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FUZZY SOFT θ- CLOSURE OPERATOR IN FUZZY SOFT TOPOLOGICAL SPACES

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Abstract. In this paper, the concept of fuzzy soft θ- closure operation is introduced and studied. We observe that fuzzy soft θ- closure operator on a fuzzy soft topological space (X, E, τ) does not satisfy the Kuratowski closure axioms. Fuzzy soft θ- continuous mapping is introduced and characterized in different ways. Finally, an attempt has made to introduce and characterize fuzzy soft θ- connectedness.

1. Introduction

Uncertainty is the faith of this world and all its creatures. Traditional mathematical tools are not sufficient to handle all the practical problems in many disciplines such as medical science, social science, economics, engineering, environment etc involving uncertainty of various types. In 1965, Zadeh [16] was the first to come up with his remarkable theory of fuzzy set for dealing these types of uncertainties where conventional tools fail. His theory brought a grand paradigmatic change in mathematics but this theory has their inherent difficulties [10]. The reason for these difficulties is possibly the inadequacy of parameterization tool of the theories as point out by Molodtsov in [10].
In 1999, Molodtsov [10] introduced the concept of soft sets as a mathematical tool for dealing with uncertainties which is free from the above mentioned difficulties. Shabir and Naz [13] studied the topological structures of soft sets.
In recent times, fuzzification of soft set theory is progress rapidly. Combining fuzzy sets and soft sets, Maji et al.[9] put forward a new model known as fuzzy soft set. Kharal and Ahmad [8]defined the concept of mapping on fuzzy soft classes. Topological structure of fuzzy soft sets was started by Tanay and Burc Kandemir [14]. The study was pursued by some others [1, 2, 3, 4, 5, 6, 7, 11, 12, 15].
In the present paper, we introduce fuzzy soft θ- closure, fuzzy soft θ- open set, fuzzy soft θ- closed set and fuzzy soft θ- interior and some of their basic properties are studied. Fuzzy soft θ- continuous mapping, fuzzy soft θ- open mapping, fuzzy

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soft θ- closed mapping is introduced and some basic properties are studied. The concept of fuzzy soft θ- connectedness is introduced and studied.

2. Preliminaries

Throughout this paper X denotes initial universe, E denotes the set of all possible parameters for X, $P(X)$ denotes the power set of X, I^X denotes the set of all fuzzy sets on X and I stands for $[0,1]$.

Definition 2.1 [16] A fuzzy set A in X is defined by a membership function $\mu_A : X \to [0,1]$ whose value $\mu_A(x)$ represents the "grade of membership" of x in A for $x \in X$.

If A, $B \in I^X$ then from [16] we have the following:
(i) $A \leq B \Leftrightarrow \mu_A(x) \leq \mu_B(x),\ \forall\ x \in X$
(ii) $A = B \Leftrightarrow \mu_A(x) = \mu_B(x),\ \forall\ x \in X$
(iii) $C = A \vee B \Leftrightarrow \mu_C(x) = \max(\mu_A(x), \mu_B(x)),\ \forall\ x \in X$
(iv) $D = A \wedge B \Leftrightarrow \mu_D(x) = \min(\mu_A(x), \mu_B(x)),\ \forall\ x \in X$
(v) $E = A^c \Leftrightarrow \mu_E(x) = 1 - \mu_A(x),\ \forall\ x \in X$

Definition 2.2 [10] Let $A \subseteq E$. A pair (F, A) is called a soft set over X if F is a mapping from A into $P(X)$, i.e. $F : A \to P(X)$.
In other words, a soft set is a parameterized family of subsets of the set X. For $e \in A$, $F(e)$ may be considered as the set of $e-$approximate elements of the soft set (F, A).

Definition 2.3 [9] Let $A \subseteq E$. A pair (f, A) is called a fuzzy soft set over X if $f : A \to I^X$ is a function, i.e., for each $a \in A$, $f(a) = f_a : X \to [0,1]$ is a fuzzy set on X.

We will use $FS(X,E)$ to denote the family of all fuzzy soft sets over X.

Modification in the above definition was done by Roy and Samanta [12],which goes as follows.

Definition 2.4 [12] Let $A \subseteq E$. A fuzzy soft set f_A over X is a mapping from the parameter set E to I^X, i.e. $f_A : E \to I^X$, where $f_A(e) \neq 0_X$ if $e \in A \subseteq E$ and $f_A(e) = 0_X$ if $e \notin A$,where 0_X denotes the empty fuzzy set on X.

Definition 2.5 [12] The fuzzy soft set $f_\phi \tilde{\in} FS(X,E)$ is called null fuzzy soft set,denoted by $\tilde{0}_E$, if for all $e \in E$, $f_A(e) = 0_X$.

Definition 2.6 [12] Let $f_E \tilde{\in} FS(X,E)$.The fuzzy soft set f_E is called universal fuzzy soft set,denoted by $\tilde{1}_E$, if for all $e \in E$, $f_E(e) = 1_X$ where $1_X(x) = 1$ for all $x \in X$.

Definition 2.7 [9] Let $A, B \subseteq E$ and $f_A, g_B \tilde{\in} FS(X,E)$. We say that f_A is a fuzzy soft subset of g_B and write $f_A \sqsubseteq g_B$ if and only if
(1) $A \subseteq B$.
(2) For every $e \in E$, $f_A(e) \leq g_B(e)$.

Definition 2.8 [9] Let $f_A, g_B \tilde{\in} FS(X,E)$. Then the union of f_A and g_B, denoted by $h_C = f_A \sqcup g_B$, where $C = A \cup B$ and h_C is defined by

$$h_C = \begin{cases} f_A(e) & \text{if e} \in A - B, \\ g_B(e) & \text{if e} \in B - A, \\ f_A(e) \vee g_B(e) & \text{if e} \in A \cap B. \end{cases}$$

Definition 2.9 [9] Let $f_A, g_B \tilde{\in} FS(X, E)$. Then the intersection of f_A and g_B is also a fuzzy soft set h_C,defined by $h_C(e) = f_A(e) \wedge g_B(e)$ for all $e \in E$,where $C = A \cap B$. Here we write $h_C = f_A \sqcap g_B$.

Definition 2.10 [9] Let $f_A \tilde{\in} FS(X, E)$. Then the complement of f_A,denoted by ${f_A}^c$,is a fuzzy soft set defined by ${f_A}^c(e) = 1_X \setminus f_A(e), \forall e \in E$.
Let us call ${f_A}^c$ to be the fuzzy soft complement of f_A in $FS(X, E)$.

Clearly $({f_A}^c)^c = f_A$, $(\tilde{1}_E)^c = \tilde{0}_E$ and $(\tilde{0}_E)^c = \tilde{1}_E$.

Definition 2.11 [2] A fuzzy soft set $f_A \tilde{\in} FS(X, E)$ is called fuzzy soft point if $A = \{e\} \subseteq E$ and $f_A(e)$ is a fuzzy point in X, i.e., there exists $x \in X$ such that $f_A(e)(x) = \alpha(0 < \alpha \leq 1)$ and $f_A(e)(y) = 0$ for all $y \in X - \{x\}$. We denoted by e_x^α.

Definition 2.12 [2] Let $f_A, g_B \tilde{\in} FS(X, E)$. Then f_A is said to be soft quasi-coincident with g_B, denoted by $f_A \tilde{q} g_B$, if there exist $e \in E$ and $x \in X$ such that $f_A(e)(x) + g_B(e)(x) > 1$.

If f_A is not soft quasi-coincident with g_B, then we write $f_A \bar{\tilde{q}} g_B$.

Definition 2.13 [8] Let $FS(X, E)$ and $FS(Y, K)$ be families of all fuzzy soft sets over X and Y, respectively. Let $u : X \to Y$ and $p : E \to K$ be two functions. Then f_{up} is called a fuzzy soft mapping from $FS(X, E)$ to $FS(Y, K)$ and denoted by $f_{up} : FS(X, E) \to FS(Y, K)$.
(1) Let $f_A \tilde{\in} FS(X, E)$. Then the image of f_A under the fuzzy soft mapping f_{up} is the fuzzy soft set over Y defined by $f_{up}(f_A)$, where

$$f_{up}(f_A)(k)(y) = \begin{cases} \bigvee\limits_{x \in u^{-1}(y)} (\bigvee\limits_{e \in p^{-1}(k) \cap A} f_A(e))(x) & \text{if } \mathrm{u}^{-1}(\mathrm{y}) \neq \phi \text{ and if } \mathrm{p}^{-1}(\mathrm{k}) \cap \mathrm{A} \neq \phi \\ 0_Y & \text{otherwise.} \end{cases}$$

(2) Let $g_B \tilde{\in} FS(X, E)$. Then the pre-image of g_B under the fuzzy soft mapping f_{up} is the fuzzy soft set over X defined by ${f_{up}}^{-1}(g_B)$, where

$${f_{up}}^{-1}(g_B)(e)(x) = \begin{cases} g_B(p(e))(u(x)) & \text{for } \mathrm{p(e)} \in \mathrm{B} \\ 0_X & \text{otherwise.} \end{cases}$$

If u and p are injective then the fuzzy soft mapping f_{up} is said to be injective. If u and p are surjective then the fuzzy soft mapping f_{up} is said to be surjective. The fuzzy soft mapping f_{up} is called constant if u and p are constant.

Definition 2.14 [14] A fuzzy soft topological space is a triple (X, E, τ) where X is a non-empty set and τ is a family of fuzzy soft sets over (X, E) satisfying the following properties:

(1) $\tilde{0}_E, \tilde{1}_E \tilde{\in} \tau$
(2) If $f_A, g_B \tilde{\in} \tau$, then $f_A \sqcap g_B \tilde{\in} \tau$
(3) If $f_{A_\alpha} \tilde{\in} \tau$ for all $\alpha \in \Lambda$, an index set, then $\bigsqcup\limits_{\alpha \in \Lambda} f_{A_\alpha} \tilde{\in} \tau$

Then τ is called a fuzzy soft topology over (X, E). Also each member of τ is called a fuzzy soft open set in (X, E, τ).

g_B is called fuzzy soft closed in (X, E, τ) if $(g_B)^c \tilde{\in} \tau$.

Definition 2.15 [15] Let (X, E, τ) be a fuzzy soft topological space and $f_A \tilde{\in} FS(X, E)$. The fuzzy soft closure of f_A denoted by $\overline{f_A}$ is the intersection of all fuzzy soft closed supersets of f_A.

Clearly, $\overline{f_A}$ is the smallest fuzzy soft closed set over (X, E) which contains f_A.

Definition 2.16 [14] Let (X, E, τ) be a fuzzy soft topological space and $f_A \tilde{\in} FS(X, E)$. The fuzzy soft interior of f_A denoted by ${f_A}^0$ is the union of all fuzzy soft open subsets of f_A.

Clearly, ${f_A}^0$ is the largest fuzzy soft open set over (X, E) which contained in f_A.

Theorem 2.17 [6] Let (X, E, τ) be a fuzzy soft topological space and $f_A \tilde{\in} FS(X, E)$. Then

(1) $({f_A}^0)^c = \overline{({f_A}^c)}$
(2) $(\overline{f_A})^c = ({f_A}^c)^0$

Definition 2.18 [2] Let (X, E, τ) be a fuzzy soft topological space and $f_A \tilde{\in} FS(X, E)$ is called fuzzy soft q-neighbourhood(briefly fuzzy soft *q-nbd*) of g_B only if there exists a fuzzy soft open set h_C in τ such that $g_B \tilde{q} h_C \sqsubseteq f_A$.

If, in addition, f_A is fuzzy soft open then f_A is called a fuzzy soft open *q-nbd* of g_B.

Definition 2.19 [3] Let (X, E, τ_1) and (X, K, τ_2) be two fuzzy soft topological spaces. A fuzzy soft function $f_{up} : (X, E, \tau_1) \to (X, K, \tau_2)$ is called a fuzzy soft continuous if ${f_{up}}^{-1}(g_B) \tilde{\in} \tau_1$ for all $g_B \tilde{\in} \tau_2$.

3. Fuzzy soft θ- open and fuzzy soft θ- closed sets

In this section, we define fuzzy soft θ- closure, fuzzy soft θ- open and fuzzy soft θ- closed sets and its related properties.

Definition 3.1 A fuzzy soft point e_x^α is called a fuzzy soft θ- cluster point of a fuzzy soft set f_A in an fuzzy soft topological space (X, E, τ) if and only if fuzzy soft closure of every fuzzy soft open *q-nbd* of e_x^α is soft quasi-coincident with f_A.

The union of all fuzzy soft θ- cluster points of f_A is called the fuzzy soft θ- closure of f_A and is denoted by $fs\theta cl(f_A)$.

Remark 3.2 For a fuzzy soft set f_A in a fuzzy soft topological space (X, E, τ), $f_A \sqsubseteq \overline{f_A} \sqsubseteq fs\theta cl(f_A)$.

Theorem 3.3 For a fuzzy soft open set f_A in a fuzzy soft topological space (X, E, τ), $\overline{f_A} = fs\theta cl(f_A)$.

Proof: From Remark 3.2 it is sufficient to show that $fs\theta cl(f_A) \sqsubseteq \overline{f_A}$. Let e_x^α be a fuzzy soft point in (X, E, τ) such that $e_x^\alpha \tilde{\notin} \overline{f_A}$. Then there exists a fuzzy soft open q-*nbd* g_B of e_x^α such that $g_B \overline{\tilde{q}} f_A$. Then $\overline{g_B} \sqsubseteq \overline{(\tilde{1}_E - f_A)} = \tilde{1}_E - f_A$. Thus $\overline{g_B} \overline{\tilde{q}} f_A$ and consequently $e_x^\alpha \tilde{\notin} fs\theta cl(f_A)$.

Theorem 3.4 Let (X, E, τ) be a fuzzy soft topological space and $f_A \tilde{\in} FS(X, E)$. Then $fs\theta cl(f_A) = \sqcap\{\overline{g_B} : f_A \sqsubseteq g_B \in \tau\}$.

Proof: Obviously, $L.H.S \sqsubseteq R.H.S$. Now if possible, let $e_x^\alpha \tilde{\in} R.H.S$ but $e_x^\alpha \tilde{\notin} fs\theta cl(f_A)$. Then there exists a fuzzy soft open q-nbd g_B of e_x^α such that $\overline{g_B}\overline{\tilde{q}}f_A$ and hence $f_A \sqsubseteq (\tilde{1}_E - \overline{g_B})$. Then $e_x^\alpha \tilde{\in} fs\theta cl(\tilde{1}_E - \overline{g_B})$ and consequently, $\overline{g_B}\tilde{q}(\tilde{1}_E - \overline{g_B})$ which is impossible and this completes the proof.

Definition 3.5 Let (X, E, τ) be a fuzzy soft topological space and $f_A \tilde{\in} FS(X, E)$. Then f_A is called a fuzzy soft θ- closed if and only if $f_A = fs\theta cl(f_A)$.

The complement of a fuzzy soft θ- closed set is a called a fuzzy soft θ- open set.

Theorem 3.6 Let (X, E, τ) be a fuzzy soft topological space. Then

(1) the fuzzy soft sets $\tilde{0}_E$ and $\tilde{1}_E$ are fuzzy soft θ- closed;

(2) for any two fuzzy soft sets f_A and g_B, if $f_A \sqsubseteq g_B$ then $fs\theta cl(f_A) \sqsubseteq fs\theta cl(g_B)$;

(3) the union of any two fuzzy soft θ- closed sets is a fuzzy soft θ- closed set in (X, E, τ);

(4) the intersection of any collection of fuzzy soft θ- closed sets is a fuzzy soft θ- closed set in (X, E, τ).

Proof: (1) and (2) are obviously true.

(3) Let f_A and g_B be any two fuzzy soft θ- closed sets in (X, E, τ). Let e_x^α be a fuzzy soft point in (X, E, τ) such that $e_x^\alpha \tilde{\notin} (f_A \sqcup g_B)$. Then there exist a fuzzy soft open q-nbd h_C and j_D of e_x^α such that $\overline{h_C}\overline{\tilde{q}}f_A$ and $\overline{j_D}\overline{\tilde{q}}g_B$. Now $h_C \sqcap j_D$ is a fuzzy soft open q-nbd of e_x^α, and for any $e \in E$ and $x \in X$ we have

$\overline{(h_C \sqcap j_D)}(e)(x) + (f_A \sqcup g_B)(e)(x) \leq (\overline{h_C} \wedge \overline{j_D})(e)(x) + \max(f_A(e)(x), g_B(e)(x))$
$\leq \min(\overline{h_C}(e)(x), \overline{j_D}(e)(x)) + \max(f_A(e)(x), g_B(e)(x))$
≤ 1

Thus $\overline{(h_C \sqcap j_D)}\overline{\tilde{q}}(f_A \sqcup g_B)$, which shows that $e_x^\alpha \tilde{\notin} fs\theta cl(f_A \sqcup g_B)$. Hence $(f_A \sqcup g_B) = fs\theta cl(f_A \sqcup g_B)$ and $(f_A \sqcup g_B)$ is fuzzy soft θ- closed.

(4) The proof is straight forward.

Remark 3.7 The fuzzy soft θ- closure operation on a fuzzy soft topological space (X, E, τ) does not satisfy the Kuratowski closure axioms, which follows from the following example.

Example 3.8 Let $X = \{a, b\}$, $E = \{e_1, e_2, e_3\}$, $A = \{e_1, e_2\} \subseteq E$, $B = \{e_1, e_3\} \subseteq E$, $C = \{e_2, e_3\} \subseteq E$, $D = \{e_1\} \subseteq E$, $F = \{e_2\} \subseteq E$, $G = \{e_3\} \subseteq E$
Let us consider the following fuzzy soft sets over (X, E).

$f_A = \{f(e_1) = \{a/0.5, b/0.6\}, f(e_2) = \{a/0.7, b/0.4\}, f(e_3) = \{a/0, b/0\}\}$

$g_B = \{g(e_1) = \{a/0.5, b/0.6\}, g(e_2) = \{a/0, b/0\}, g(e_3) = \{a/0.4, b/0.7\}\}$

$h_C = \{h(e_1) = \{a/0, b/0\}, h(e_2) = \{a/0.7, b/0.4\}, h(e_3) = \{a/0.4, b/0.7\}\}$

$i_E = \{i(e_1) = \{a/0.5, b/0.6\}, i(e_2) = \{a/0.7, b/0.4\}, i(e_3) = \{a/0.4, b/0.7\}\}$

$j_D = \{j(e_1) = \{a/0.5, b/0.6\}, j(e_2) = \{a/0, b/0\}, j(e_3) = \{a/0, b/0\}\}$

$k_F = \{k(e_1) = \{a/0, b/0\}, k(e_2) = \{a/0.7, b/0.4\}, k(e_3) = \{a/0, b/0\}\}$

$u_G = \{u(e_1) = \{a/0, b/0\}, u(e_2) = \{a/0, b/0\}, u(e_3) = \{a/0.4, b/0.7\}\}$

Let us consider the fuzzy soft topology $\tau = \{\tilde{0}_E, \tilde{1}_E, f_A, g_B, h_C, i_E, j_D, k_F, u_G\}$ over (X, E).
Now,${f_A}^c = \{f^c(e_1) = \{a/0.5, b/0.4\}, f^c(e_2) = \{a/0.3, b/0.6\}, f^c(e_3) = \{a/1, b/1\}\}$

$g_B{}^c = \{g^c(e_1) = \{a/0.5, b/0.4\}, g^c(e_2) = \{a/1, b/1\}, g^c(e_3) = \{a/0.6, b/0.3\}\}$

$h_C{}^c = \{h^c(e_1) = \{a/1, b/1\}, h^c(e_2) = \{a/0.3, b/0.6\}, h^c(e_3) = \{a/0.6, b/0.3\}\}$

$i_E{}^c = \{i^c(e_1) = \{a/0.5, b/0.4\}, i^c(e_2) = \{a/0.3, b/0.6\}, i^c(e_3) = \{a/0.6, b/0.3\}\}$

$j_D{}^c = \{j^c(e_1) = \{a/0.5, b/0.4\}, j^c(e_2) = \{a/1, b/1\}, j^c(e_3) = \{a/1, b/1\}\}$

$k_F{}^c = \{k^c(e_1) = \{a/1, b/1\}, k^c(e_2) = \{a/0.3, b/0.6\}, k^c(e_3) = \{a/1, b/1\}\}$

$u_G{}^c = \{u^c(e_1) = \{a/1, b/1\}, u^c(e_2) = \{a/1, b/1\}, u^c(e_3) = \{a/0.6, b/0.3\}\}$

Clearly, $\overline{f_A} = u_G{}^c, \overline{g_B} = k_F{}^c, \overline{h_C} = j_D{}^c, \overline{i_E} = \tilde{1}_E, \overline{j_D} = h_C{}^c, \overline{k_F} = g_B{}^c, \overline{u_G} = f_A{}^c$

Let us consider the following fuzzy soft set over (X, E).
$w_A = \{w(e_1) = \{a/0.4, b/0.3\}, w(e_2) = \{a/0.5, b/0.2\}, w(e_3) = \{a/0, b/0\}\}$
Now, $fs\theta cl(w_A) = u_G{}^c$
Again, $fs\theta cl(fs\theta cl(w_A)) = fs\theta cl(u_G{}^c) = g_B{}^c \neq fs\theta cl(w_A)$.

Definition 3.9 For a fuzzy soft set f_A in a fuzzy soft topological space (X, E, τ), we define a fuzzy soft θ- interior of f_A(denoted by $fs\theta int(f_A)$), which is defined by $fs\theta int(f_A) = \tilde{1}_E - fs\theta cl(\tilde{1}_E - f_A)$.

Remark 3.10 A fuzzy soft set f_A in a fuzzy soft topological space (X, E, τ) is fuzzy soft θ- open if and only if $f_A = fs\theta int(f_A)$.

Remark 3.11 For any fuzzy soft set f_A in a fuzzy soft topological space (X, E, τ), $fs\theta int(f_A) \sqsubseteq f_A{}^0$.

Remark 3.12 The set of all fuzzy soft θ- open sets in a fuzzy soft topological space (X, E, τ) forms a topology, called the fuzzy soft θ- topology and denoted by τ_θ.

Theorem 3.13 For any fuzzy soft closed set f_A in a fuzzy soft topological space (X, E, τ), $f_A{}^0 = fs\theta int(f_A)$.

Proof: If f_A is a fuzzy soft closed set. Then $f_A{}^c$ is fuzzy soft open, then from Theorem 3.3 $\overline{f_A{}^c} = fs\theta cl(f_A{}^c)$, and now $fs\theta int(f_A) = \tilde{1}_E - \overline{f_A{}^c} = f_A{}^0$, by Theorem 2.17.

Theorem 3.14 For any fuzzy soft set f_A in a fuzzy soft topological space (X, E, τ), $fs\theta cl(f_A{}^c) = (fs\theta int(f_A))^c$.

Proof: The proof is obvious.

Definition 3.15 A fuzzy soft set f_A in a fuzzy soft topological space (X, E, τ) is said to be a fuzzy soft θ- *nbd* of a fuzzy soft point e_x^α if and only if there exists a fuzzy soft open *q*-*nbd* g_B of e_x^α such that $g_B \bar{\tilde{q}} f_A{}^c$.

4. Fuzzy soft θ- continuous mapping

In this section, we introduce fuzzy soft θ- continuous mapping, fuzzy soft θ- open mapping, fuzzy soft θ- closed mapping and investigate their related properties.

Definition 4.1 A function $f_{up} : (X, E, \tau_1) \to (Y, K, \tau_2)$ is said to be a fuzzy soft θ- continuous if and only if for each fuzzy soft point e_x^α in $FS(X, E)$ and each fuzzy soft open *q*-*nbd* g_B of $f_{up}(e_x^\alpha)$ in $FS(Y, K)$, there exists a fuzzy soft open *q*-*nbd* f_A of e_x^α such that $f_{up}(\overline{f_A}) \sqsubseteq \overline{g_B}$.

Definition 4.2 Let $f_{up} : (X, E, \tau_1) \to (Y, K, \tau_2)$ be a fuzzy soft mapping. Then

(i) f_{up} is said to be a fuzzy soft θ- open mapping if for each fuzzy soft θ- open set f_A in (X, E, τ_1), $f_{up}(f_A)$ is fuzzy soft θ- open in (Y, K, τ_2);

(ii) f_{up} is said to be a fuzzy soft θ- closed mapping if for each fuzzy soft θ- closed set g_B in (X, E, τ_1), $f_{up}(g_B)$ is fuzzy soft θ- closed in (Y, K, τ_2).

Theorem 4.3 A function $f_{up} : (X, E, \tau_1) \to (Y, K, \tau_2)$ is fuzzy soft θ- continuous if and only if for each fuzzy soft point e_x^α in $FS(X, E)$ and each fuzzy soft θ- *nbd* g_B of $f_{up}(e_x^\alpha)$, ${f_{up}}^{-1}(g_B)$ is a fuzzy soft θ- *nbd* of e_x^α.

Proof: Let $e_x^\alpha \tilde{\in} FS(X, E)$ and g_B be a fuzzy soft θ- *nbd* of $f_{up}(e_x^\alpha)$. Then there exists a fuzzy soft open *q-nbd* h_C of $f_{up}(e_x^\alpha)$ such that $\overline{h_C \tilde{\bar{q}} g_B}^c$. Since f_{up} is fuzzy soft θ- continuous, there exists a fuzzy soft open *q-nbd* j_A of e_x^α such that $f_{up}(\overline{j_A}) \sqsubseteq \overline{h_C} \sqsubseteq g_B$, and hence $\overline{j_A} \sqsubseteq {f_{up}}^{-1}(f_{up}(\overline{j_A})) \sqsubseteq {f_{up}}^{-1}(h_C)$. Therefore, since ${f_{up}}^{-1}(h_C) \sqsubseteq {f_{up}}^{-1}(g_B)$, ${f_{up}}^{-1}(g_B)$ is a fuzzy soft θ- *nbd* of e_x^α.

Conversely, let $e_x^\alpha \tilde{\in} FS(X, E)$ and g_B be a fuzzy soft open *q-nbd* of $f_{up}(e_x^\alpha)$. Then $\overline{g_B}$ is a fuzzy soft θ- *nbd* of $f_{up}(e_x^\alpha)$. By hypothesis, ${f_{up}}^{-1}(\overline{g_B})$ is a fuzzy soft θ- *nbd* of e_x^α. Therefore, there exists a fuzzy soft open *q-nbd* j_A of e_x^α such that $\overline{j_A} \sqsubseteq {f_{up}}^{-1}(\overline{g_B})$ and hence $f_{up}(\overline{j_A}) \sqsubseteq {f_{up}}^{-1}(f_{up}(\overline{g_B})) \sqsubseteq \overline{g_B}$. Hence f_{up} is fuzzy soft θ- continuous.

Theorem 4.4 Let $f_{up} : (X, E, \tau_1) \to (Y, K, \tau_2)$ be a fuzzy soft θ- continuous mapping. Then the following hold:
i) $f_{up}(fs\theta cl(f_A)) \sqsubseteq fs\theta cl[f_{up}(f_A)]$ for each fuzzy soft set f_A in $FS(X, E)$.
ii) $fs\theta cl[{f_{up}}^{-1}(f_A)] \sqsubseteq {f_{up}}^{-1}[fs\theta cl(f_A)]$ for each fuzzy soft set $f_A$in $FS(Y, K)$.
iii) For each fuzzy soft θ- closed set f_A in (Y, K, τ_2), ${f_{up}}^{-1}(f_A)$ is a fuzzy soft θ- closed set in (X, E, τ_1).
iv) For each fuzzy soft θ- open set f_A in (Y, K, τ_2), ${f_{up}}^{-1}(f_A)$ is a fuzzy soft θ- open set in (X, E, τ_1).
v) For each fuzzy soft open set f_A in (Y, K, τ_2), $fs\theta cl[{f_{up}}^{-1}(f_A)] \sqsubseteq {f_{up}}^{-1}[\overline{f_A}]$.

Proof: (i) Let $e_x^\alpha \tilde{\in} fs\theta cl(f_A)$ and g_B be any fuzzy soft open *q-nbd* of $f_{up}(e_x^\alpha)$. Then there exists a fuzzy soft open *q-nbd* h_C of e_x^α such that $f_{up}(\overline{h_C}) \sqsubseteq \overline{g_B}$. Since $e_x^\alpha \tilde{\in} fs\theta cl(f_A)$, we have $\overline{h_C} \tilde{q} f_A$. Then $f_{up}(\overline{h_C}) \tilde{q} f_{up}(f_A)$. Thus $\overline{g_B} \tilde{q} f_{up}(f_A)$ and hence $f_{up}(e_x^\alpha) \tilde{\in} fs\theta cl(f_A)$. So $f_{up}(fs\theta cl(f_A)) \sqsubseteq fs\theta cl[f_{up}(f_A)]$.

(ii) By (i), $f_{up}(fs\theta cl({f_{up}}^{-1}(f_A))) \sqsubseteq fs\theta cl[f_{up}({f_{up}}^{-1}(f_A))] \sqsubseteq fs\theta cl(f_A)$. Hence $fs\theta cl[{f_{up}}^{-1}(f_A)] \sqsubseteq {f_{up}}^{-1}[fs\theta cl(f_A)]$.

(iii) Let f_A be a fuzzy soft θ- closed set in (Y, K, τ_2). Then $fs\theta cl(f_A) = f_A$. By (ii), $fs\theta cl[{f_{up}}^{-1}(f_A)] \sqsubseteq {f_{up}}^{-1}[fs\theta cl(f_A)] = {f_{up}}^{-1}(f_A)$. Hence ${f_{up}}^{-1}(f_A)$ is fuzzy soft θ- closed in (X, E, τ_1).

(iv) Let f_A be a fuzzy soft θ- open set in (Y, K, τ_2). Then ${f_A}^c$ is a fuzzy soft θ- closed set in (Y, K, τ_2). By (iii), ${f_{up}}^{-1}({f_A}^c)$ is fuzzy soft θ- closed in (X, E, τ_1). Since ${f_{up}}^{-1}({f_A}^c) = \tilde{1}_E - {f_{up}}^{-1}(f_A)$, ${f_{up}}^{-1}(f_A)$ is a fuzzy soft θ- open set in

(X, E, τ_1).

(v) Since f_A is fuzzy soft open in (Y, K, τ_2), $\overline{f_A} = fs\theta cl(f_A)$ and we have, from (ii), $fs\theta cl[{f_{up}}^{-1}(f_A)] \sqsubseteq {f_{up}}^{-1}[fs\theta cl(f_A)] = {f_{up}}^{-1}(\overline{f_A})$.

Example 4.5 Let $X = \{a, b\}$, $E = \{e_1, e_2\}$
Let us consider the following fuzzy soft sets over (X, E).
$f_E = \{f(e_1) = \{a/0.5, b/0.3\}, f(e_2) = \{a/0.6, b/0.4\}\}$
$g_E = \{g(e_1) = \{a/0.6, b/0.7\}, g(e_2) = \{a/0.2, b/0.7\}$
Let us consider the following fuzzy soft topology $\tau_1 = \{\tilde{0}_E, \tilde{1}_E, f_E\}$, $\tau_2 = \{\tilde{0}_E, \tilde{1}_E, g_E\}$ over (X, E). We define the fuzzy soft mapping $f_{up} : (X, E, \tau_1) \to (X, E, \tau_2)$ where $u : X \to X$ and $p : E \to E$ are mappings, defined by $u(a) = a, u(b) = b, p(e_1) = e_1, p(e_2) = e_2$. Obviously, the fuzzy soft θ- open set in (X, E, τ_1) is $\tilde{0}_E$, $\tilde{1}_E$ ${f_{up}}^{-1}(\tilde{0}_E) = \tilde{0}_E \tilde{\in} (X, E, \tau_1)$ and ${f_{up}}^{-1}(\tilde{1}_E) = \tilde{1}_E \tilde{\in} (X, E, \tau_1)$. Thus $f_{up} : (X, E, \tau_1) \to (X, E, \tau_2)$ is fuzzy soft θ- continuous, but not fuzzy soft continuous, since ${f_{up}}^{-1}(g_E) = g_E$ and g_E is τ_2- open but not τ_2- open.

Theorem 4.6 Let X, Y and Z be fuzzy soft topological spaces and $f_{up} : X \to Y$ and $g_{up} : Y \to Z$ be fuzzy soft θ- continuous. Then the composite mapping $g_{up} \circ f_{up} : X \to Z$ is fuzzy soft θ- continuous.

Proof: Let $e_x^\alpha \tilde{\in} FS(X, E)$ and g_B be a fuzzy soft θ- nbd of $g_{up}(f_{up}(e_x^\alpha))$. Since g_{up} is fuzzy soft θ- continuous, ${g_{up}}^{-1}(g_B)$ is a fuzzy soft θ- nbd of $f_{up}(e_x^\alpha)$. Also since f_{up} is fuzzy soft θ- continuous, ${f_{up}}^{-1}({g_{up}}^{-1}(g_B))$ is a fuzzy soft θ- nbd e_x^α. But $(g_{up} \circ f_{up})^{-1}(g_B) = {f_{up}}^{-1}({g_{up}}^{-1}(g_B))$. Therefore, $g_{up} \circ f_{up}$ is fuzzy soft θ- continuous.

5. Fuzzy soft θ- connectedness

In this section, we introduce fuzzy soft θ- connectedness, characterised in terms of fuzzy soft θ- continuous mapping and investigate their related properties.

Definition 5.1 A pair (f_A, g_B) of non-null fuzzy soft sets f_A and g_B in a fuzzy soft topological space (X, E, τ) is said to be a fuzzy soft θ- separation relative to (X, E) if and only if $fs\theta cl(f_A)\bar{\tilde{q}}g_B$ and $f_A\bar{\tilde{q}}fs\theta cl(g_B)$.

If, in addition, $f_A = \tilde{1}_E$, then (X, E, τ) is called a θ- connected space.

Remark 5.2 From $fs\theta cl(f_A) \sqsupseteq \overline{f_A}$ and $fs\theta cl(g_B) \sqsupseteq \overline{g_B}$, it follows that every fuzzy soft θ- separation is a weak fuzzy soft separation.

Definition 5.3 A fuzzy soft set f_A in a fuzzy soft topological space (X, E, τ) is said to be a fuzzy soft θ- connected if and only if there does not exist any fuzzy soft θ- separation (g_B, h_C) relative to (X, E) such that $f_A = g_B \sqcup h_C$.

Remark 5.4 Every fuzzy soft connected set is a fuzzy soft θ- connected set but converse may not be true.

Theorem 5.5 If (f_A, g_B) is a fuzzy soft θ- separation relative to (X, E, τ) and h_C, j_D are two non- null fuzzy soft sets such that $h_C \sqsubseteq f_A$ and $j_D \sqsubseteq g_B$, then (h_C, j_D) is also a fuzzy soft θ- separation relative to (X, E).

Proof: The proof is obvious.

Theorem 5.6 Let f_A be a non-null fuzzy soft set in a fuzzy soft topological space (X, E, τ). If f_A is fuzzy soft θ- connected, then for every fuzzy soft θ- separation (g_B, h_C) relative to (X, E) with $f_A \sqsubseteq g_B \sqcup h_C$, exactly one of the following holds:

1. $f_A \sqsubseteq g_B$ and $f_A \sqcap h_C = \tilde{0}_E$,
2. $f_A \sqsubseteq h_C$ and $f_A \sqcap g_B = \tilde{0}_E$.

Conversely, if for every fuzzy soft θ- separation (g_B, h_C) relative to (X, E) with $f_A \sqsubseteq g_B \sqcup h_C$ either $f_A \sqcap g_B = \tilde{0}_E$ or $f_A \sqcap h_C = \tilde{0}_E$ holds, then f_A is fuzzy soft θ- connected.

Proof: Let f_A be fuzzy soft θ- connected in (X, E, τ). Since $f_A \sqsubseteq g_B \sqcup h_C$, both of $f_A \sqcap h_C = \tilde{0}_E$ and $f_A \sqcap g_B = \tilde{0}_E$ do not hold simultaneously. If $f_A \sqcap h_C \neq \tilde{0}_E$ and $f_A \sqcap g_B \neq \tilde{0}_E$, then $(f_A \sqcap g_B, f_A \sqcap h_C)$ is a fuzzy soft θ- separation relative to (X, E) such that $f_A = (f_A \sqcap g_B) \sqcup (f_A \sqcap h_C)$, which contradicts the fuzzy soft θ- connected of f_A. Thus exactly one of $f_A \sqcap g_B$ and $f_A \sqcap h_C$ is a non-null fuzzy soft set. Now, whenever $f_A \sqcap g_B = \tilde{0}_E$, we have $f_A \sqsubseteq h_C$, since $f_A \sqsubseteq g_B \sqcup h_C$. Similarly, when $f_A \sqcap h_C = \tilde{0}_E$, we have $f_A \sqsubseteq g_B$.

Conversely, if f_A is not fuzzy soft θ- connected, then there is a fuzzy soft θ- separation (g_B, h_C) relative to (X, E) such that $f_A = g_B \sqcup h_C$. By hypothesis, either $f_A \sqcap g_B = \tilde{0}_E$, which implies that $g_B = \tilde{0}_E$ (since $g_B \sqsubseteq f_A$), or $f_A \sqcap h_C = \tilde{0}_E$ implying $h_C = \tilde{0}_E$ and so none of which is true.

Theorem 5.7 Let $f_{up} : (X, E, \tau_1) \to (Y, K, \tau_2)$ be a fuzzy soft θ- continuous mapping and f_A be fuzzy soft θ- connected relative to (X, E). Then $f_{up}(f_A)$ is fuzzy soft θ- connected relative to (Y, K).

Proof: If possible, let $f_{up}(f_A)$ be not fuzzy soft θ- connected in (Y, K, τ_2) , then there exists a fuzzy soft θ- separation (g_B, h_C) relative to (Y, K) such that $f_{up}(f_A) = g_B \sqcup h_C$. Put $j_D = f_A \sqcap {f_{up}}^{-1}(g_B)$ and $w_F = f_A \sqcap {f_{up}}^{-1}(h_C)$. Now $f_{up}(f_A) \sqcap g_B \neq \tilde{0}_E \Longrightarrow {f_{up}}^{-1}(f_{up}(f_A) \sqcap g_B) \neq \tilde{0}_E \Longrightarrow f_A \sqcap {f_{up}}^{-1}(g_B) \neq \tilde{0}_E \Longrightarrow j_D \neq \tilde{0}_E$. Similarly, $w_F \neq \tilde{0}_E$.
If possible, let $j_D \tilde{q} w_F$, then for some $e \in E$ and $x \in X$, $({f_{up}}^{-1}(g_B))(e)(x) + ({f_{up}}^{-1}(h_C))(e)(x) > 1$ and so $g_B(f_{up}(e)(x)) + h_C(f_{up}(e)(x)) > 1$, a contradiction since $g_B \bar{\tilde{q}} h_C$. Thus $j_D \tilde{q} w_F$. Again, $fs\theta cl({f_{up}}^{-1}(h_C)) \sqsubseteq {f_{up}}^{-1}(fs\theta cl(h_C))$ and $w_F \sqsubseteq {f_{up}}^{-1}(h_C) \Longrightarrow fs\theta cl(w_F) \sqsubseteq {f_{up}}^{-1}(fs\theta cl(h_C))$. Then $g_B \bar{\tilde{q}} fs\theta cl(h_C) \Longrightarrow {f_{up}}^{-1}(g_B) \bar{\tilde{q}} {f_{up}}^{-1}(fs\theta cl(h_C)) \Longrightarrow j_D \bar{\tilde{q}} {f_{up}}^{-1}(fs\theta cl(h_C))$ (as $\mathrm{j_D \sqsubseteq {f_{up}}^{-1}(fs\theta cl(h_C))}$). This implies that $j_D \bar{\tilde{q}} fs\theta cl(w_F)$. Similarly, $w_F \bar{\tilde{q}} fs\theta cl(j_D)$. Thus (j_D, w_F) is a fuzzy soft θ- separation relative to (X, E), and so $f_{up}(f_A)$ is fuzzy soft θ- connected.

6. Conclusion

Topology is an important area of mathematics with many applications in the domain of computer science and physical sciences. Fuzzy soft topology [14] is a relatively new and promising domain which can lead to the development of new mathematical models and innovative approaches that will significantly contribute to the solution of complex problems in natural sciences.

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