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EXISTENCE AND STABILITY OF SOLUTIONS FOR SOME CLASS OF NEUTRAL STOCHASTIC PARTIAL INTEGRO-DIFFERENTIAL SYSTEMS WITH INFINITE DELAY

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Abstract. This paper is concerned with the existence, uniqueness and stability of mild solution to neutral stochastic functional integrodifferential equations with non-Lipschitz condition and Lipschitz condition. Furthermore, we discuss the existence of mild solutions for the equations are discussed by means of semigroup theory and theory of resolvent operator. Under some suitable assumptions, the results are obtained by using the method of successive approximation and Bihari's inequality. Moreover, an example is given to illustrate our results.

1. Introduction

Neutral stochastic differential equation occurs in many areas of science and engineering and has attained much attention in the past decades. The partial integrodifferential equations has wide applications in the field of mechanical, electrical and so on., and refer [14]. For abstract model of partial integrodifferential equations with resolvent operators, see for instance [7, 9, 13]. The deterministic model often fluctuate due to noise. Under this circumstance we move the deterministic model problems to stochastic model problems, for more details reader may refer [4, 8, 10, 12, 19]. The existence and uniqueness of the neutral stochastic differential equations with infinite delay have been studied by many authors [6, 17]. Recently, the authors have established the problem with Lipschitz and non-Lipschitz condition, we suggest [2, 3, 11, 18, 21, 22, 24] and reference therein.

In [1] Anguraj et al. studied the impulsive stochastic neutral functional differential equations under non-Lipschitz condition and Lipschitz condition, whereas A. Lin et al. [18] has established on neutral impulsive stochastic integrodifferential equations with infinite delay via fractional operators and H. Bin Chen [6] has proved the existence and uniqueness for the solution of neutral stochastic functional differential equations with infinite delay, then A. Vinodkumar [24] has examined the existence, uniqueness and stability results of impulsive stochastic semilinear functional differential equations with infinite delay. Recently, Y. Ren [23] has described

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the existence, uniqueness and stability of mild solutions for time-dependent stochastic evolution equations with poisson jums and infinite delay. Moreover, we study the stability through the continuous dependence on the initial values by means of Bihari's inequality. For more details reader may refer [2, 12, 20].

Inspired by the above mentioned works [6, 9, 23, 24], the purpose of this paper is to study the existence, uniqueness and stability for neutral stochastic functional integrodifferential equations with infinite delay in a real separable Hilbert space:

$$d[x(t)-g(t,x_t)] = A\left[x(t)+\int_0^t f(t-s)x(s)ds\right]dt + h(t,x_t)dt + \sigma(t,x_t)dw(t), \quad t \in J := [0,T], \tag{1.1}$$

$$x_0 = \varphi \in \mathcal{B}. \tag{1.2}$$

Here, the state $x(\cdot)$ takes the values in a real separable Hilbert space H with inner product $(\cdot,\cdot)$ and the norm $\|\cdot\|$, A is the infinitesimal generator of strongly continuous semigroup of bounded linear operator $\{R(t),\, t \geq 0\}$ on H, and $f(t), t \in J$ is a bounded linear operator. The history $x_t : (-\infty, 0] \to H$, $x_t(\theta) = x(t+\theta)$, for $t \geq 0$, belongs to the phase space $\mathcal{B}$, which will be described axiomatically in Preliminaries. Suppose $\{w(t); t \geq 0\}$ is a given K-valued Brownian motion with a finite trace nuclear covariance operator $Q \geq 0$ defined on a complete probability space $(\Omega, \mathcal{F}, P)$ equipped with a normal filtration $\{\mathcal{F}_t\}_{t\geq 0}$, which is generated by Wiener process w. we are also employing the same notation $\|\cdot\|$ for the norm $\mathcal{L}(K,H)$, where $\mathcal{L}(K,H)$ denotes the space of all bounded linear operator from K into H. Assume that $h : \mathbb{R}^+ \times \mathcal{B} \to H$ and $\sigma : \mathbb{R}^+ \times \mathcal{B} \to \mathcal{L}_Q(K,H)$, where $\mathbb{R}^+ = [0,\infty)$ are Borel measurable and $g : \mathbb{R}^+ \to \mathcal{B}$ is continuous. Here, $L_Q(K,H)$ denotes the space of all Q-Hilbert-Schmidt operator from K into H, which will be defined in Section 2.

The substance of the paper is organised as follows. In Section 2, we recapitulate some basic definitions, lemmas, notations, and theorems which will be used to develop our results. In Section 3 and 4, we give several sufficient conditions to prove the existence, uniqueness and stability for the problem (1.1)-(1.2) respectively. Section 5 is reserved for an example is to illustrate the efficiency of the obtained results.

2. Preliminaries

Let $(K, \|\cdot\|_K)$ and $(H, \|\cdot\|_H)$ be the two real separable Hilbert space with inner product $\langle\cdot,\cdot\rangle_K$ and $\langle\cdot,\cdot\rangle_H$, respectively. We denote $\mathcal{L}(K,H)$ be the set of all linear bounded operator from K into H, equipped with the usual operator norm $\|\cdot\|$. In this article, we use the symbol $\|\cdot\|$ to denote norms of operator regardless of the space involved when no confusion possibly arises.

Let $(\Omega, \mathcal{F}, P; H)$ be the complete probability space furnished with a complete family of right continuous increasing σ- algebra $\{\mathcal{F}_t, t \in J\}$ satisfying $\mathcal{F}_t \subset \mathcal{F}$. An H- valued random variable is an $\mathcal{F}$- measurable function $x(t) : \Omega \to H$ and a collection of random variables $S = \{x(t,\omega) : \Omega \to H \setminus t \in J\}$ is called stochastic process. Usually we write $x(t)$ instead of $x(t,\omega)$ and $x(t) : J \to H$ in the space of S. Let $\{e_i\}_{i=1}^{\infty}$ be a complete orthonormal basis of K. Suppose that $\{w(t) : t \geq 0\}$ is a cylindrical K-valued wiener process with a finite trace nuclear covariance operator $Q \geq 0$, denote $\mathrm{Tr}(Q) = \sum_{i=1}^{\infty} \lambda_i = \lambda < \infty$, which satisfies that $Qe_i = \lambda_i e_i$. So, actually, $\omega(t) = \sum_{i=1}^{\infty} \sqrt{\lambda_i}\omega_i(t)e_i$, where $\{\omega_i(t)\}_{i=1}^{\infty}$ are mutually independent one-dimensional standard Wiener processes. We assume that $\mathcal{F}_t = \sigma\{\omega(s) : 0 \leq s \leq t\}$ is the σ-algebra generated by ω and $\mathcal{F}_t = \mathcal{F}$. Let $\Psi \in \mathcal{L}(K, H)$ and define

$$\|\Psi\|_Q^2 = \mathrm{Tr}(\Psi Q \Psi^*) = \sum_{n=1}^{\infty} \|\sqrt{\lambda_n}\Psi e_n\|^2.$$

If $\|\Psi\|_Q < \infty$, then Ψ is called a Q-Hilbert-Schmidt operator. Let $\mathcal{L}_Q(K, H)$ denote the space of all Q-Hilbert-Schmidt operators $\Psi : K \to H$. The completion $\mathcal{L}_Q(K, H)$ of $\mathcal{L}(K, H)$ with respect to the topology induced by the norm $\|\cdot\|_Q$ where $\|\Psi\|_Q^2 = \langle \Psi, \Psi \rangle$ is a Hilbert space with the above norm topology.

In this work we will employ an axiomatic definition of the phase space $\mathcal{B}$ introduced by Hale et al. [15]. To establish the axioms of the phase space $\mathcal{B}$, we use the following terminology used in Hinto et al. [16]. The axioms of the space $\mathcal{B}$ are established for $\mathcal{F}_0$-measurable functions from $(-\infty, 0]$ into H, endowed with a seminorm $\|\cdot\|_{\mathcal{B}}$ which satisfies the following axioms:

(A1) If $x : (-\infty, T] \to H$, $T > 0$ is such that $x_0 \in \mathcal{B}$ then for every $t \in [0, T]$, the following conditions hold:

- (i) $x_t \in \mathcal{B}$;
- (ii) $\|x(t)\| \leq L\|x_t\|_{\mathcal{B}}$;
- (iii) $\|x_t\|_{\mathcal{B}} \leq M(t) \sup\limits_{0 \leq s \leq t} \|x(s)\| + N(t)\|x_0\|_{\mathcal{B}}$, where $L > 0$ is a constant; $M(\cdot)$, $N(\cdot) : [0, +\infty) \to [1, +\infty)$, is continuous $N(\cdot)$ is locally bounded, and $L, M(\cdot), N(\cdot)$ are independent of $x(\cdot)$.

(A2) For the function, $x(\cdot)$ in (A1), x_t is a $\mathcal{B}$-valued continuous function on $[0, b]$.

(A3) The space $\mathcal{B}$ is complete.

The $\mathcal{B}$- valued stochastic process $x_t : \Omega \to \mathcal{B}$, $t \geq 0$, is defined by $x_t(s) = \{x(t+s)(\omega) : s \in (-\infty, 0]\}$. The collection of all strongly measurable, square integrable, H-valued random variables, denoted by $L_2(\Omega, \mathcal{F}, P; H) \equiv L_2(\Omega; H)$, is a Banach space equipped with norm $\|x(\cdot)\|_{L_2}^2 = E\|x(\cdot, w)\|_H^2$, where E denotes expectation defined by $E(h) = \int_\Omega h(w)dP$. Let $C(J, L_2(\Omega; H))$ be the Banach space of all continuous map from J into $L_2(\Omega; H)$ satisfying the condition $\sup\limits_{t \in J} E\|x(t)\|^2 < \infty$. An important subspace is given by $L_2^0(\Omega, H) = \{f \in L_2(\Omega, H) : f \,\mathrm{is}\, \mathcal{F}_0 - \text{is measurable}\}$.

Let $\mathcal{Z}$ be the closed subspace of all continuously differentiable process x that belongs to the space $C(J, L_2(\Omega; H))$ consisting of $\mathcal{F}_t$- adapted measurable process such that the $\mathcal{F}_0$-adapted process $\varphi \in L_2^0(\Omega, \mathcal{B})$. Let $\|\cdot\|_{\mathcal{Z}}$ be a seminorm in $\mathcal{Z}$ defined by

$$\|x\|_{\mathcal{Z}} = \left(\sup_{t \in J} \|x_t\|_{\mathcal{B}}^2\right)^{\frac{1}{2}},$$

where

$$\|x_t\|_{\mathcal{B}} \leq N_T E\|\varphi\|_{\mathcal{B}} + M_T \sup\{E\|x(s)\| : 0 \leq s \leq T\},$$

$M_T = \sup_{t \in J}\{M(t)\}$, $N_T = \sup_{t \in J}\{N(t)\}$. It is easy to verify that $\mathcal{Z}$ furnished with the norm topology as defined above, is a Banach space.

The resolvent operator plays an important role in the study of the existence of solutions and to give a variation of constant formula for linear systems. However, we need to know when the linear system (2.1) has a resolvent operator. For more details on resolvent operator, reader may refer [13].

To obtain our results, consider the integrodifferential abstract Cauchy problem

$$dx(t) = \left[Ax(t) + \int_0^t f(t-s)x(s)ds\right]dt,\ t \geq 0, \tag{2.1}$$
$$x(0) = x_0 \in H.$$

Definition 2.1. [13] *A family of bounded linear operator $R(t) \in \mathcal{P}(H)$, $t \in J$ is called a resolvent operator for*

$$\frac{dx}{dt} = A\left[x(t) + \int_0^t f(t-s)x(s)ds\right],$$

if

(i) *$R(0) = I$, the identity operator on H;*
(ii) *for all $x \in H$, $R(t)x$ is continuous for $t > 0$;*
(iii) *$R(t) \in \mathcal{P}(Y)$, $t \in J$. For $x \in Y$, $R(\cdot)x \in C^1(J,H) \cap C(J,Y)$ and*

$$\begin{aligned}\frac{d}{dt}R(t)x &= A\left[R(t)x + \int_0^t f(t-s)R(s)xds\right]\\ &= R(t)Ax + \int_0^t R(t-s)Af(s)xds,\ for\, t \geq 0.\end{aligned}$$

In what we make the following assumptions:

(H1) A is the infinitesimal generator of a strongly continuous semigroup on H.
(H2) For all $t \geq 0$, $f(t)$ is a closed linear operator $D(A)$ to X, and $f(t) \in f(Y,H)$. For any $y \in Y$, the map $t \to f(t)y$ is bounded differentiable and the derivative $t \to f'(t)y$ is bounded and uniformly continuous on $\mathbb{R}^+$.

Let $0 \in \rho(A)$, then it is possible to define the fractional power (A^α), $0 < \alpha \leq 1$, as a closed linear operator with its domain $D(A^\alpha)$ being dense in H. If H_α represent the space $D(A^\alpha)$ endowed with the norm $\|\cdot\|$, which is equivalent to the graph norm of A^α, then we have the following properties:

Lemma 2.1. [20] *Assume that the following properties hold:*

(i) *If $A^\alpha : H_\alpha \to H_\alpha$, then H_α is a Banach space for $0 \leq \alpha \leq 1$.*
(ii) *If the resolvent operator operator of A is compact, then the embedding $H_\beta \subset H_\alpha$ is continuous and compact for $0 < \alpha \leq \beta$.*
(iii) *There exists a constant $M_\alpha > 0$ depending on $0 < \alpha \leq 1$ such that*

$$\|A^\alpha R(t)\| \leq \frac{M_\alpha}{t^\alpha},\ t > 0.$$

Remark 2.1. *Let us give some concrete functions $K(\cdot)$. Let $\gamma_0 >$ and $\delta(0,1)$ be sufficiently small. Define*

$$K_1(u) = \gamma_0 u, u \geq 0.$$

$$K_2(u) = \begin{cases} u\,log(u^{-1}), & 0 \leq u \leq \delta,\\ \delta\,log(\delta^{-1}) + K_2'(\delta-)(u-\delta), & u > \delta.\end{cases}$$

$$K_3(u) = \begin{cases} u\,log(u^{-1})loglog(u^{-1}), & 0 \leq u \leq \delta,\\ \delta\,log(\delta^{-1})loglog(\delta^{-1}) + K_3'(\delta-)(u-\delta), & u > \delta.\end{cases}$$

They are all concave nondecreasing functions satisfying $\int_{0^+} \frac{du}{K_j(u)} = +\infty (j = 1,2,3)$. In particular, we see that the Lipschitz condition is a special case of the proposed conditions.

In order to obtain the uniqueness of solutions, we give the Bihari inequality which appeared in reference [5].

Lemma 2.2. [5] *Let $T > 0$ and $u_0 \geq 0$, $u(t), v(t)$ be the continuous function on $[0,T]$. Let $K : \mathbb{R}^+ \to \mathbb{R}^+$ be a concave continuous and nondecreasing function such that $K(r) > 0$ for $r > 0$. If*

$$u(t) \leq u_0 + \int_0^t v(s)K(u(s))ds \, \textit{for all} \, 0 \leq t \leq T,$$

then

$$u(t) \leq G^{-1}\big(G(u_0) + \int_0^t v(s)ds\big) \, \textit{for all} \, t \in [0,T] \, \textit{such that}$$
$$G(u_0) + \int_0^t v(s)ds \in Dom(G^{-1}),$$

where $G(r) = \int_1^r \frac{ds}{K(s)}$ for $r \geq 0$ and G^{-1} is the inverse function of G. In particular, moreover if, $u_0 = 0$ and $\int_{0^+} \frac{ds}{K(s)} = \infty$, then $u(t) = 0$ for all $t \in [0,T]$.

Lemma 2.3. [22] *Let the assumption of Lemma 2.2 holds. If*

$$u(t) \leq u_0 + \int_0^t v(s)K(u(s))ds \, \textit{for all} \, 0 \leq t \leq T,$$

then

$$u(t) \leq G^{-1}\big(G(u_0) + \int_0^t v(s)ds\big) \, \textit{for all} \, t \in [0,T] \, \textit{such that}$$
$$G(u_0) + \int_0^t v(s)ds \in Dom(G^{-1}),$$

where $G(r) = \int_1^r \frac{ds}{K(s)}$ for $r \geq 0$ and G^{-1} is the inverse function of G.

Corollary 2.1. [22] *Let the assumption of Lemma 2.2 hold and $v(t) \geq 0$ for $t \in [0,T]$. If for all $\epsilon > 0$, there exists $t_1 \geq 0$ such that for $0 \leq u_0 \leq \epsilon$, $\int_{t_1}^T v(s)ds \leq \int_{u_0}^{\epsilon} \frac{ds}{K(s)}$ holds, then for every $t \in [t_1, T]$, the estimate $u(t) \leq \epsilon$ holds.*

Lemma 2.4. [8] *For any $r \geq 1$ and for arbitrary L_2^0-valued predictable process $\Psi(\cdot)$*

$$\sup_{s\in[0,t]} E\| \int_0^s \Psi(u)dw(u)\|_X^{2r} = (r(2r-1))^r \bigg(\int_0^t (E\|\Psi(s)\|_{L_2^0}^{2r})ds \bigg)^r.$$

Now, we present the definition of the mild solution of the system (1.1)-(1.2).

Definition 2.2. *A stochastic process $\{x(t) \in C(J, L_2(\Omega; H)), t \in (-\infty, T]\}$, $(0 < T < \infty)$, is said to be a mild solution of the equation (1.1)-(1.2) if*

(i) *$x(t) \in H$ is $\mathcal{F}_t$-adapted;*

(ii) *for each $t \in J$, $x(t)$ satisfies the following integral equation*

$$x(t) = \begin{cases} \varphi(t), \quad \textit{for} \quad t \in (-\infty, 0], \\ R(t)\big[\varphi(0) - g(0,\varphi)\big] + g(t, x_t) + \displaystyle\int_0^t AR(t-s)g(s, x_s)ds \\ + \displaystyle\int_0^t R(t-s)h(s, x_s)ds + \displaystyle\int_0^t R(t-s)\sigma(s, x_s)dw(s), \quad \textit{for a.s} \quad t \in [0, T]. \end{cases} \tag{2.2}$$

3. Existence and Uniqueness

In this section, we discuss the existence and uniqueness of mild solution of the system (1.1)-(1.2). We will work under the following assumptions:

(H3) A is the infinitesimal generator of a strongly continuous semigroup $R(t)$, whose domain $D(A)$ is dense in H such that $\|R(t)\|^2 \le M_1$, for all $t \in J$.

(H4) For each $x, y \in \mathcal{B}$ and for all $t \in [0, T]$ such that

$$\|h(t, x_t) - h(t, y_t)\|^2 \vee \|\sigma(t, x_t) - \sigma(t, y_t)\|^2 \le K(\|x_t - y_t\|^2_{\mathcal{B}}),$$

where $K(\cdot)$ is a concave non-decreasing function from $\mathbb{R}^+$ to $\mathbb{R}^+$, $K(0) = 0$, $K(u) > 0$, for $u > 0$ and $\int_{0^+} \frac{du}{K(u)} = \infty$.

(H5) Assuming that there exists a positive number $M_g > 0$ such that for any $x, y \in \mathcal{B}$ and for $t \in [0, T]$, we have

$$\|A^\alpha g(t, x_t) - A^\alpha g(t, y_t)\|^2 \le M_g \|x_t - y_t\|^2_{\mathcal{B}}.$$

(H6) For all $t \in [0, T]$, it follows that $G(t, 0), h(t, 0), A^\alpha g(t, 0) \in L^2$ such that

$$\|\sigma(t, 0)\|^2 \vee \|h(t, 0)\|^2 \vee \|A^\alpha g(t, 0)\|^2 \le \gamma_0,$$

where $\kappa_0 > 0$ is a constant.

Let us now introduce the successive approximation to equation (2.2) as follows

$$x^0(t) = \begin{cases} \varphi(t) \quad \text{for} \quad t \in (-\infty, 0], \\ R(t)\varphi(0) \quad \text{for} \quad t \in [0, T]. \end{cases} \tag{3.1}$$

and, for $n = 1, 2, \dots,$

$$x^n(t) = \begin{cases} \varphi(t), \quad \text{for} \quad t \in (-\infty, 0], \\ R(t)\big[\varphi(0) - g(0,\varphi)\big] + g(t, x^n_t) + \displaystyle\int_0^t AR(t-s)g(s, x^n_s)ds \\ + \displaystyle\int_0^t R(t-s)h(s, x^{n-1}_s)ds + \displaystyle\int_0^t R(t-s)\sigma(s, x^{n-1}_s)dw(s), \text{ for a.s, } t \in J. \end{cases} \tag{3.2}$$

with an arbitrary non-negative initial approximation $x^0 \in C(J, L_2(\Omega; H))$.

Theorem 3.1. *Assume that $(H3) - (H6)$ hold. Then the system (1.1)-(1.2) has unique mild solution $x(t)$ in $C(J, L_2(\Omega; H))$provided that $E\|x^n(t)\|^2 \le \widetilde{Q}$.*

Proof. Let $x^0 \in C(J, L_2(\Omega; H))$ be a fixed initial approximation to (3.2). To begin with our assumptions $(H3) - (H6)$ and observing that $\|R(t)\|^2 \le M_1$ for some

$M_1 \geq 1$ and for all $t \in [0, T]$. Then for any $n \geq 1$, we have

$$\begin{aligned}\|x^n(t)\|^2 &\leq 5M_1\|\big[\varphi(0) - g(0,\varphi)\big]\|^2 + 10\|A^{-\alpha}\|^2\big[\|A^\alpha g(t, x_t^n) - A^\alpha g(t,0)\|^2 \\ &\quad + \|A^\alpha g(t,0)\|^2\big] + 5T\int_0^t \|A^{1-\alpha}R(t-s)\|^2\big[\|A^\alpha g(s, x_s^n) - A^\alpha g(s,0)\|^2 \\ &\quad + \|A^\alpha g(s,0)\|^2\big]ds + 10M_1T\int_0^t [\|h(s, x_s^{n-1}) - h(s,0)\|^2 + \|h(s,0)\|^2]ds \\ &\quad + 10M_1\int_0^t [\|\sigma(s, x_s^{n-1}) - \sigma(s,0)\|^2 + \|\sigma(s,0)\|^2]ds.\end{aligned}$$

From the Lemma 2.1, (H5) and (H6), we get

$$\begin{aligned}&E\|\int_0^t AR(t-s)g(s, x_s^n)ds\|^2 \\ &= E\|\int_0^t A^{1-\alpha}R(t-s)A^\alpha g(t, x_t^n)ds\|^2 \\ &\leq 2T\int_0^t \frac{M_{1-\alpha}^2}{(t-s)^{2(1-\alpha)}}E\big[\|A^\alpha g(s, x_s^n) - A^\alpha g(s,0)\|^2\big] + 2E\|A^\alpha g(s,0)\|^2 ds \\ &\leq 2T^{2\alpha}\frac{M_{1-\alpha}^2}{2\alpha-1}\bigg[M_g E\|x_t^n\|_{\mathcal{B}}^2 + \kappa_0\bigg].\end{aligned}$$

Thus from the above, we have

$$\begin{aligned}E\|x_t^n\|_{\mathcal{B}}^2 &\leq 10M_1\big[E\|\varphi(0)\| + E\|g(0,\varphi)\|^2\big] + 10\|A^{-\alpha}\|^2\bigg[M_g E\|x_t^n\|_{\mathcal{B}}^2 + \kappa_0\bigg] \\ &\quad + 10T^{2\alpha}\frac{M_{1-\alpha}^2}{2\alpha-1}\bigg[M_g E\|x_t^n\|_{\mathcal{B}}^2 + \kappa_0\bigg] \\ &\quad + 10M_1(T+1)E\int_0^t [K(\|x_s^{n-1}\|_{\mathcal{B}}^2) + \kappa_0]ds \\ E\|x_t^n\|_{\mathcal{B}}^2 &\leq \frac{Q_1}{1-Q_2} + \frac{10M_1(T+1)}{1-Q_2}E\int_0^t K(\|x_s^{n-1}\|_{\mathcal{B}}^2)ds,\end{aligned}$$

where $Q_1 = 10M_1\big[E\|\varphi(0)\| + E\|g(0,\varphi)\|^2\big] + 10\big[M_1T(T+1) + \|A^{-\alpha}\|^2 + T^{2\alpha}\frac{M_{1-\alpha}^2}{2\alpha-1}\big]\kappa_0$ and $Q_2 = 10\bigg[\|A^{-\alpha}\|^2 + T^{2\alpha}\frac{M_{1-\alpha}^2}{2\alpha-1}\bigg]M_g$.

Given that $K(\cdot)$ is concave and $K(0) = 0$, we can find a pair of positive constants a and b such that

$$K(u) \leq a + bu, \text{ for all}\, u \geq 0.$$

Then, we have

$$
\begin{aligned}
E\|x_t^n\|_{\mathcal{B}}^2 &\le Q_3 + \frac{10M_1(T+1)b}{1-Q_2}\int_0^t E\|x_s^{n-1}\|_{\mathcal{B}}^2 ds \\
&\le Q_3 + \frac{10M_1(T+1)b}{1-Q_2}\int_0^t \big[N(t)E\|\varphi\|_{\mathcal{B}}^2 + M(t)\sup_{0\le s\le T} E\|x^{n-1}(s)\|^2\big]ds \\
&\le Q_3 + \frac{10M_1T(T+1)b}{1-Q_2}N_T E\|\varphi\|_{\mathcal{B}}^2 \\
&\quad + \frac{10M_1M_T(T+1)b}{1-Q_2}\int_0^t \sup_{0\le s\le T} E\|x^{n-1}(s)\|^2 ds,
\end{aligned}
$$

where $Q_3 = \frac{Q_1}{1-Q_2} + \frac{10M_1(T+1)Ta}{1-Q_2}$.

Therefore,

$$
E\|x_t^n\|_{\mathcal{B}}^2 \le Q_4 + \frac{10M_1M_T(T+1)b}{1-Q_2}\int_0^t \sup_{0\le s\le T} E\|x^{n-1}(s)\|^2 ds,\ n = 1, 2, \ldots, \quad (3.3)
$$

where, $Q_4 = Q_3 + \frac{10M_1T(T+1)b}{1-Q_2}N_T E\|\varphi\|_{\mathcal{B}}^2$.

For any $k \ge 1$, it follows from equation (3.3),

$$
\begin{aligned}
\max_{1\le n\le k} E\|x_t^n\|_{\mathcal{B}}^2 &\le Q_4 + \frac{10M_1M_T(T+1)b}{1-Q_2}\int_0^t \left[E\|x^0(s)\|^2 + E\max_{1\le n\le k}\left(\sup_{0\le s\le T}\|x^n(s)\|^2\right)\right]ds \\
&\le Q_4 + \frac{10M_1M_TT(T+1)b}{1-Q_2}E\|\varphi\|_{\mathcal{B}}^2 \\
&\quad + \frac{10M_1M_TT(T+1)b}{1-Q_2}\int_0^t E\max_{1\le n\le k}\left(\sup_{0\le s\le T}\|x^n(s)\|^2\right)ds \\
&\le Q_5 + \frac{10M_1M_TT(T+1)b}{1-Q_2}\int_0^t E\max_{1\le n\le k}\left(\sup_{0\le s\le T}\|x^n(s)\|^2\right)ds,
\end{aligned}
$$

where $Q_5 = Q_4 + \frac{10M_1M_TT(T+1)b}{1-Q_2}E\|\varphi\|_{\mathcal{B}}^2$ Therefore,

$$
\begin{aligned}
\max_{1\le n\le k} E\|x_t^n\|_{\mathcal{B}}^2 &\le Q_5 + \int_0^t E\max_{1\le n\le k}\left(\sup_{0\le s\le T}\|x^n(s)\|^2\right)ds \\
&\le Q_5 + Q_6\int_0^t E\max_{1\le n\le k}\left(\sup_{0\le s\le T}\|x^n(s)\|^2\right)ds,
\end{aligned}
$$

where $Q_6 = \frac{10M_1M_TT(T+1)b}{1-Q_2}$.

From the Gronwall inequality, we obtain that

$$
\max_{1\le n\le k} E\|x_t^n\|_{\mathcal{B}}^2 \le Q_5 e^{Q_6T}.
$$

While k is arbitrary, we have

$$
E\|x_t^n\|_{\mathcal{B}}^2 \le Q_5 e^{Q_6T},\ \text{for all},\ 0\le t\le T,\ n\ge 1.
$$

As a result, which holds with $\widetilde{Q} = \max\{Q_5e^{Q_6T}, E\|\varphi\|_{\mathcal{B}}^2\}$.

Lemma 3.5. *Under the assumption, there exists a positive constant N_1 such that*

$$
E\|x_t^{n+m} - x_t^n\|_{\mathcal{B}}^2 \le N_1\int_0^t K\big(E\|x_s^{n+m-1} - x_s^{n-1}\|_{\mathcal{B}}^2\big)ds
$$

for all $0\le t\le T$, n, $m\ge 1$.

Proof. From (3.2), n, $m \geq 1$ and $0 \leq t \leq T$, we derive that

$$\begin{aligned}
&x^{n+m}(t) - x^n(t)\\
&= R(t-s)\big[g(s, x_t^{n+m}) - g(s, x_t^n)\big] + \int_0^t R(t-s)\Big[h(s, x_s^{n+m-1} - h(s, x_s^{n-1}))\Big]ds\\
&\quad + \int_0^t R(t-s)\Big[\sigma(s, x_s^{n+m-1}) - \sigma(s, x_s^{n-1})\Big]dw(s).
\end{aligned}$$

Therefore, we have

$$\begin{aligned}
E\|x_t^{n+m} - x_t^n\|_{\mathcal{B}}^2 &\leq 3M_g E\|x_t^{n+m} - x_t^n\|_{\mathcal{B}}^2 + 3M_1(T+1)E\int_0^t K(\|x_s^{n+m-1} - x_s^{n-1}\|_{\mathcal{B}}^2)\\
&\leq \frac{3M_1(T+1)}{1-M_g} E\int_0^t K(\|x_s^{n+m-1} - x_s^{n-1}\|_{\mathcal{B}}^2)ds.
\end{aligned}$$

From the Jensen inequality, we get

$$E\|x_t^{n+m} - x_t^n\|_{\mathcal{B}}^2 \leq \frac{3M_1(T+1)}{1-M_g}\int_0^t K\Big(E\|x_s^{n+m-1} - x_s^{n-1}\|_{\mathcal{B}}^2\Big)ds.$$

If we prefer $N_1 = \frac{3M_1(T+1)}{1-M_g}$, we acquire the desired results. ☐

Lemma 3.6. *Under assumptions, there exists a positive constant N_2 such that*

$$E\|x_t^{n+m} - x_t^n\|^2 \leq N_2 t \tag{3.4}$$

for all $0 \leq t \leq T$, n, $m \geq 1$.

Proof. From Theorem 3.1 and Lemma 3.5, we have

$$\begin{aligned}
E\|x_t^{n+m} - x_t^n\|^2 &\leq N_1\int_0^t K\Big(E\|x_s^{n+m-1} - x_s^{n-1}\|_{\mathcal{B}}^2\Big)ds\\
&\leq N_1\int_0^t K\Big(2\widetilde{Q}\Big)ds\\
&\leq N_1 K(2N_1)t = N_2 t.
\end{aligned}$$

Hence the proof.

Define

$$\begin{aligned}
\zeta_1(t) &= N_2 t,\\
\zeta_{n+1}(t) &= N_1\int_0^t K(\zeta_n(s))ds,\ n \geq 1,\\
\zeta_{n,m}(t) &= E\|x_t^{n+m} - x_t^n\|^2,\ n, m \geq 1.
\end{aligned}$$

Choose $T_1 \in [0, T[$ such that

$$N_1 K(N_2 t) \leq N_2, \text{ for all } 0 \leq t \leq T_1.$$

☐

Lemma 3.7. *There exists a positive $0 \leq T_1 < T$ such that for all n, $m \geq 1$,*

$$0 \leq \zeta_{n,m}(t) \leq \zeta_n(t) \leq \zeta_{n-1}(t) \leq \ldots \leq \zeta_1(t) \tag{3.5}$$

for all $0 \leq t \leq T_1$.

Proof. We prove this Lemma by induction with respect to n. By Lemma 3.6, we have

$$\zeta_{1,m}(t) = E\|x_t^{1+m} - x_t^1\|^2 \leq N_2 t = \zeta_1(t)$$

By Lemma 3.6,

$$\begin{aligned}\zeta_{2,m}(t) &= E\|x_t^{1+m} - x_t^1\|^2 \\ &\leq N_1 \int_0^t K\big(E\|x_t^{2+m} - x_t^2\|^2\big)ds \\ &\leq N_1 \int_0^t K\big(\zeta_{1,m}(s)\big)ds \\ &\leq N_1 \int_0^t K\big(\zeta_1(s)\big)ds = \zeta_1(t).\end{aligned}$$

Thus, we also have

$$\begin{aligned}\zeta_2(t) = N_1 \int_0^t K\big(\zeta_1(s)\big)ds &\leq N_1 \int_0^t K\big(N_2 s\big)ds \\ \leq N_1 \int_0^t N_2 ds = \zeta_1(t).\end{aligned}$$

We have previously shown that

$$\zeta_{2,m}(t) \leq \zeta_2(t) \leq \zeta_1(t), \text{ for all } 0 \leq t \leq T_1.$$

Now, we assume that (3.5) holds for some $n \geq 1$. Then, using the same inequalities as above capitulate that

$$\begin{aligned}\zeta_{n+1,m}(t) &= N_1 \int_0^t K\big(E\|x_t^{2+m} - x_t^2\|^2\big)ds \\ &\leq N_1 \int_0^t K\big(\zeta_{n,m}(s)\big)ds \\ &\leq N_1 \int_0^t K\big(\zeta_n(s)\big)ds = \zeta_{n+1}(t)\end{aligned}$$

for all $0 \leq t \leq T_1$. On other hand, we have

$$\zeta_{n+1}(t) = N_1 \int_0^t K\big(\zeta_n(s)\big)ds \leq N_1 \int_0^t K\big(\zeta_{n-1}(s)\big)ds = \zeta_n(t),$$

for all $0 \leq t \leq T_1$. Hence the proof. □

Now, we prove the uniqueness of the solutions of (2.2). Let $x, y \in C(J, L_2(\Omega; H))$ be the two solution of (1.1)-(1.2) on some interval $(-\infty, T]$. Then, for $t \in (-\infty, 0]$, the uniqueness is obvious and for $0 \leq t \leq T$, we have

$$\begin{aligned}E\|x(t) - y(t)\|^2 \leq 4\Big[\|A^{-\alpha}\|^2 + T^{2\alpha}\frac{M_{1-\alpha}^2}{2\alpha - 1}\Big] M_g E\|x_t - y_t\|_{\mathcal{B}}^2 \\ + 4M_1(T+1)\int_0^t K(E\|x_s - y_s\|_{\mathcal{B}}^2)ds.\end{aligned}$$

Thus,

$$E\|x_t - y_t\|_{\mathcal{B}}^2 \leq Q_7 E\|x_t - y_t\|_{\mathcal{B}}^2 + Q_8 \int_0^t K(E\|x_s - y_s\|_{\mathcal{B}}^2) ds$$
$$\leq \frac{Q_8}{1 - Q_7} \int_0^t K(E\|x_s - y_s\|_{\mathcal{B}}^2) ds.$$

Thus, Bihari's inequality yield that

$$\sup_{t \in [0,T]} E\|x_t - y_t\|_{\mathcal{B}}^2 = 0, \; 0 \leq t \leq T. \tag{3.6}$$

Thus, $x(t) = y(t)$, for all $0 \leq t \leq T$. Therefore, for all $-\infty \leq t \leq T$, $x(t) = y(t)$. This completes the proof.

Existence: We claim that

$$E\|x_t^{n+m} - x_t^n\|^2 \to 0 \tag{3.7}$$

for all $0 \leq t \leq T_1$, as n, $m \to \infty$. Note that ζ_n is continuous on $[0, T_1]$. Note also that for each $n \geq 1$, $\zeta_n(\cdot)$ is decreasing on $[0, T_1]$ and for each t, $\zeta_n(t)$ is decreasing sequence. Thus we define the function $\zeta(t)$ as

$$\zeta(t) = \lim_{n\to\infty} \zeta_n(t) = N_1 \lim_{n\to\infty} \int_{t_0}^t K(\zeta_{n-1}(s)) ds = N_1 \int_{t_0}^t K(\zeta(s)) ds$$

for all $0 \leq t \leq T_1$. The Bihari inequality implies that $\zeta(t) = 0$ for all $0 \leq t \leq T_1$. Now from Lemma 3.7, we have

$$\zeta_{n,n}(t) \leq \sup_{t_0 \leq t \leq T_1} \zeta_n(t) \leq \zeta_n(T_1) \to 0$$

as $n \to \infty$. That is $x^n(t)$ is a Cauchy sequence in L^2 on $]-\infty, T_1]$. From Theorem 3.1, we derive that

$$\|x(t)\|^2 \leq Q,$$

where Q is a positive constant. □

On the other hand, by (H4) then, letting $n \to \infty$, we can also claim that for $t \in [0, T]$

$$E\|\int_0^t R(t-s)\Big[h(t, x_s^{n-1}) - h(t, x_s)\Big] ds\|_{\mathcal{B}}^2 \to 0,$$
$$E\|\int_0^t R(t-s)\Big[\sigma(t, x_s^{n-1}) - \sigma(t, x_s)\Big] dw(s)\|_{\mathcal{B}}^2 \to 0.$$

On further, by applying (H5), we can also assert, for $t \in [0, T]$, that

$$E\|g(s, x_s^n) - g(s, x_s)\|^2 \leq M_g E \sup_{0 \leq s \leq t} \|x^n(s) - x(s)\|_{\mathcal{B}}^2 \to 0$$

$$E\|\int_0^t AR(t-s)\Big[g(t, x_s^n) - g(t, x_s)\Big] ds\|^2 \leq 2T^{2\alpha} \frac{M_{1-\alpha}^2}{2\alpha - 1} E\|x^n(s) - x(s)\|_{\mathcal{B}}^2 \to 0.$$

At this instant, taking limits in both sides of (3.2) leads, for $t \geq 0$, to

$$x(t) = R(t)\big[\varphi(0) - g(0, \varphi)\big] + g(t, x_t) + \int_0^t AR(t-s) g(s, x_s) ds$$
$$+ \int_0^t R(t-s) h(s, x_s) ds + \int_0^t R(t-s)\sigma(s, x_s) dw(s).$$

Thus, the above term exhibit that $x(t)$ is a mild solution of (1.1)-(1.2) on the interval $[0, T_1]$. By iteration, the existence of solutions (1.1)-(1.2) on $[0, T]$ can be obtained.

4. Stability

In this section, we study the stability through the continuous dependence on initial values.

Definition 4.3. A mild solution $x(t)$ of the system (1.1)-(1.2) with initial value φ is said to be stable in the mean square if for all $\epsilon > 0$, there exists $\delta > 0$ such that

$$E\|x_t - \widehat{x}_t\|^2_{\mathcal{B}} \le \epsilon \quad \text{whenever} \quad E\|\varphi - \widehat{\varphi}\|^2 \le \delta \quad \text{for all} \quad t \in [0, T],$$

where $\widehat{x}(t)$ is another mild solution of the system (1.1)-(1.2) with initial data $\widehat{\varphi}$.

Theorem 4.2. *Let $x(t)$ and $y(t)$ be the mild solution of the system (1.1)-(1.2) with initial values φ_1 and φ_2 respectively. If the assumption of Theorem 3.1 are satisfied, then the mild solution of the system (1.1)-(1.2) is stable in the mean square.*

Proof. By the assumption, $x(t)$ and $y(t)$ are two mild solutions of equations (1.1)-(1.2) with initial values φ_1 and φ_2 respectively, then for $0 \le t \le T$,

$$\begin{aligned} x(t) - y(t) &= R(t)\big[\varphi_1(0) - \varphi_2(0) + g(0, \varphi_1) - g(0, \varphi_2)\big] + \big[g(t, x_t) - g(t, y_t)\big] \\ &\quad + \int_0^t AR(t-s)\big[g(s, x_s) - g(s, y_s)\big]ds + \int_0^t R(t-s)\big[h(s, x_s) - h(s, y_s)\big]ds \\ &\quad + \int_0^t R(t-s)\big[\sigma(s, x_s) - \sigma(s, y_s)\big]dw(s). \end{aligned}$$

So, estimating as before, we get

$$\begin{aligned} &E\|x(t) - y(t)\|^2 \\ &\le 6M_1\big[1 + \|A^{-\alpha}\|^2 M_g\big]E\|\varphi_1 - \varphi_2\|^2 + 6\left[\|A^{-\alpha}\|^2 + T^{2\alpha}\frac{M^2_{1-\alpha}}{2\alpha - 1}\right]M_g E\|x_t - y_t\|^2_{\mathcal{B}} \\ &\quad + 6M_1(T+1)\int_0^t K(E\|x_s - y_s\|^2_{\mathcal{B}})ds. \end{aligned}$$

Thus,

$$\begin{aligned} E\|x_t - y_t\|^2_{\mathcal{B}} &\le \frac{6M_1\big[1 + \|A^{-\alpha}\|^2 M_g\big]}{1 - Q_{10}}E\|\varphi_1 - \varphi_2\|^2 \\ &\quad + \frac{6M_1(T+1)}{1 - Q_{10}}\int_0^t K(E\|x_s - y_s\|^2_{\mathcal{B}})ds, \end{aligned}$$

where $Q_{10} = 6\left[\|A^{-\alpha}\|^2 + T^{2\alpha}\frac{M^2_{1-\alpha}}{2\alpha-1}\right]M_g$.

Let $K_1(u) = \frac{6M_1(T+1)}{1-Q_{10}}K(u)$, where K is concave increasing function from $\mathbb{R}^+$ to $\mathbb{R}^+$ such that $K(0) = 0$, $K(u) > 0$ for $u > 0$ and $\int_{0+}\frac{du}{K(u)} = +\infty$. So, $K_1(u)$ is obviously, a concave function from $\mathbb{R}^+$ to $\mathbb{R}^+$ such that $K_1(0) = 0$, $K_1(u) \ge K(u)$, for $0 \le u \le 1$ and $\int_{0+}\frac{du}{K_1(u)} = +\infty$. Now for any $\epsilon > 0$, $\epsilon_1 = \frac{1}{2}\epsilon$, we have $\lim\limits_{s\to 0}\displaystyle\int_s^{\epsilon_1}\frac{du}{K_1(u)} = \infty$. So, there is a positive constant $\delta < \epsilon_1$, such that $\int_\delta^{\epsilon_1}\frac{du}{K_1(u)} \ge T$.

Let

$$u_0 = \frac{6M_1\big[1+\|A^{-\alpha}\|^2 M_g\big]}{1-Q_{10}} E\|\varphi_1-\varphi_2\|^2,$$
$$u(t) = E\|x_t-y_t\|^2_{\mathcal{B}},\ v(t)=1,$$

when $u_0 \le \delta \le \epsilon_1$. From Corollary 2.1 we have

$$\int_{u_0}^{\epsilon_1} \frac{du}{K_1(u)} \ge \int_{\delta}^{\epsilon_1} \frac{du}{K_1(u)} \ge T = \int_0^T v(s)ds.$$

So, for any $t\in[0,T]$, we estimate $u(t)\le \epsilon_1$ holds. This completes the proof. □

5. Example

Consider the following stochastic partial integrodifferential equation of the form

$$d\Big[u(t,\xi)+\int_0^\pi a(y,\xi)u(tsint,y)dy\Big] = \frac{\partial^2}{\partial\xi^2}\Big[u(t,\xi)+\int_0^t f(t-s)u(s,\xi)ds\Big]dt$$
$$+H(t,u(tsint,\xi))dt+G(t,u(tsint,\xi))d\beta(t),$$
$$0\le\xi\le\pi,\ \tau>0, t\in J=[0,T], \tag{5.1}$$
$$u(t,0)=u(t,\pi)=0,\quad t\in J, \tag{5.2}$$
$$u(\theta,\xi)=\varphi(\theta,\xi),\quad \theta\in(-\infty,0],\quad 0\le\xi\le\pi, \tag{5.3}$$

where $\beta(t)$ denotes a standard cylindrical Wiener process in H defined on a stochastic process $(\Omega,\mathcal{F},P)$ and $H=L^2([0,\pi])$.

To rewrite (5.1)-(5.3) into the form (1.1)-(1.2), define $A: H\to H$ by $Az=z''$ with domain
$$D(A)=\Big\{z\in H, z, z'\text{are absolutely continuous} z''\in H, z(0)=z(\pi)=0\Big\}.$$

Then, A generates a strongly continuous semigroup $R(t)$ on H, thus (H1) is true. Moreover, the operator A can be expressed as

$$Az=\sum_{n=1}^{\infty} n^2 < z, z_n > z_n,\quad z\in D(A),$$

where $z_n(s)=\sqrt{\frac{2}{\pi}}sin(ns)$, $n=1,2,\ldots$, is orthonormal set of eigenvectors of A.

We assume the following condition hold:

(i) The function b is measurable and

$$\int_0^\pi a^2(y,\xi)dyd\xi<\infty.$$

(ii) The function $\frac{\partial}{\partial t}b(y,\xi)$ is measurable $b(y,0)=b(y,\pi)=0$ and let

$$M_g=\Big[\int_0^\pi\int_0^\pi \big(\frac{\partial}{\partial t}a(t,\xi)\big)^2 dyd\xi\Big]^{\frac{1}{2}}<\infty.$$

Let $\alpha<0$, define the phase space

$$\mathcal{B}=\Big\{\phi\in C((-\infty,0],H): \lim_{\theta\to -r} e^{\alpha\theta}\phi(\theta)\quad \text{exists in H}\Big\},$$

and let $\|\phi\|_{\mathcal{B}} = \sup_{\theta\in(-\infty,0]} \{e^{\alpha\theta}\|\phi(\theta)\|_{L_2}\}$. Then, $(\mathcal{B}, \|\cdot\|_{\mathcal{B}})$ is a Banach space and satisfied axioms(1)-(2) with $L = 1$, $N(t) = e^{-\alpha t}$, $M(t) = \max\{1, e^{-\alpha t}\}$. Thus for $(t,\phi) \in J \times \mathcal{B}$, where $\phi(\theta)(\xi) = \varphi(\theta,\xi)$, $(\theta,\xi) \in (-\infty,0] \times [0,\pi]$.

Suppose that conditions (i) and (ii) are verified, then the problem (5.1)-(5.3) can be represent as the abstract neutral stochastic integrodifferential equation of the form (1.1)-(1.2), as follows

$$g(t,x_t) = \int_0^\pi a(y,\xi)u(tsint,y)dy,\ h(t,x_t) = H(t,u(tsint,\xi)),$$
$$\sigma(t,x_t) = G(t,u(tsint,\xi))$$

Hence, we can conclude that the system (5.1)-(5.3) has unique mild solution on J.

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