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EXISTENCE OF MULTIPLE SOLUTIONS FOR A HOLLING II TYPE FUNCTIONAL RESPONSE SYSTEM

LIANG DING, JINLONG WEI, BIN ZHANG

Abstract. Applying the Mawhin continuation theorem, the homotopy invariance of topological degree and some construction skills, we obtain the existence of multiple positive periodic solutions for our new Holling II type functional response system. Moreover, an interesting result is discovered: when the male prey species population density is a constant, the existence of multiple positive periodic solutions is independent of $\alpha(t)$ in some special domain $\Omega_i, (i \in A$, in Section 2). At last, we take an example illustrate our interesting results.

1. Introduction

In recent years, the existence of periodic solutions for predatory-prey system has been widely investigated by many researchers. For example, in [1], Wu considers a prey-predator model with sex-structure:

$$\begin{cases} \dot{x}(t) = \beta(t)x(t) - k(t)\big(x(t)+y(t)\big)x(t) - \xi_1(t)x(t)z(t), \\ \dot{y}(t) = \gamma_1(t)x(t) - d_1(t)y(t) - k(t)\big(x(t)+y(t)\big)y(t) - \xi_1(t)y(t)z(t), \\ \dot{z}(t) = z(t)\big(-d(t) - \gamma_2(t)z(t) + \xi_2(t)y(t) + \xi_2(t)x(t)\big), \end{cases} \tag{1.1}$$

where $x(t)$ and $y(t)$ represent female and male prey species population densities respectively, $z(t)$ stands for predator species population density. All the parameters in system (1.1), are positive continuous ω-periodic functions, with $\omega > 0$. Then by virtue of coincidence degree theory, the sufficient conditions, which are depended on $\xi_2(t)$, are gained for guaranteeing the existence of at least one positive periodic solution.

Besides, Zhang and Hou [2] discuss the following Holling II type functional response system with harvesting terms:

$$\begin{cases} \dot{s}(t) = s(t)(a(t) - b(t)s(t) - \frac{c(t)y(t)}{m(t)z(t)+s(t)}) - h_1(t), \\ \dot{z}(t) = z(t)(-d(t) + \frac{f(t)s(t)}{m(t)z(t)+s(t)}) - h_2(t), \end{cases}$$

where $s(t)$ stands for prey species population density, $z(t)$ is predator species population density, and $h_i(t)$ $(i = 1, 2)$ is a harvesting term. Using coincidence degree

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theory, the existence of four positive periodic solutions was obtained. For more details in this direction, one can consult to [3-7] and the references cited up there.

In this paper, we consider the following Holling II type response function model with sex-structure and new multiple harvesting terms:

$$\begin{cases} \dot{x}(t) = x(t)\big[\frac{a(t)}{2} - b_1(t)x(t) - \frac{c(t)z(t)}{m(t)z(t)+\alpha(t)x(t)+(1-\alpha(t))y(t)}\big] - h_1, \\ \dot{y}(t) = y(t)\big[-b_2(t)y(t) - \frac{c_1(t)z(t)}{m(t)z(t)+\alpha(t)x(t)+(1-\alpha(t))y(t)}\big] + \frac{a(t)}{2}x - h_2 \equiv 0, \\ \dot{z}(t) = z(t)[-d(t) + \frac{f(t)\alpha(t)x(t)+(1-\alpha(t))y(t)f(t)}{m(t)z(t)+\alpha(t)x(t)+(1-\alpha(t))y(t)}] - h_3, \end{cases} \quad (1.2)$$

where $h_1 = h_1(t, x, \frac{x}{y}), h_2 = h_2(t, y, \frac{x}{y}), h_3 = h_3(t, z)$, and all the parameters in (1.2) are positive continuous ω-periodic functions.

Predator eats both male and female prey species, and at different times and/or at different locations, the male and female prey species may be eaten differently. So we investigate the new Holling II type functional response system (1.2) involving a ratio parameter $\alpha(t)$ with $\alpha(t) \in [0, 1]$, and now the food of predator $z(t)$ contains two parts: $\alpha(t)x(t)$ and $(1 - \alpha(t))y(t)$.

Note that when x is very big, then h increases and vice versa. So the harvesting term h should be x-dependent and it motivates us to substitute $h(t, x)$ for $h(t)$, which has been studied by many scholars in the past few years. Moreover, we discover that when sex-ratio $\frac{x(t)}{y(t)}$ is large enough, one harvests female prey species more and vice versa. This stipulates us to study the gender ratio of prey species and replace $h(t, x)$ by $h(t, x, \frac{x}{y})$.

In model (1.2), we assume that the males have the same birth rate as females, and both of them are $\frac{1}{2}$. By using the Mawhin continuation theorem ([8]) and the homotopy invariance of topological degree, we discuss the existence of multiple positive periodic solutions for a improved Holling II type functional response system. In this paper, the definitions of $N(u, \lambda)$ and the operator $G(u)$ are quite different from the above mentioned papers, and an interesting result is revealed as well: when x is a constant, no matter what $\alpha(t)$ takes on, (1.2) always has at least four positive periodic solutions in Ω_i $(i \in A)$, with $A = \{1, 2, 3, 4\}$.

2. Main results

Firstly, let us review some notions and make some preparations.

Let X and Z be Banach spaces, that $N : X \times [0, 1] \to Z$ be a continuous mapping, that L: Dom$L \subset X \to Z$ be a linear mapping, which is called to be a Fredholm mapping of index zero if dim Ker$L$ = codimIm$L < \infty$ and ImL is closed in Z.

If L is a Fredholm mapping of index zero, then there exists continuous projectors $P : X \to X$ and $Q : Z \to Z$ such that ImP = Ker$L$ and Ker$Q$ = ImL = Im$(I - Q)$, and $X = \mathrm{Ker}L \bigoplus \mathrm{Ker}P, Z = \mathrm{Im}L \bigoplus \mathrm{Im}Q$. Therefore, $L|_{\mathrm{Dom}L \bigcap \mathrm{Ker}P} : (I - P)X \to \mathrm{Im}L$ is invertible and its inverse is denoted by K_P.

Let $\Omega \subset X$ be bounded and open, then N is called L-compact on $\bar{\Omega} \times [0, 1]$, if $QN(\bar{\Omega} \times [0, 1]$ is bounded and $K_P(I - Q)N : \bar{\Omega} \times [0, 1] \to X$ is compact.

ForImQ is isomorphic to Ker$L$, there exists an isomorphism $J : \mathrm{Im}Q \to \mathrm{Ker}L$.

Besides, we need Mawhin's continuous theorem below.

Lemma 2.1. ([8, p_{40}]) Let L be a Fredholm mapping of index zero, that N be L-compact on $\bar{\Omega}\times$ $[0, 1]$. Assume

(a) for each $\lambda \in (0,1), x \in \partial\Omega \cap \mathrm{Dom}\, L, Lx \neq \lambda N(x,\lambda)$;
(b) for each $x \in \partial\Omega \cap \mathrm{Ker}L,\ QN(x,0) \neq 0$;
(c) $\deg(JQN(x,0), \Omega \cap \mathrm{Ker}L, 0) \neq 0$.

Then $Lx = Nx$ has at least one solution in $\overline{\Omega} \cap \mathrm{Dom}L$.

For the sake of convenience, we introduce notations as follows:

$$a^L := \min_{t\in[0,\omega]} a(t),\ a^M := \max_{t\in[0,\omega]} a(t),\ h_1^L := \min_{t\in[0,\omega]} h_1(t,x\frac{x}{y}),\ h_1^M := \max_{t\in[0,\omega]} h_1(t,x,\frac{x}{y}),$$

$$l^{\pm} := \frac{\frac{a^M}{2} \pm \sqrt{\frac{(a^M)^2}{4} - 4b_1^L h_1^L}}{2b_1^L}, \qquad H^{\pm} := \frac{\frac{a^L}{2} - \frac{c^M}{m^L} \pm \sqrt{(\frac{a^L}{2} - \frac{c^M}{m^L})^2 - 4b_1^M h_1^M}}{2b_1^M},$$

$$S_3 := \max\{\ln l^+, u_2(t)\}, \qquad S_4 := \min\{\ln l^-, u_2(t)\},$$

$$u^{\pm} := \frac{e^{S_4} f^L - d^M e^{S_3} - h_3^M m^M \pm \sqrt{(d^M e^{S_3} + h_3^M m^M - e^{S_4} f^L)^2 - 4d^M m^M h_3^M e^{S_3}}}{2m^M d^M}.$$

Throughout this paper, we need the following assumptions:

$$A_1:\ \frac{a^L}{2} - \frac{c^M}{m^L} > 2\sqrt{b_1^M h_1^M},\quad A_2:\ e^{S_4} f^L - e^{S_3} d^M - h_3^M m^M > 2\sqrt{m^M d^M h_3^M e^{S_3}}.$$

Lemma 2.2. $l^- < H^- < H^+ < l^+$.

Proof. In fact, from above assumptions, we have:

$$l^- = 4b_1^L h_1^L \left[2b_1^L\left(\frac{a^M}{2} + \sqrt{\frac{(a^M)^2}{4} - 4b_1^L h_1^L}\right)\right]^{-1} < H^-, \tag{2.1}$$

and

$$H^+ < \left[\frac{a^M}{2} + \sqrt{\frac{(a^M)^2}{4} - 4b_1^L h_1^L}\right]\left[2b_1^L\right]^{-1} = l^+. \tag{2.2}$$

From (2.1), (2.2) and $l^- < l^+$, $H^- < H^+$, one verifies the conclusion.

Theorem 2.1. Let A_1 and A_2 hold. Then, (1.2) has at least four positive ω-periodic solutions.

Proof. For $\dot{y}(t) \equiv 0$, $y(t)$ is a constant. By making the change of variables $x(t) = e^{u_1(t)}$, $y(t) = e^{u_2(t)}$ and $z(t) = e^{u_3(t)}$, (1.2) can be reformulated as:

$$\begin{cases} \dot{u}_1(t) = \frac{a(t)}{2} - b_1(t)e^{u_1(t)} - \frac{c(t)e^{u_3(t)}}{m(t)e^{u_3(t)} + \alpha(t)e^{u_1(t)} + (1-\alpha(t))e^{u_2(t)}} \\ \qquad\quad -h_1\big(t, e^{u_1(t)}, e^{u_1(t)-u_2(t)}\big)e^{-u_1(t)}, \\ \dot{u}_3(t) = -d(t) + \frac{f(t)\alpha(t)e^{u_1(t)} + (1-\alpha(t))e^{u_2(t)}f(t)}{m(t)e^{u_3(t)} + \alpha(t)e^{u_1(t)} + (1-\alpha(t))e^{u_2(t)}} - h_3\big(t, e^{u_3(t)}\big)e^{-u_3(t)}, \end{cases} \tag{2.3}$$

where $u_2(t)$ is a constant. Let

$$X = Z = \{u = (u_1, u_3)^\top \in C(\mathbb{R}, \mathbb{R}^2) : u(t+\omega) = u(t),\ t \in \mathbb{R}\}$$

and define

$$\|u\| = \max_{t\in[0,\omega]} |u_1(t)| + \max_{t\in[0,\omega]} |u_3(t)|,\ u \in X \ or\ Z.$$

Equipped with the above norm $\|\cdot\|$, then X and Z are Banach spaces. Let the mappings be that

$$N(u,\lambda): X\times[0,1] \to Z,\ L: \mathrm{Dom}L \subset X \to Z,\ P: \mathrm{Dom}L \cap X \to \mathrm{Ker}L, Q: Z \to Z/\mathrm{Im}L,$$

$$N(u,\lambda)=\begin{pmatrix}\frac{a(t)}{2}-b_1(t)e^{u_1(t)}-\lambda\frac{c(t)e^{u_3(t)}}{m(t)e^{u_3(t)}+\alpha(t)e^{u_1(t)}+(1-\alpha(t))e^{u_2(t)}}-h_1e^{-u_1(t)}\\ \\ -d(t)+\frac{f(t)\alpha(t)e^{u_1(t)}+(1-\alpha(t))e^{u_2(t)}f(t)}{m(t)e^{u_3(t)}+\alpha(t)e^{u_1(t)}+(1-\alpha(t))e^{u_2(t)}}-h_3e^{-u_3(t)}\end{pmatrix},$$

$$Lu=\dot{u}=\frac{du(t)}{dt},\ Pu=\frac{1}{\omega}\int_0^\omega u(t)dt,\quad u\in X,\ Qz=\frac{1}{\omega}\int_0^\omega z(t)dt,\ z\in Z.$$

Then $\mathrm{Ker}L=\mathbb{R}^2,\mathrm{Im}L=\{z\in Z:\int_0^\omega z(t)dt=0\}$ is closed in $Z,\mathrm{dimKer}L=2=\mathrm{codimIm}L$, and P,Q are continuous projectors such that $\mathrm{Im}P=\mathrm{Ker}L$ and $\mathrm{Ker}Q=\mathrm{Im}L=\mathrm{Im}(I-Q)$. Hence, L is a Fredholm mapping of index zero. Furthermore, the generalized inverse (to L) $K_p:\mathrm{Im}L\to\mathrm{Ker}P\cap\mathrm{Dom}\,L$ is given by

$$K_p(z)=\int_0^t z(s)ds-\frac{1}{\omega}\int_0^\omega\int_0^s z(t)dtds.$$

Thus

$$QN(u,\lambda)=(\frac{1}{\omega}\int_0^\omega F_1(s,\lambda)ds,\quad\frac{1}{\omega}\int_0^\omega F_3(s,\lambda)ds)^\top$$

and

$$K_p(I-Q)N(u,\lambda)=\begin{pmatrix}\int_0^t F_1(s,\lambda)ds-\frac{1}{\omega}\int_0^\omega\int_0^t F_1(s,\lambda)dsdt+(\frac{1}{2}-\frac{t}{\omega})\int_0^\omega F_1(s,\lambda)ds\\ \int_0^t F_3(s,\lambda)ds-\frac{1}{\omega}\int_0^\omega\int_0^t F_3(s,\lambda)dsdt+(\frac{1}{2}-\frac{t}{\omega})\int_0^\omega F_3(s,\lambda)ds\end{pmatrix},$$

where

$$\begin{aligned}F_1(s,\lambda)=&\ \frac{a(s)}{2}-b_1(s)e^{u_1(s)}-h_1\big(s,e^{u_1(s)},e^{u_1(s)-u_2(s)}\big)e^{-u_1(s)}\\ &-\lambda\frac{c(s)e^{u_3(s)}}{m(s)e^{u_3(s)}+\alpha(s)e^{u_1(s)}+(1-\alpha(s))e^{u_2(s)}},\\ F_3(s,\lambda)=&\ -d(s)+\frac{f(s)\alpha(s)e^{u_1(s)}+(1-\alpha(s))e^{u_2(s)}f(s)}{m(s)e^{u_3(s)}+\alpha(s)e^{u_1(s)}+(1-\alpha(s))e^{u_2(s)}}-h_3\big(s,e^{u_3(s)}\big)e^{-u_3(s)}.\end{aligned}$$

Evidently, QN and $K_p(I-Q)N$ are continuous and it is easy to derive the compactness of $K_p(I-Q)N(\bar{\Omega})$ for any open bounded set $\Omega\subset X$ by using the Arzela-Ascoli theorem. Moreover, $QN(\bar{\Omega})$ is bounded. Therefore, N is L-compact on $\bar{\Omega}$ for any open bounded set Ω.

In order to use Lemma 2.1, we must find at least four appropriate open bounded subsets in X. Applying the operator equation $Lu=\lambda N(u,\lambda),\lambda\in(0,1)$, one gains

$$\begin{cases}\dot{u}_1(t)=\lambda\bigg[\frac{a(t)}{2}-b_1(t)e^{u_1(t)}-h_1\big(t,e^{u_1(t)},e^{u_1(t)-u_2(t)}\big)e^{-u_1(t)}\\ \qquad\qquad-\frac{\lambda c(t)e^{u_3(t)}}{m(t)e^{u_3(t)}+\alpha(t)e^{u_1(t)}+(1-\alpha(t))e^{u_2(t)}}\bigg],\\ \dot{u}_3(t)=\lambda\bigg[\frac{f(t)\alpha(t)e^{u_1(t)}+(1-\alpha(t))e^{u_2(t)}f(t)}{m(t)e^{u_3(t)}-d(t)+\alpha(t)e^{u_1(t)}+(1-\alpha(t))e^{u_2(t)}}-h_3\big(t,e^{u_3(t)}\big)e^{-u_3(t)}\bigg].\end{cases}\tag{2.4}$$

Assume that $u=u(t)\in X$ is an ω-periodic solution of (2.4) for some $\lambda\in(0,1)$. Then for $i=1,3$, there exist $\xi_i,\eta_i\in[0,\omega]$ such that $u_i(\xi_i)=\max_{t\in[0,\omega]}u_i(t),u_i(\eta_i)=$

$\min_{t\in[0,\omega]} u_i(t)$. Obviously, $\dot{u}_i(\xi_i)=0, \dot{u}_i(\eta_i)=0$. From this and (2.4), we have

$$\begin{cases} \frac{a(\xi_1)}{2} - b_1(\xi_1)e^{u_1(\xi_1)} - h_1\big(\xi_1, e^{u_1(\xi_1)}, \frac{e^{u_1(\xi_1)}}{e^{u_2(\xi_1)}}\big)e^{-u_1(\xi_1)} \\ \quad -\lambda \frac{c(\xi_1)e^{u_3(\xi_1)}}{m(\xi_1)e^{u_3(\xi_1)}+\alpha(\xi_1)e^{u_1(\xi_1)}+(1-\alpha(\xi_1))e^{u_2(\xi_1)}} = 0, \\ \frac{f(\xi_3)\alpha(\xi_3)e^{u_1(\xi_3)}+(1-\alpha(\xi_3))e^{u_2(\xi_3)}f(\xi_3)}{m(\xi_3)e^{u_3(\xi_3)}+\alpha(\xi_3)e^{u_1(\xi_3)}+(1-\alpha(\xi_3))e^{u_2(\xi_3)}} - d(\xi_3) - h_3\big(\xi_3, e^{u_3}\big)e^{-u_3} = 0, \end{cases} \tag{2.5}$$

and

$$\begin{cases} \frac{a(\eta_1)}{2} - b_1(\eta_1)e^{u_1(\eta_1)} - h_1\big(\eta_1, e^{u_1(\eta_1)}, \frac{e^{u_1(\eta_1)}}{e^{u_2(\eta_1)}}\big)e^{-u_1(\eta_1)} \\ \quad -\lambda \frac{c(\eta_1)e^{u_3(\eta_1)}}{m(\eta_1)e^{u_3(\eta_1)}+\alpha(\eta_1)e^{u_1(\eta_1)}+(1-\alpha(\eta_1))e^{u_2(\eta_1)}} = 0, \\ \frac{f(\eta_3)\alpha(\eta_3)e^{u_1(\eta_3)}+(1-\alpha(\eta_3))e^{u_2(\eta_3)}f(\eta_3)}{m(\eta_3)e^{u_3(\eta_3)}+\alpha(\eta_3)e^{u_1(\eta_3)}+(1-\alpha(\eta_3))e^{u_2(\eta_3)}} - d(\eta_3) - h_3\big(\eta_3, e^{u_3}\big)e^{-u_3} = 0. \end{cases} \tag{2.6}$$

From $(2.5)_1$, it leads to

$$\begin{aligned} & \frac{a^L}{2} - b_1^M e^{u_1(\xi_1)} - \frac{c^M}{m^L} - h_1^M e^{-u_1(\xi_1)} \\ \leq\ & \frac{a(\xi_1)}{2} - b_1(\xi_1)e^{u_1(\xi_1)} - \frac{c(\xi_1)}{m(\xi_1)} - h_1\big(\xi_1, e^{u_1(\xi_1)}, \frac{e^{u_1(\xi_1)}}{e^{u_2(\xi_1)}}\big)e^{-u_1(\xi_1)} \\ \leq\ & \frac{a(\xi_1)}{2} - b_1(\xi_1)e^{u_1(\xi_1)} - \frac{c(\xi_1)e^{u_3(\xi_1)}}{m(\xi_1)e^{u_3(\xi_1)}} - h_1\big(\xi_1, e^{u_1(\xi_1)}, \frac{e^{u_1(\xi_1)}}{e^{u_2(\xi_1)}}\big)e^{-u_1(\xi_1)} \\ <\ & 0. \end{aligned}$$

Therefore, $b_1^M e^{2u_1(\xi_1)} - (\frac{a^L}{2} - \frac{c^M}{m^L})e^{u_1(\xi_1)} + h_1^M > 0$, which implies that

$$u_1(\xi_1) < \ln H^- \quad or \quad u_1(\xi_1) > \ln H^+. \tag{2.7}$$

Thanks to $(2.6)_1$, it yields

$$\frac{a(\eta_1)}{2} - b_1(\eta_1)e^{u_1(\eta_1)} - h_1\big(\eta_1, e^{u_1(\eta_1)}, \frac{e^{u_1(\eta_1)}}{e^{u_2(\eta_1)}}\big)e^{-u_1(\eta_1)} > 0,$$

that is

$$b_1(\eta_1)e^{2u_1(\eta_1)} - \frac{a(\eta_1)}{2}e^{u_1(\eta_1)} + h_1\big(\eta_1, e^{u_1(\eta_1)}, \frac{e^{u_1(\eta_1)}}{e^{u_2(\eta_1)}}\big) < 0.$$

Moreover,

$$b_1^L e^{2u_1(\eta_1)} - \frac{a^M}{2}e^{u_1(\eta_1)} + h_1^L < 0. \tag{2.8}$$

By condition A_1,

$$\frac{a^M}{2} > \frac{a^L}{2} - \frac{c^M}{m^L} > 2\sqrt{b_1^M h_1^M} \geq 2\sqrt{b_1^L h_1^L}.$$

So $\frac{a^{2M}}{4} - 4b_1^L h_1^L > 0$, and from (2.8), one gets

$$\ln l^- < u_1(\eta_1) < \ln l^+. \tag{2.9}$$

Due to (2.7), (2.9) and Lemma 2.1, then for any $t \in [0,\omega]$, one has

$$\ln l^- < u_1(t) < \ln H^- \quad or \quad \ln H^+ < u_1(t) < \ln l^+. \tag{2.10}$$

Note that u_2 is constant and

$$u_1(\xi_1) = \max_{t\in[0,\omega]} u_1(t),\ S_3 = \max\{\ln l^+, u_2(t)\} > \max\{u_1(t), u_2(t)\}.$$

A direct computation from $(2.5)_2$ gives

$$\begin{aligned} d^L &\le d(\xi_3) < \frac{f^M\alpha(\xi_3)e^{u_1(\xi_3)} + (1-\alpha(\xi_3))e^{u_2(\xi_3)}f^M}{m^L e^{u_3(\xi_3)}} \\ &< \frac{\alpha(\xi_3)f^M e^{S_3} + (1-\alpha(\xi_3))f^M e^{S_3}}{m^L e^{u_3(\xi_3)}} \\ &= \frac{f^M e^{S_3}}{m^L e^{u_3(\xi_3)}}. \end{aligned}$$

It means that

$$u_3(\xi_3) < \ln\frac{f^M e^{S_3}}{m^L d^L}. \tag{2.11}$$

From $(2.6)_2$ and by $S_4 = \min\{\ln l^-, u_2(t)\} < \min\{u_1(t), u_2(t)\}$, it follows that

$$h_3^L e^{-u_3(\eta_3)} \le h_3\big(\eta_3, e^{u_3(\eta_3)}\big)e^{-u_3(\eta_3)} < \frac{f^M e^{S_3}}{\alpha(\eta_3)e^{u_1(\eta_3)} + (1-\alpha(\eta_3))e^{u_2(\eta_3)}} < \frac{f^M e^{S_3}}{e^{S_4}}.$$

Then

$$u_3(\eta_3) > \ln\frac{h_3^L e^{S_4}}{f^M e^{S_3}}. \tag{2.12}$$

On account of $(2.5)_2$, we have

$$\frac{f(\xi_3)\alpha(\xi_3)e^{u_1(\xi_3)} + (1-\alpha(\xi_3))e^{u_2(\xi_3)}f(\xi_3)}{m(\xi_3)e^{u_3(\xi_3)} + \alpha(\xi_3)e^{u_1(\xi_3)} + (1-\alpha(\xi_3))e^{u_2(\xi_3)}} = d(\xi_3) + h_3\big(\xi_3, e^{u_3(\xi_3)}\big)e^{-u_3(\xi_3)},$$

thus $\frac{e^{S_4}f^L}{m^M e^{u_3(\xi_3)} + e^{S_3}} < d^M + h_3^M e^{-u_3(\xi_3)}$. It hints that

$$m^M d^M e^{2u_3(\xi_3)} + (e^{S_3}d^M + h_3^M m^M - e^{S_4}f^L)e^{u_3(\xi_3)} + h_3^M e^{S_3} > 0.$$

In view of A_2, one gains $u^- > 0, u^+ > 0$ and

$$u_3(\xi_3) < \ln u^- \quad or \quad u_3(\xi_3) > \ln u^+. \tag{2.13}$$

Analogously, by $(2.6)_2$, we get

$$u_3(\eta_3) < \ln u^- \quad or \quad u_3(\eta_3) > \ln u^+. \tag{2.14}$$

From (2.11) – (2.14), one concludes

$$\ln\frac{h_3^L e^{S_4}}{f^M e^{S_3}} < u_3(t) < \ln u^- \quad or \quad \ln u^+ < u_3(t) < \ln\frac{f^M e^{S_3}}{m^L d^L}. \tag{2.15}$$

Indeed, by A_2, then $e^{S_4}f^L - e^{S_3}d^M - h_3^M m^M > 0$, and

$$\begin{aligned}
u^- &= \frac{e^{S_4}f^L - d^M e^{S_3} - h_3^M m^M - \sqrt{(d^M e^{S_3} + h_3^M m^M - e^{S_4}f^L)^2 - 4d^M m^M h_3^M e^{S_3}}}{2m^M d^M} \\
&= (4d^M m^M h_3^M e^{S_3})\Big[2m^M d^M (e^{S_4}f^L - d^M e^{S_3} - h_3^M m^M \\
&\quad + \sqrt{(d^M e^{S_3} + h_3^M m^M - e^{S_4}f^L)^2 - 4d^M m^M h_3^M e^{S_3}})\Big]^{-1} \\
&= \frac{2h_3^M e^{S_3}}{e^{S_4}f^L - d^M e^{S_3} - h_3^M m^M + \sqrt{(d^M e^{S_3} + h_3^M m^M - e^{S_4}f^L)^2 - 4d^M m^M h_3^M e^{S_3}}} \\
&= (2h_3^M e^{S_3})\Big[e^{S_4}f^L - d^M e^{S_3} - h_3^M m^M + \sqrt{(e^{S_4}f^L - d^M e^{S_3} - h_3^M m^M)^2}\Big]^{-1} \\
&> (2h_3^L e^{S_4})\Big[e^{S_4}f^L - d^M e^{S_3} - h_3^M m^M + \sqrt{(e^{S_3}f^L + d^M e^{S_3} + h_3^M m^M)^2}\Big]^{-1} \\
&= \frac{h_3^L e^{S_4}}{e^{S_3}f^L}.
\end{aligned}$$

Thus $\ln \frac{h_3^L e^{S_4}}{f^M e^{S_3}} < u_3(t) < \ln u^-$. Furthermore,

$$\begin{aligned}
u^+ &= \frac{e^{S_4}f^L - d^M e^{S_3} - h_3^M m^M + \sqrt{(d^M e^{S_3} + h_3^M m^M - e^{S_4}f^L)^2 - 4d^M m^M h_3^M e^{S_3}}}{2m^M d^M} \\
&= \Big[e^{S_4}f^L - d^M e^{S_3} - h_3^M m^M + \sqrt{(e^{S_4}f^L - d^M e^{S_3} - h_3^M m^M)^2}\Big]\big(2m^M d^M\big)^{-1} \\
&< \Big[e^{S_3}f^M - d^M e^{S_3} - h_3^M m^M + \sqrt{(e^{S_3}f^M + d^M e^{S_3} + h_3^M m^M)^2}\Big]\big(2m^L d^L\big)^{-1} \\
&= \Big[e^{S_3}f^M - d^M e^{S_3} - h_3^M m^M + e^{S_3}f^M + d^M e^{S_3} + h_3^M m^M\Big]\big(2m^L d^L\big)^{-1} \\
&= \frac{e^{S_3}f^M}{m^L d^L},
\end{aligned}$$

and $\ln u^+ < u_3(t) < \ln \frac{f^M e^{S_3}}{m^L d^L}$. Clearly, $\ln l^\pm, \ln H^\pm, u^\pm, S_3, S_4$ are independent of λ. Now, let $u = (u_1, u_2, u_3)^\top$,

$$\Omega_1 = \{u \in X : \ln l^- - C_1 < u_1(t) < \ln H^- + 2C_2,\ \ln \frac{h_3^L e^{S_4}}{f^M e^{S_3}} < u_3(t) < \ln u^-\},$$

$$\Omega_2 = \{u \in X : \ln l^- - C_1 < u_1(t) < \ln H^- + 2C_2,\ \ln u^+ < u_3(t) < \ln \frac{f^M e^{S_3}}{m^L d^L}\},$$

$$\Omega_3 = \{u \in X : \ln H^+ - C_2 < u_1(t) < \ln l^+ + C_3,\ \ln \frac{h_3^L e^{S_4}}{f^M e^{S_3}} < u_3(t) < \ln u^-\},$$

$$\Omega_4 = \{u \in X : \ln H^+ - C_2 < u_1(t) < \ln l^+ + C_3,\ \ln u^+ < u_3(t) < \ln \frac{f^M e^{S_3}}{m^L d^L}\}.$$

where $C_1 = \frac{\ln H^- - \ln l^-}{2}, C_2 = \frac{\ln H^+ - \ln H^-}{4}, C_3 = \frac{\ln l^+ - \ln H^+}{2}$. For $i, j \in A$, $\Omega_i \subset X$ is bounded and open, and $\Omega_i \cap \Omega_j = \emptyset$ $(i \neq j)$. For this reason, Ω_i satisfies the requirement (a) in Lemma 2.1.

Now we check that (b) in Lemma 2.1 holds, i.e., when $u \in \partial\Omega_i \cap \mathrm{Ker}L = \partial\Omega_i \cap \mathbb{R}^2, QN(u,0) \neq (0,0)^\top,\ (i \in A)$. If not, then when $u \in \partial\Omega_i \cap \mathrm{Ker}L = \partial\Omega_i \cap \mathbb{R}^2$, is a constant vector $u = (u_1, u_3)^\top$ with $u \in \partial\Omega_i\ (i \in A)$ meets:

$$\begin{cases} \int_0^\omega \Big(\frac{a(t)}{2} - b_1(t)e^{u_1(t)} - h_1\big(t, e^{u_1(t)}, e^{u_1(t)-u_2(t)}\big)e^{-u_1(t)}\Big)dt = 0, \\ \int_0^\omega \Big(-d(t) + \frac{f(t)\alpha(t)e^{u_1(t)}+f(t)(1-\alpha(t))e^{u_2(t)}}{m(t)e^{u_3(t)}+\alpha(t)e^{u_1(t)}+(1-\alpha(t))e^{u_2(t)}} - h_3\big(t, e^{u_3(t)}\big)e^{-u_3(t)}\Big)dt = 0. \end{cases}$$

Then there exist $t_j \in [0,\omega]$ $(j = 1,3)$ such that

$$\begin{cases} \frac{a(t_1)}{2} - b_1(t_1)e^{u_1(t_1)} - h_1\big(t_1, e^{u_1(t_1)}, \frac{e^{u_1(t_1)}}{e^{u_2(t_1)}}\big)e^{-u_1(t_1)} = 0, \\ \frac{f(t_3)\alpha(t_3)e^{u_1(t_3)}+f(t_3)(1-\alpha(t_3))e^{u_2(t_3)}}{m(t_3)e^{u_3(t_3)}+\alpha(t_3)e^{u_1(t_3)}+(1-\alpha(t_3))e^{u_2(t_3)}} - d(t_3) - h_3\big(t_3, e^{u_3}\big)e^{-u_3} = 0. \end{cases} \tag{2.16}$$

Akin to obtain $(2.10), (2.15)$, it is clear that the solution of (2.16) does not satisfy $u \in \partial\Omega_i \cap \mathbb{R}^2,\ (i \in I)$. Hence, condition (b) in Lemma 2.1 holds.

Finally, let us show (c) in Lemma 2.1 is valid. Since $\mathrm{Ker}L = \mathrm{Im}Q$, we can take $J = I$. To this end, we define the homotopy mapping $\phi : \mathrm{Dom}\, L \times [0,1] \to X$ by

$$\phi(u_1, u_3, \mu) = \mu QN(u,0) + (1-\mu)G(u), \quad \mu \in [0,1],$$

where $G(u)$ is

$$G(u) = \begin{pmatrix} \int_0^\omega \big[\frac{a(t)}{2} - b_1^M e^{u_1(t)} - h_1^L e^{-u_1(t)}\big]dt \\ \int_0^\omega \big[-d(t) + \frac{f^L e^{S_4}}{(m^M + 2e^{S_3})e^{u_3(t)}} - h_3^M e^{S_4} e^{-u_3(t)}\big]dt \end{pmatrix}.$$

For $u \in \partial\Omega_i \cap \mathrm{Ker}L = \partial\Omega_i \cap \mathbb{R}^2$ and $\mu \in [0,1]$, then they fulfill

$$\phi(u_1, u_3, \mu) = \mu QN(u,0) + (1-\mu)G(u) \neq (0,0)^\top.$$

Otherwise, there are μ and constant vector $u = (u_1, u_3)^\top$ such that $\phi(u_1, u_3, \mu) = (0,0)^\top$. Thanks to the mean value theorem, there exist $\bar{t}_4, \bar{t}_6 \in [0,\omega]$, so that

$$\begin{cases} \mu\Big[\frac{a(\bar{t}_4)}{2} - b_1(\bar{t}_4)e^{u_1(\bar{t}_4)} - h_1\big(\bar{t}_4, e^{u_1(\bar{t}_4)}, \frac{e^{u_1(\bar{t}_4)}}{e^{u_2(\bar{t}_4)}}\big)e^{-u_1(\bar{t}_4)}\Big] \\ \qquad\qquad + (1-\mu)\Big[\frac{a(\bar{t}_4)}{2} - b_1^M e^{u_1(\bar{t}_4)} - h_1^L e^{-u_1(\bar{t}_4)}\Big] = 0, \\ \mu\Big[\frac{f(\bar{t}_6)\alpha(\bar{t}_6)e^{u_1(\bar{t}_6)}+f(\bar{t}_6)(1-\alpha(\bar{t}_6))e^{u_2(\bar{t}_6)}}{m(t_6)e^{u_3(\bar{t}_6)}+\alpha(\bar{t}_6)e^{u_1(\bar{t}_6)}+(1-\alpha(\bar{t}_6))e^{u_2(\bar{t}_6)}} - h_3(\bar{t}_6, u_3(\bar{t}_6))e^{-u_3(\bar{t}_6)} \\ \quad -d(\bar{t}_6)\Big] + (1-\mu)\Big[\frac{f^L e^{S_4}}{(m^M+2e^{S_3})e^{u_3(\bar{t}_6)}} - h_3^M e^{S_4} e^{-u_3(\bar{t}_6)} - d(\bar{t}_6)\Big] = 0. \end{cases} \tag{2.17}$$

We make the following Claims.

Claim 1. $\ln l^- \leq u_1(\bar{t}_4) < \ln H^-$ *or* $\ln H^+ < u_1(\bar{t}_4) \leq \ln l^+$. (2.18)

From $(2.17)_1$, we obtain

$$\begin{aligned} & \frac{a^L}{2} - b_1^M e^{u_1(\bar{t}_4)} - h_1^M e^{-u_1(\bar{t}_4)} \\ \leq\ & \frac{a(\bar{t}_4)}{2} - b_1^M e^{u_1(\bar{t}_4)} - \mu h_1\big(\bar{t}_4, e^{u_1(\bar{t}_4)}, \frac{e^{u_1(\bar{t}_4)}}{e^{u_2(\bar{t}_4)}}\big)e^{-u_1(\bar{t}_4)} - (1-\mu)h_1^L e^{-u_1(\bar{t}_4)} \\ =\ & -\mu\Big(b_1^M e^{u_1(\bar{t}_4)} - b_1(\bar{t}_4)e^{u_1(\bar{t}_4)}\Big) \\ \leq\ & 0. \end{aligned}$$

It means that
$$\frac{a^L}{2}-b_1^Me^{u_1(\bar t_4)}-\frac{c^M}{m^L}-h_1^Me^{-u_1(\bar t_4)}<0.$$
So $u_1(\bar t_4)<\ln H^-$ or $u_1(\bar t_4)>\ln H^+$. In like manner, from $(2.17)_1$, one achieves
$$\begin{aligned}0 &= \frac{a(\bar t_4)}{2}-b_1^Me^{u_1(\bar t_4)}-\mu h_1\big(\bar t_4,e^{u_1(\bar t_4)},\frac{e^{u_1(\bar t_4)}}{e^{u_2(\bar t_4)}}\big)e^{-u_1(\bar t_4)}-(1-\mu)h_1^Le^{-u_1(\bar t_4)}\\ &\quad +\mu\big(b_1^M-b_1(\bar t_4)\big)e^{u_1(\bar t_4)}\\ &\le \frac{a(\bar t_4)}{2}-b_1^Me^{u_1(\bar t_4)}-h_1^Le^{-u_1(\bar t_4)}+\mu\big(b_1^M-b_1(\bar t_4)\big)e^{u_1(\bar t_4)}\\ &\le \frac{a(\bar t_4)}{2}-(1-\mu)b_1^Me^{u_1(\bar t_4)}-\mu b_1(\bar t_4)e^{u_1(\bar t_4)}-h_1^Le^{-u_1(\bar t_4)}\\ &\le \frac{a(\bar t_4)}{2}-b_1(\bar t_4)e^{u_1(\bar t_4)}-h_1^Le^{-u_1(\bar t_4)}\\ &\le \frac{a^M}{2}-b_1^Le^{u_1(\bar t_4)}-h_1^Le^{-u_1(\bar t_4)},\end{aligned}$$
which suggests
$$b_1^Le^{2u_1(\bar t_4)}-\frac{a^M}{2}e^{u_1(\bar t_4)}+h_1^L\le 0.$$
Thus $\ln l^-\le u_1(\bar t_4)\le \ln l^+$, and according to $l^-<H^-<H^+<l^+$, the desired result follows.

Claim 2. $\ln\frac{h_3^Le^{S_4}}{f^Me^{S_3}}<u_3(\bar t_6)<\ln u^-$ *or* $\ln u^+<u_3(\bar t_6)<\ln\frac{f^Me^{S_3}}{m^Ld^L}$.

Employing $S_3=\max\{\ln l^+,u_2(t)\}$, $S_4=\min\{\ln l^-,u_2(t)\}$, and $(2.17)_2$, we fulfill the conclusion easily so we omit the details.

From Claim 1 and Claim 2, when $\mu\in[0,1]$ and $\phi(u_1,u_3,\mu)=0$, then $u\bar\in\partial\Omega_i\cap KerL(\partial\Omega_i\cap\mathbb{R}^2)$, so when $u\in\partial\Omega_i\cap KerL=\partial\Omega_i\cap\mathbb{R}^2$ and $\mu\in[0,1]$, $\phi(u_1,u_3,\mu)\neq 0$. By the homotopy invariance of topological degree and note the system below
$$\begin{cases}\frac{a(\bar t_4)}{2}-b_1^Me^{u_1(\bar t_4)}-h_1^Le^{-u_1(\bar t_4)}=0,\\ -d(\bar t_4)+\frac{f^Le^{S_4}}{(m^M+2e^{S_3})e^{u_3(\bar t_6)}}-h_3^Me^{S_4}e^{-u_3(\bar t_6)}=0,\end{cases}$$
one gains
$$\begin{aligned}&\deg(JQN(u,0),\Omega_1\cap\operatorname{Ker}L,(0,0)^\top)\\ =\;&\deg(\phi(u,1),\Omega_1\cap\operatorname{Ker}L,(0,0)^\top)\\ =\;&\deg(\phi(u,0),\Omega_1\cap\operatorname{Ker}L,(0,0)^\top)\\ =\;&\deg(JG(u),\Omega_1\cap\operatorname{Ker}L,(0,0)^\top)\\ =\;&\operatorname{sign}\big[-b_1^Me^{u_1(\bar t_4)}+h_1^Le^{-u_1(\bar t_4)}\big]\times\operatorname{sign}\big[\frac{-f^Le^{S_4}}{(m^M+2e^{S_3})e^{u_3(\bar t_6)}}+h_3^Me^{S_4}e^{-u_3(\bar t_6)}\big].\end{aligned}$$
For $\operatorname{sign}\big[-b_1^Me^{u_1(\bar t_4)}+h_1^Le^{-u_1(\bar t_4)}\big]$, from $a(\bar t_4)-b_1^Me^{u_1(\bar t_4)}-h_1^Le^{-u_1(\bar t_4)}=0$, then
$$-b_1^Me^{u_1(\bar t_4)}+h_1^Le^{-u_1(\bar t_4)}=a(\bar t_4))-2b_1^Me^{u_1(\bar t_4)}.$$
Employing
$$e^{u_1^\pm(\bar t_4)}=\ln\frac{a(\bar t_4)\pm\sqrt{(a(\bar t_4))^2-4b_1^Mh_1^L}}{2b_1^M},$$

one ascertains

$$\mathrm{sign}\big[-b_1^M e^{u_1(\bar t_4)} + h_1^L e^{-u_1(\bar t_4)}\big] = \mathrm{sign}\big[a(\bar t_4)) - 2b_1^M e^{u_1(\bar t_4)}\big] = \pm 1.$$

For $\mathrm{sign}\big[\frac{-f^L e^{S_4}}{(m^M+2e^{S_3})e^{u_3(\bar t_6)}} + h_3^M e^{-u_3(\bar t_6)}\big]$, according to

$$-d(\bar t_6) + \frac{f^L e^{S_4}}{(m^M + 2e^{S_3})e^{u_3(\bar t_6)}} - h_3^M e^{S_4} e^{-u_3(\bar t_6)} = 0,$$

it yields

$$-\frac{f^L e^{S_4}}{(m^M + 2e^{S_3})e^{u_3(\bar t_6)}} = -d(\bar t_6) - h_3^M e^{S_4} e^{-u_3(\bar t_6)}.$$

Thus

$$\mathrm{sign}\big[\frac{-f^L e^{S_4}}{(m^M + 2e^{S_3})e^{u_3(\bar t_6)}} + h_3^M e^{S_4} e^{-u_3(\bar t_6)}\big] = \mathrm{sign}\big[-d(\bar t_6)\big] = -1.$$

Therefore,

$$\begin{aligned} & deg(JQN(u,0), \Omega_1 \cap \mathrm{Ker}\, L, (0,0)^\top) \\ = \; & \mathrm{sign}\big[-b_1^M e^{u_1(\bar t_4)} + h_1^L e^{-u_1(\bar t_4)}\big] \times \mathrm{sign}\big[\frac{-f^L e^{S_4}}{(m^M + 2e^{S_3})e^{u_3(\bar t_6)}} + h_3^M e^{S_4} e^{-u_3(\bar t_6)}\big] \\ = \; & \pm 1 \times (-1) \\ = \; & \pm 1 \neq 0. \end{aligned} \tag{2.19}$$

The argument appeals above for (2.19), adapted to $G(u)$ now yields that the topological degree $\deg(JQN(x,0), \Omega_i \cap \mathrm{Ker} L, (0,0)^\top)$ should be independent of Ω_i. Consequently, for $i = 2,3,4$,

$$\deg(JQN(u,0), \Omega_2 \cap \mathrm{Ker}\, L, (0,0)^\top) = \pm 1.$$

So far, we have confirmed that Ω_i $(i \in A)$ satisfies all the assumptions in Lemma 2.1. So, (2.3) has at least four different ω-periodic solutions. For this reason, (1.2) has at least four different positive ω-periodic solutions. This completes the proof.

Remark 2.1. (i) The definition for $G(u)$ is quite different from the previous papers, especially in $G(u)_2$. And in $G(u)_1$, we omit a complex term and make some special terms.

(ii) In the definition of the operator $N(u,\lambda)$, the first equation involves λ, but the second equation is independent of λ.

(iii) When defining $\Omega_i (i \in A)$, we use a priori bounds for $u_1(t)$, so that we can supersede the particular $G(u)$ for $QN(u,0)$ to compute $\deg(JQN(u,0), \Omega_i \cap \mathrm{Ker} L, (0,0)^\top)$.

3. An example

Consider the following Holling II type functional response system with a ratio parameter $\alpha(t)$ with $\alpha(t) \in [0, 1]$, and new harvesting terms:

$$\begin{cases} \dot{x}(t) = x(t)\Big[\frac{1}{2}(30 + 2\cos t) - \frac{\frac{2+\cos t}{3} z(t)}{(2+\sin t)z(t)+\alpha(t)x(t)+(1-\alpha(t))y(t)} \\ \qquad\qquad -(5+\cos t)x(t)\Big] - \frac{17}{4} - \sin t - \frac{\cos^2 t}{1+2^{-\frac{x(t)}{y(t)}+1} \times 3^{-x(t)}}, \\ y(t) \equiv 2, \\ \dot{z}(t) = z(t)\Big[\frac{(61+\sin t)\alpha(t)x(t)+(1-\alpha(t))y(t)(11+\sin t)}{(2+\sin t)z(t)+\alpha(t)x(t)+(1-\alpha(t))y(t)-\frac{3+\cos t}{2}}\Big] - 2 - \frac{\sin t}{1+2^{-z(t)}}, \end{cases} \tag{3.1}$$

Now, $a(t) = 30+2\cos t, b_1(t) = 5+\cos t, c(t) = \frac{2+\cos t}{3}, m(t) = 2+\sin t, h_1(t, x, \frac{x}{y}) = \frac{17}{4} + \sin t + \frac{\cos^2 t}{1+2^{-\frac{x(t)}{y(t)}+1} \times 3^{-x(t)}}, d(t) = \frac{3+\cos t}{2}, f(t) = 61 + \sin t, h_3(t, z) = 2 + \frac{\sin t}{1+2^{-z(t)}}$.

$$\frac{a^L}{2} - \frac{c^M}{m^L} = 14 - 1 = 13 > 2\sqrt{b_1^M h_1^M}.$$

This proves A_1. Next, we check that A_2 is valid and by virtue of $f^L = 60, b_1^L = 4, h_3^M < 3, a^M = 32, d^M = 2, h_1^L = \frac{13}{4}$,

$$l^+ = \frac{\frac{a^M}{2} + \sqrt{\frac{(a^M)^2}{4} - 4b_1^L h_1^L}}{2b_1^L} = \frac{16 + \sqrt{196 - 4 \times 4 \times \frac{13}{4}}}{8} = \frac{7}{2},$$

$$l^- = \frac{\frac{a^M}{2} - \sqrt{\frac{(a^M)^2}{4} - 4b_1^L h_1^L}}{2b_1^L} = \frac{16 - \sqrt{196 - 4 \times 4 \times \frac{13}{4}}}{8} = \frac{1}{2},$$

one deduces $S_3 = \max\{\ln l^+, \ln 2\} = \ln \frac{7}{2}$, $S_4 = \min\{\ln l^-, \ln 2\} = \ln \frac{1}{2}$ and

$$e^{S_4} f^L - e^{S_3} d^M - h_3^M m^M > \frac{1}{2} \times 60 - \frac{7}{2} \times 2 - 3 \times 2 = 17,$$

$$2\sqrt{m^M d^M h_3^M e^{S_3}} < 2 \times \sqrt{3 \times 2 \times 3 \times \frac{7}{2}} < 17 < e^{S_4} f^L - e^{S_3} d^M - h_3^M m^M.$$

Hence A_2 is legitimate. By Theorem 2.1, (3.1) possesses at least four positive ω-periodic solutions.

Remark 3.1. In the example, from the expression of the harvesting term $h_1(t, x, \frac{x}{y})$, it is easy to find that when prey species population density $x(t)$ is very large, h_1 increases. In addition, in the first equation, we also discover that when the sex-ratio $\frac{x(t)}{y(t)}$ is large enough, h_1 increases as well and vice versa.

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Liang Ding

Department of Basis Education Dehong Vocational College Dehong, Yunnan, 678400, P.R. China

E-mail address: lovemathlovemath@126.com

Jinlong Wei (corresponding author)

School of Mathematics and Statistics, Huazhong University of Science and Technology, Wuhan, China

E-mail address: weijinlong.hust@gmail.com

Bin Zhang

Department of Basis Education Dehong Vocational College Dehong, Yunnan, 678400, P.R. China

E-mail address: 1498988524@qq.com