.007E-03	8.514E-04	4.651E-04	3.239E-05
	ECBS	6.068E-04	6.800E-04	5.476E-04	2.746E-04	6.339E-05
0.4	UCBS	1.226E-03	1.520E-03	1.302E-03	7.334E-04	4.932E-06
	TCBS	1.222E-03	1.514E-03	1.292E-03	7.194E-04	2.342E-05
	ECBS	9.013E-04	1.023E-03	8.289E-04	4.221E-04	8.366E-05
0.6	UCBS	1.226E-03	1.520E-03	1.302E-03	7.334E-04	4.932E-06
	TCBS	1.222E-03	1.514E-03	1.292E-03	7.194E-04	2.342E-05
	ECBS	9.013E-04	1.023E-03	8.289E-04	4.221E-04	8.366E-05
0.8	UCBS	8.323E-04	1.011E-03	8.581E-04	4.745E-04	1.991E-05
	TCBS	8.295E-04	1.007E-03	8.514E-04	4.651E-04	3.239E-05
	ECBS	6.068E-04	6.800E-04	5.476E-04	2.746E-04	6.339E-05



(a)



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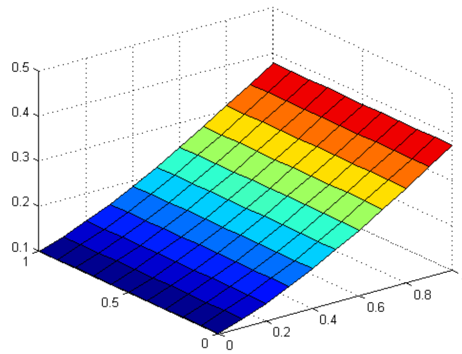

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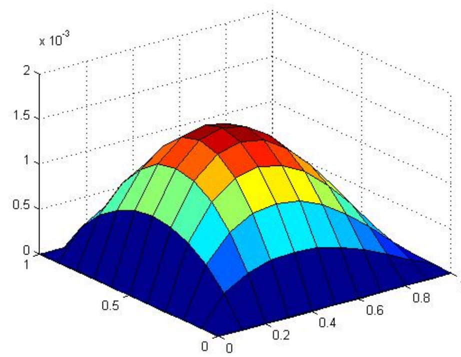


(d)

Fig. 3. Approximate solution (a) Uniform, (b) Trigonometric, (c) Extended, (d) Error (UCBS)

5.2. **Problem(2).** In (1) $a = -2, b = 0, q = 1, k = 1$ and $\psi(x, t, u, u_x) = 0$ then the NWS equation is written with class no (1) of boundary conditions as in [11]

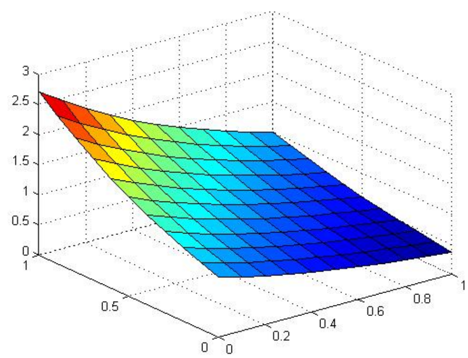
$$u_t - u_{xx} + 2u = 0,$$

$$u(x, 0) = e^x, u(0, t) = e^{-t},$$

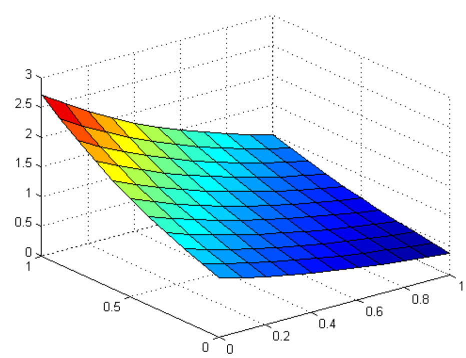
$$u(1, t) = e^{(1-t)}, u(x, t) = e^{(x-t)}$$

Table 5: Absolute maximum error for $0\leq x \leq 1$,$0\leq t \leq 1$

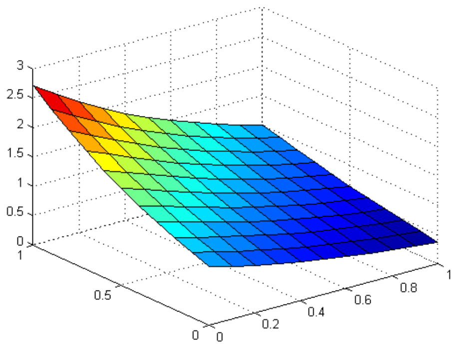
x/t	Method	0.2	0.4	0.6	0.8	1.0
0.2	UCBS	3.426E-03	3.544E-03	3.058E-03	2.536E-03	2.080E-03
	TCBS	3.258E-03	3.372E-03	2.910E-03	2.413E-03	1.982E-03
	ECBS	3.258E-03	3.372E-03	2.910E-03	2.413E-03	1.982E-03
0.4	UCBS	5.426E-03	5.651E-03	4.879E-03	4.048E-03	3.325E-03
	TCBS	5.160E-03	5.376E-03	4.643E-03	3.851E-03	3.164E-03
	ECBS	5.160E-03	5.376E-03	4.643E-03	3.851E-03	3.164E-03
0.6	UCBS	5.940E-03	6.094E-03	5.243E-03	4.345E-03	3.569E-03
	TCBS	5.649E-03	5.798E-03	4.989E-03	4.135E-03	3.396E-03
	ECBS	5.649E-03	5.798E-03	4.989E-03	4.135E-03	3.396E-03
0.8	UCBS	4.474E-03	4.438E-03	3.791E-03	3.136E-03	2.575E-03
	TCBS	4.255E-03	4.223E-03	3.607E-03	2.985E-03	2.450E-03
	ECBS	4.255E-03	4.223E-03	3.607E-03	2.985E-03	2.450E-03



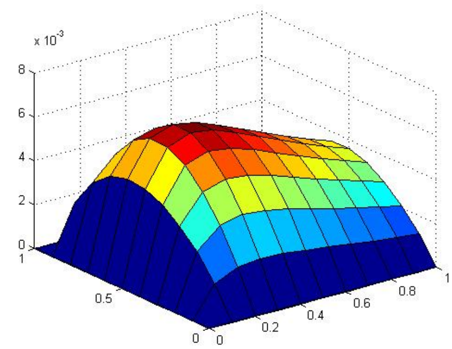
(a)



(b)



(c)



(d)

Fig. 4. Approximate solution (a) Uniform, (b) Trigonometric, (c) Extended, (d) Error (UCBS)

5.3. **Problem(3).** In (1) $a = 1, b = -1, q = 2, k = 1$ and $\psi(x, t, u, u_x) = 0$ then the NWS equation is written with class no (1) of boundary conditions as in [11]

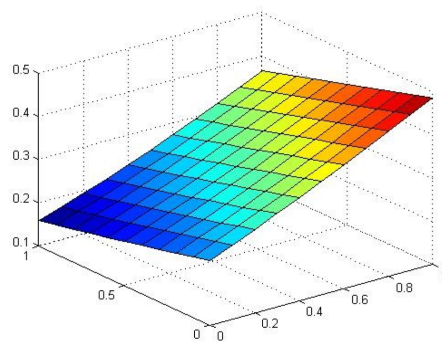
$$u_t - u_{xx} - u + u^2 = 0$$

$$u(x, 0) = (1 - e^{(\frac{x}{\sqrt{6}})})^{-2},$$

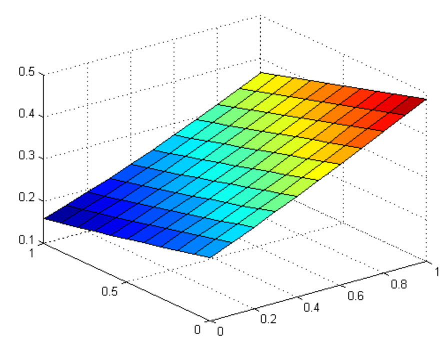
$$u(0, t) = (1+e^{(-\frac{5}{6t})})^{-2} \qquad , u(1, t) = (1+e^{(\frac{1}{\sqrt{6}}-\frac{5}{6t})})^{-2} \qquad , u(x, t) = (1+e^{(\frac{x}{\sqrt{6}}-\frac{5}{6t})})^{-2}$$

Table 6: Absolute maximum error for $0\leq x \leq 1$,$0\leq t \leq 1$

x/t	Method	0.2	0.4	0.6	0.8	1.0
0.2	UCBS	2.810E-04	3.030E-04	2.480E-04	1.660E-04	7.410E-05
	TCBS	2.716E-04	3.249E-04	2.341E-04	1.506E-04	5.793E-05
	ECBS	6.909E-04	6.760E-04	5.184E-04	3.196E-04	1.094E-04
0.4	UCBS	4.270E-04	4.740E-04	3.970E-04	2.760E-04	1.390E-04
	TCBS	4.126E-04	4.551E-04	3.764E-04	2.536E-04	1.148E-04
	ECBS	1.056E-03	1.059E-03	8.328E-04	5.384E-04	2.226E-04
0.6	UCBS	4.380E-04	4.910E-04	4.180E-04	2.990E-04	1.620E-04
	TCBS	4.247E-04	4.723E-04	3.972E-04	2.765E-04	1.381E-04
	ECBS	1.085E-03	1.099E-03	8.799E-04	5.897E-04	2.744E-04
0.8	UCBS	3.050E-04	3.370E-04	2.890E-04	2.110E-04	1.200E-04
	TCBS	2.959E-04	2.905E-04	2.755E-04	1.962E-04	1.044E-04
	ECBS	7.482E-04	7.547E-04	6.123E-04	4.219E-04	2.127E-04



(a)



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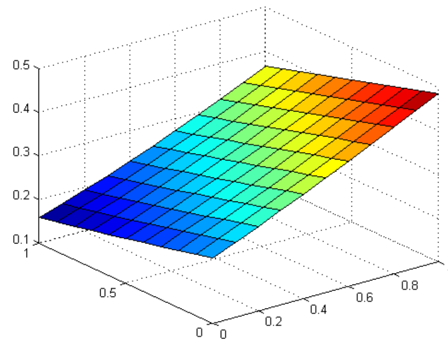

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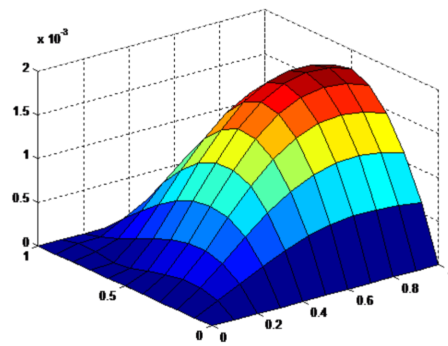


(d)

Fig. 5. Approximate solution (a) Uniform, (b) Trigonometric, (c) Extended, (d) Error (UCBS)

5.4. **Problem(4).** In (1) $a = 1, b = -1, q = 4, k = 1$ and $\psi(x, t, u, u_x) = 0$ then the NWS equation is written with class no (1) of boundary conditions as in [11]

$$u_t - u_{xx} - u + u^4 = 0$$

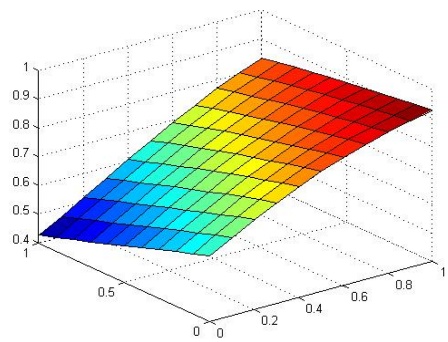
$$u(x, 0) = (1 - e^{\frac{3x}{\sqrt{10}}})^{\frac{-2}{3}},$$

$$u(0, t) = (\frac{1}{2}\tanh((\frac{21}{20t^2}) + \frac{1}{2}))^{\frac{2}{3}}, \quad u(1, t) = (\frac{1}{2}\tanh(\frac{-3}{2\sqrt{10}}(1 - \frac{7}{\sqrt{10}}t) + \frac{1}{2}))^{\frac{2}{3}},$$

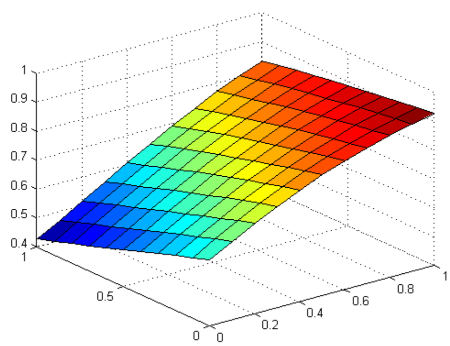
$$u(x, t) = (\frac{1}{2}\tanh(\frac{-3}{2\sqrt{10}}(x - \frac{7}{\sqrt{10}}t) + \frac{1}{2}))^{\frac{2}{3}}$$

Table 7: Absolute maximum error for $0\leq x \leq 1$,$0\leq t \leq 1$

x/t	Method	0.2	0.4	0.6	0.8	1.0
0.2	UCBS	3.800E-04	8.190E-04	1.112E-03	1.184E-03	1.080E-03
	TCBS	3.951E-04	8.362E-04	1.130E-03	1.204E-03	1.103E-03
	ECBS	9.673E-04	1.900E-03	2.446E-03	2.519E-03	2.251E-03
0.4	UCBS	4.230E-04	1.111E-03	1.613E-03	1.777E-03	1.658E-03
	TCBS	4.448E-04	1.137E-03	1.640E-03	1.806E-03	1.688E-03
	ECBS	1.159E-03	2.641E-03	3.587E-03	3.805E-03	3.462E-03
0.6	UCBS	2.890E-04	1.008E-03	1.569E-03	1.789E-03	1.703E-03
	TCBS	3.111E-04	1.034E-03	1.596E-03	1.818E-03	1.734E-03
	ECBS	8.863E-04	2.444E-03	3.512E-03	3.838E-03	3.558E-03
0.8	UCBS	1.120E-04	6.120E-04	1.024E-03	1.206E-03	1.172E-03
	TCBS	1.270E-04	6.300E-04	1.042E-03	1.225E-03	1.192E-03
	ECBS	4.224E-04	1.505E-03	2.293E-03	2.583E-03	2.441E-03



(a)



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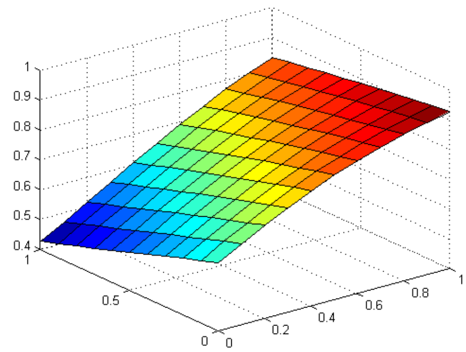

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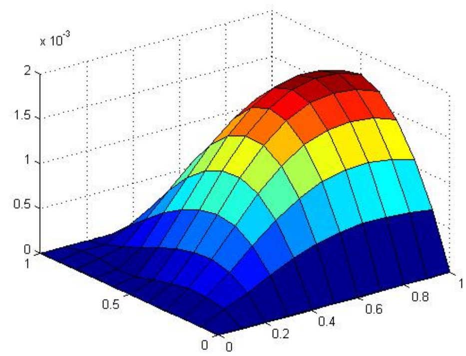


(d)

Fig. 6. Approximate solution (a) Uniform, (b) Trigonometric, (c) Extended, (d) Error (UCBS)

5.5. **Problem(5).** In (1) $a = 3, b = -4, q = 3, k = 1$ and $\psi(x, t, u, u_x) = 0$ then the NWS equation is written with class no (1) of boundary conditions as in [11]

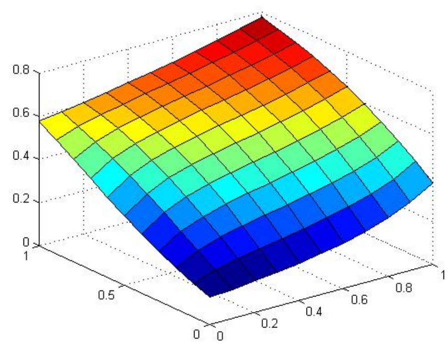
$$u_t - u_{xx} - 3u + 4u^3 = 0$$

$$u(x, 0) = \sqrt{\frac{3}{4}}\left(\frac{e^{\sqrt{6x}}}{e^{\sqrt{6x}} + e^{(\frac{\sqrt{6x}}{2})}}\right)$$

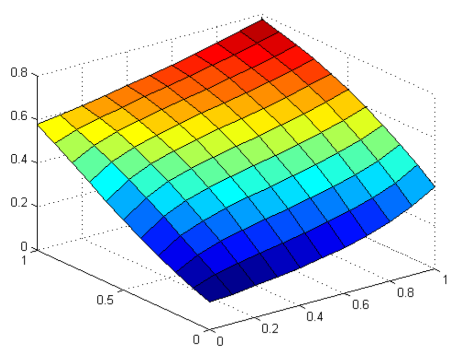
$$u(0, t) = \sqrt{\frac{3}{4}}\left(\frac{1}{1 + e^{(1-\frac{9t}{2})}}\right), \quad u(1, t) = \sqrt{\frac{3}{4}}\left(\frac{e^{\sqrt{6}}}{e^{\sqrt{6}} + e^{(\frac{\sqrt{6}}{2}-\frac{9t}{2})}}\right), \quad u(x, t) = \sqrt{\frac{3}{4}}\left(\frac{e^{\sqrt{6x}}}{e^{\sqrt{6x}} + e^{(\frac{\sqrt{6x}}{2}-\frac{9t}{2})}}\right)$$

Table 8: Absolute maximum error for $0 \leq x \leq 1$,$0 \leq t \leq 1$

x/t	Method	0.2	0.4	0.6	0.8	1.0
0.2	UCBS	6.129E-02	7.120E-02	4.312E-02	4.661E-02	1.560E-02
	TCBS	6.134E-02	7.129E-02	6.430E-02	4.670E-02	1.563E-02
	ECBS	5.166E-02	5.402E-02	4.511E-02	2.860E-02	1.518E-03
0.4	UCBS	8.711E-02	1.005E-01	8.771E-02	6.001E-02	1.640E-02
	TCBS	8.718E-02	1.006E-01	8.786E-02	6.015E-02	1.650E-02
	ECBS	7.224E-02	7.421E-02	5.888E-02	3.321E-02	4.323E-03
0.6	UCBS	8.030E-02	9.072E-02	7.637E-02	4.899E-02	9.878E-03
	TCBS	8.036E-02	9.085E-02	7.651E-02	4.911E-02	9.969E-03
	ECBS	6.461E-02	6.456E-02	4.822E-02	2.328E-02	9.679E-03
0.8	UCBS	4.857E-02	5.319E-02	6.420E-02	2.582E-02	2.862E-03
	TCBS	4.862E-02	5.328E-02	4.321E-02	2.590E-02	2.915E-03
	ECBS	3.721E-02	3.588E-02	2.500E-02	9.644E-03	9.118E-03



(a)



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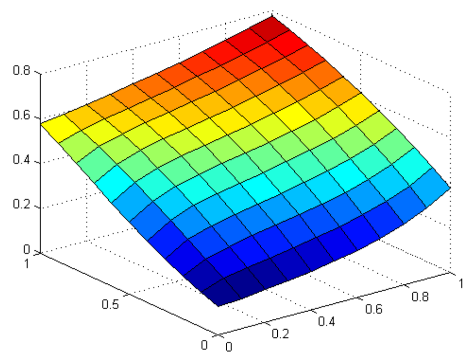

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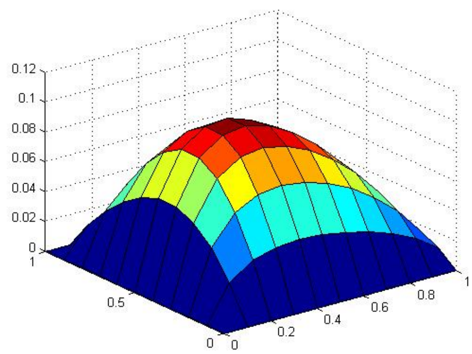


(d)

Fig. 7. Approximate solution (a) Uniform, (b) Trigonometric, (c) Extended, (d) Error (UCBS)

5.6. **Problem(6).** In (1) setting $k = 1$ and $\psi(x,t,u,u_x) = u(\alpha-1)(1-u)$ then the equation becomes the famous Nagumo reaction diffusion equation in [12] under class no (2) of boundary conditions as

$$u_t - u_{xx} - u(\alpha - u)(1-u) = 0$$

subject to initial conditions $u(x,0) = \rho$

And boundary conditions

$$u_x(0,t) = 0, u_x(1,t) = 0,$$

Where matrix **A** is the same and **B** and **C** in (15)becomes

$$\mathbf{B}=\begin{pmatrix} \frac{-3}{6h} & 0 & \frac{3}{6h} & 0 & 0 & 0 & 0 & 0 & 0 \\ d^{'} & e^{'} & d^{'} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & d^{'} & e^{'} & d^{'} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & d^{'} & e^{'} & d^{'} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \ddots & \ddots & \ddots & \ddots & \ddots & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & d^{'} & e^{'} & d^{'} \\ 0 & 0 & 0 & 0 & 0 & 0 & \frac{-3}{6h} & 0 & \frac{3}{6h} \end{pmatrix},$$

Where

$$d' = 1 + X - Y, e' = 4 + 4X - 16Y$$
$$X = \tau(1+\alpha), \qquad Y = \frac{\tau}{3}$$

And

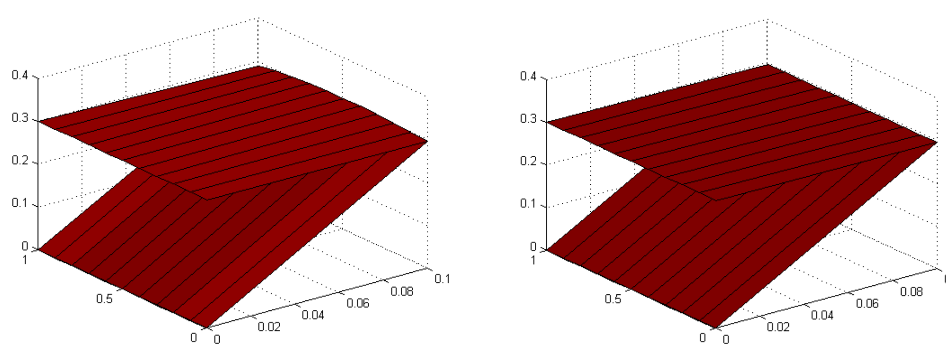
$$\mathbf{C} = [g_1(t_n), 0, 0, \cdots\cdots, 0, g_2(t_n)]^T$$

Solving the above system gives the solution of the equation. Table(9) gives the approximate solution of the equation compared to the solution in [12]. The analytical solution is given in [12]by

$$u(x,t) = 0.3 - 0.04201052049t + 0.002794271318t^2 + 0.0001237074665t^3 + (6.498086711(10^{-10})t + 2.063100590(10^{-9})t^2 - 1.502107790(10^{-9})t^3)x^2(l-x)^2 + (5.530644662(10^{-9})t - 1.727330167(10^{-8})t^2 + 1.237824484(10^{-8})t^3)x^3(l-x)^3$$

Table 9: Approximate solution for 0≤x≤1 ,0≤t≤0.1 using uniform cubic B-spline with$\alpha = 0.5, l = 1, \rho = 0.3$

x/t	0.02		0.04		0.06		0.08	
	UCBS	Ref. [12]	UCBS	Ref. [12]	UCBS	Ref. [12]	UCBS	Ref. [12]
0	0.299161	0.302659	0.298324	0.30534	0.297489	0.308053	0.296657	0.310788
0.1	0.299161	0.302659	0.298324	0.30534	0.297489	0.308053	0.296657	0.310788
0.2	0.299161	0.302659	0.298324	0.30534	0.297489	0.308053	0.296657	0.310788
0.3	0.299161	0.302659	0.298324	0.30534	0.297489	0.308053	0.296657	0.310788
0.4	0.299161	0.302659	0.298324	0.30534	0.297489	0.308053	0.296657	0.310788
0.5	0.299161	0.302659	0.298324	0.30534	0.297489	0.308053	0.296657	0.310788
0.6	0.299161	0.302659	0.298324	0.30534	0.297489	0.308053	0.296657	0.310788
0.7	0.299161	0.302659	0.298324	0.30534	0.297489	0.308053	0.296657	0.310788
0.8	0.299161	0.302659	0.298324	0.30534	0.297489	0.308053	0.296657	0.310788
0.9	0.299161	0.302659	0.298324	0.30534	0.297489	0.308053	0.296657	0.310788
1.0	0.299161	0.302659	0.298324	0.30534	0.297489	0.308053	0.296657	0.310788



(a) (b)

Fig. 8. (a) Our Method, (b) Ref. [12]

As seen from the results of the previous problems, that UCBS method and TCBS method are in good agreement with each other sense they have the same value of the maximum error where on the other hand the ECBS method has a bigger error at some nodal points of the solution domain. Even by changing the value of λ it does not affect the accuracy of the solution over the domain proving that the ECBS method is not a good technique for solving such problems unlike the two other methods.

6. Conclusion

In this paper, we presented a numerical scheme for solving the NWS equation. The method employed to find the solution of this equation is based on the B-spline functions of different types. The methods applied to several test problems from the literature to equations with constant coefficients. The computational results are found to be in good agreement with the exact solutions. Applying von-Neumann stability analysis proves the scheme to be unconditionally stable. The proposed method can be used to solve a large class of both linear and nonlinear partial differential equations with no computational effort.

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