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# ON THE PSEUDO-QUASI-CONFORMAL CURVATURE TENSOR OF P-SASAKIAN MANIFOLDS

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Abstract. In this paper, we consider P-Sasakian manifold satisfying certain conditions on the pseudo-quasi-conformal curvature tensor. We study pseudo-quasi-conformally flat, pseudo-quasi-conformally semisymmetric and $\phi$-pseudo-quasi-conformally flat P-Sasakian manifolds.

## 1. Introduction

On the analogy of almost contact Riemannian manifolds, in 1976, Sato [11] introduced the nottion of almost paracontact Riemannian manifolds. An almost contact manifold is always odd-dimensional but an almost paracontact manifold could be even dimensional as well. The notion of para-Sasakian manifold was introduced by Adati and Matsumoto [1]. This structure is an analogy of Sasakian manifold in paracontact geometry. Para-Sasakian (briefly, P-Sasakian) and special para-Sasakian (briefly, SP-Sasakian) manifolds were studied by many authors such as Tarafdar and De [14], De and Pathak [6], Matsumoto [9], Mandal and De [8], Prakasha [10], Barman [4] and others.

In 2005, Shaikh and Jana [13] introduced and studied a tensor field, called pseudo-quasi-coformal curvature tensor $\widetilde{C}$ on a Riemannian manifold of dimension greater than or equal to 3. This curvature tensor includes the projective, quasi-conformal, Weyl conformal and concircular curvature tensor as special cases. Recently, Kundu [7], studied pseudo-quasi-conformal curvature tensor on P-Sasakian manifolds.

In this paper, we consider P-Sasakian manifold satisfying certain conditions on the pseudo-quasi-conformal curvature tensor. After preliminaries, in section 3, we study pseudo-quasi-conformally flat P-Sasakian manifold. A pseudo-quasi-conformally semisymmetric P-Sasakian manifold is studeid in section 4. Section 5 is devoted to the study of $\phi$-pseudo-quasi-conformmaly flat P-Sasakian manifold.

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## 2. Preliminaries

An $n$-dimensional differentiable manifold $M$ is called an almost paracontact structure $(\phi, \xi, \eta, g)$ ([1, 2, 12]), where $\phi$ is a (1,1) tensor field, $\xi$ is a vector field, $\eta$ is a 1-form and $g$ is a Riemannian metric on $M$ such that

$$\phi^2 X = X - \eta(X)\xi, \quad \eta(\xi) = 1, \quad \eta \circ \phi = 0, \tag{1}$$

$$g(X,Y) = g(\phi X, \phi Y) + \eta(X)\eta(Y), \; g(X, \phi Y) = g(\phi X, Y), \; g(X, \xi) = \eta(X), \tag{2}$$

for all vector fields $X$ and $Y$ on $\chi(M)$, where $\chi$(M) is the collection of all smooth vector fields on $M$.

In addition, if $(\phi, \xi, \eta, g)$, satisfy the equations:

$$d\eta = 0, \quad \nabla_X \xi = \phi X, \tag{3}$$

$$(\nabla_X \phi)(Y) = -g(X,Y)\xi - \eta(Y)X + 2\eta(X)\eta(Y)\xi, \; \forall X, Y \in \chi(M), \tag{4}$$

where $\nabla$ is the Levi-Civita connection of the Riemannian manifold, then $M$ is called a para-Sasakian manifold or briefly a P-Sasakian manifold [1]. Especially, a P-Sasakian manifold $M$ is called a special para-Sasakian manifold or briefly a SP-Sasakian manifold [12] if $M$ admits a 1-form $\eta$ satisfying

$$(\nabla_X \eta)(Y) = -g(X,Y) + \eta(X)\eta(Y). \tag{5}$$

Furthermore, in an $n$-dimensional P-Sasakian manifold $M$ the following relations hold [2, 11, 12]:

$$\eta(R(X,Y)Z) = g(X,Z)\eta(Y) - g(Y,Z)\eta(X), \tag{6}$$

$$R(\xi, X)Y = \eta(Y)X - g(X,Y)\xi, \tag{7}$$

$$R(X,Y)\xi = \eta(X)Y - \eta(Y)X, \tag{8}$$

$$S(X, \xi) = -(n-1)\eta(X), \tag{9}$$

$$S(\phi X, \phi Y) = S(X,Y) + (n-1)\eta(X)\eta(Y), \tag{10}$$

for all vector fields $X, Y, Z \in \chi(M)$, where $R$ and $S$ are the Riemannian curvature tensor and Ricci tensor of the manifold, respectively.

A P-Sasakian manifold is said to be an Einstein manifold if there exists a real constant $\lambda$ such that the Ricci tensor $S$ is of the form

$$S(X,Y) = \lambda g(X,Y), \tag{11}$$

for any $X, Y \in \chi(M)$.

The pseudo-quasi-conformal curvature tensor $\widetilde{C}$ is defined by

$$\begin{aligned} \widetilde{C}(X,Y)Z &= (p+d)R(X,Y)Z + \left(q - \frac{d}{n-1}\right)[S(Y,Z)X - S(X,Z)Y] \\ &+ q\,[g(Y,Z)QX - g(X,Z)QY] \\ &- \frac{r}{n(n-1)}\{p + 2(n-1)q\}[g(Y,Z)X - g(X,Z)Y], \end{aligned} \tag{12}$$

where $X, Y, Z \in \chi(M)$, $S$ is the Ricci tensor, $r$ is the scalar curvature, $Q$ is the symmetric endomorphism of the tangent space at each point corresponding to the Ricci tensor $S$ [5], i.e. $g(QX, Y) = S(X,Y)$ and $p, q, d$ are real constants such that $p^2 + q^2 + d^2 > 0$.

In particular, if *(1)* $p = q = 0, d = 1$; *(2)* $p \neq 0, q \neq 0, d = 0$; *(3)* $p = 1, q = -\frac{1}{n-2}, d = 0$; *(4)* $p = 1, q = d = 0$; then $\widetilde{C}$ reduces to the projective curvature tensor; quasi-conformal curvature tensor; conformal curvature tensor and concircular curvature tensor, respectively.

## 3. Pseudo-quasi-conformally flat P-Sasakian manifold

**Definition 1.** *An $n$-dimensional $(n \geq 3)$ P-Sasakian manifold $M$ is called pseudo-quasi-conformally flat, if the condition*

$$\widetilde{C}(X, Y)Z = 0,$$

*holds on $M$.*

Let the manifold $M$ under consideration is pseudo-quasi-conformally flat, then we have from Definition 1 and relation (12) that

$$\begin{aligned}(p + d)R(X, Y)Z &= \left(q - \frac{d}{n-1}\right)[S(X, Z)Y - S(Y, Z)X] \\ &+ q[g(X, Z)QY - g(Y, Z)QX] \\ &- \frac{r}{n(n-1)}\{p + 2(n-1)q\}[g(X, Z)Y - g(Y, Z)X]. \quad (13)\end{aligned}$$

Taking the inner product on both sides of (13) with $W$, we obtain

$$\begin{aligned}(p + d)'R(X, Y, Z, W) &= \left(q - \frac{d}{n-1}\right)[S(X, Z)g(Y, W) - S(Y, Z)g(X, W)] \\ &+ q[g(X, Z)S(Y, W) - g(Y, Z)S(X, W)] \\ &- \frac{r}{n(n-1)}\{p + 2(n-1)q\}[g(X, Z)g(Y, W) \\ &- g(Y, Z)g(X, W)], \quad (14)\end{aligned}$$

where $'R(X, Y, Z, W) = g(R(X, Y)Z, W)$. Let $\{e_i\}$, $i = 1, 2, ..., n$, is an orthonormal basis of the tangent space at each point of the manifold. Then putting $X = W = e_i$ in (14) and taking the summation over $i$, $1 \leq i \leq n$, we get

$$\begin{aligned}(p + d)S(Y, Z) &= \left[q - (n-1)\left(q - \frac{d}{n-1}\right)\right] S(Y, Z) \\ &+ \left[\frac{r}{n}\{p + 2(n-1)q\} - rq\right] g(Y, Z).\end{aligned}$$

This gives

$$\{p + (n-2)q\} S(Y, Z) = \left[\frac{r}{n}\{p + 2(n-1)q\} - rq\right] g(Y, Z). \quad (15)$$

Putting $Y = Z = \xi$ in (15), we have

$$r = -n(n-1).$$

provided $\{p + (n-2)q\} \neq 0$. Substituting this value of $r$ in (15), we obtain

$$S(Y, Z) = -(n-1)g(Y, Z). \quad (16)$$

This shows that, $M$ is an Einstein manifold and we are able to state the following theorem:

**Theorem 1.** *An $n$-dimensional $(n \geq 3)$ pseudo-quasi-conformally flat P-Sasakian manifold is an Einstein manifold with constant scalar curvature $-n(n-1)$, provided that $\{p + (n-2)q\} \neq 0$.*

Next, using the value of $S$ and $r$ in (13), we obtain

$$R(X,Y)Z = -\{g(Y,Z)X - g(X,Z)Y\}, \tag{17}$$

provided that $p + d \neq 0$. This implies that $M$ is of constant curvature $-1$ and consequently it is locally isometric to the Hyperbolic space $H^n(-1)$.

Conversely, if $M$ is of constant curvature $-1$, then we get $\widetilde{C}(X,Y)Z = 0$, that is, $M$ is pseudo-quasi-conformally flat. This helps us to state the following:

**Theorem 2.** *An $n$-dimensional ($n \geq 3$) P-Sasakian manifold is pseudo-quasi-conformally flat if and only if it is the manifold of constant curvature $-1$ and consequently, locally isometric to the Hyperbolic space $H^n(-1)$, provided that $\{p + (n-2)q\} \neq 0$ and $p + d \neq 0$.*

Also, it is known that (see [3]) if a P-Sasakian manifold is of constant curvature then the manifold is an SP-Sasakian manifold. Hence, we have the following result:

**Theorem 3.** *An $n$-dimensional ($n \geq 3$) pseudo-quasi-conformally flat P-Sasakian manifold is an SP-Sasakian manifold, provided that $\{p+(n-2)q\} \neq 0$ and $p+d \neq 0$.*

Next, for an $n$-dimensional ($n \geq 3$) pseudo-quasi-conformally flat P-Sasakian manifold, consider

$$\begin{aligned} R(X,Y)\cdot R &= R(X,Y)R(Z,U)V - R(R(X,Y)Z,U)V \\ &- R(Z,R(X,Y)U)V - R(Z,U)R(X,Y)V, \end{aligned} \tag{18}$$

for any vector fields $X, Y, Z, U, V \in \chi(M)$. Hence, it follows from (17) that

$$\begin{aligned} R(X,Y)R(Z,U)V &= -\{g(U,V)g(Y,Z)X - g(Z,V)g(Y,U)X \\ &- g(U,V)g(X,Z)Y + g(Z,V)g(X,U)Y\}, \end{aligned} \tag{19}$$

$$\begin{aligned} R(R(X,Y)Z,U)V &= -\{g(U,V)g(Y,Z)X - g(Y,Z)g(X,V)U \\ &- g(X,Z)g(U,V)Y + g(Y,V)g(X,Z)U\}, \end{aligned} \tag{20}$$

$$\begin{aligned} R(Z,R(X,Y)U)V &= -\{g(U,Y)g(X,V)Z - g(U,Y)g(Z,V)X \\ &- g(U,X)g(Y,V)Z + g(X,U)g(Z,V)Y\} \end{aligned} \tag{21}$$

and

$$\begin{aligned} R(Z,U)R(X,Y)V &= -\{g(U,X)g(Y,V)Z - g(X,Z)g(Y,V)U \\ &- g(X,V)g(U,Y)Z + g(X,V)g(Z,Y)U\}. \end{aligned} \tag{22}$$

Thus, in view of (19)-(22), we obtain from (18) that

$$R(X,Y)\cdot R = 0, \tag{23}$$

that is, the manifold is semisymmetric. This leads to the following statement:

**Theorem 4.** *An $n$-dimensional ($n \geq 3$) pseudo-quasi-conformally flat P-Sasakian manifold is semisymmetric, provided that $\{p + (n-2)q\} \neq 0$ and $p + d \neq 0$.*

## 4. Pseudo-quasi-conformally semisymmetric P-Sasakian manifold

**Definition 2.** *An $n$-dimensional $(n \geq 3)$ P-Sasakian manifold $M$ is called pseudo-quasi-conformally semisymmetric, if the condition*

$$R(X,Y)\cdot\widetilde{C}=0,$$

*holds on $M$.*

Let us suppose that $M$ is a pseudo-quasi-conformally semisymmetric P-Sasakian manifold. Then it can be easily seen that

$$(R(X,Y)\cdot\widetilde{C})(U,V)W=0, \tag{24}$$

for any vector fields $X,Y,U,V,W \in \chi(M)$.
From (24), we have

$$\begin{aligned} & R(X,Y)\widetilde{C}(U,V)W-\widetilde{C}(R(X,Y)U,V)W \\ - & \widetilde{C}(U,R(X,Y)V)W-\widetilde{C}(U,V)R(X,Y)W=0. \end{aligned} \tag{25}$$

Taking $X=\xi$ in (25) and using relation (7) we find

$$\begin{aligned} & {}'\widetilde{C}(U,V,W,Y)-\eta(Y)\eta(\widetilde{C}(U,V)W) \\ + & \eta(U)\eta(\widetilde{C}(Y,V)W)+\eta(V)\eta(\widetilde{C}(U,Y)W) \\ + & \eta(W)\eta(\widetilde{C}(U,V)Y)-g(Y,U)\eta(\widetilde{C}(\xi,V)W) \\ - & g(Y,V)\eta(\widetilde{C}(U,\xi)W)-g(Y,W)\eta(\widetilde{C}(U,V)\xi)=0, \end{aligned} \tag{26}$$

where ${}'\widetilde{C}(U,V,W,Y)=g(\widetilde{C}(U,V)W,Y)$. Replacing $Y=U$ in (26), we obtain

$$\begin{aligned} & {}'\widetilde{C}(U,V,W,U)+\eta(V)\eta(\widetilde{C}(U,U)W) \\ + & \eta(W)\eta(\widetilde{C}(U,V)U)-g(U,U)\eta(\widetilde{C}(\xi,V)W) \\ - & g(U,V)\eta(\widetilde{C}(U,\xi)W)-g(U,W)\eta(\widetilde{C}(U,V)\xi)=0. \end{aligned} \tag{27}$$

Using (6) and (9) in (12), we get

$$\begin{aligned} \eta(\widetilde{C}(X,Y)Z) &= g(\widetilde{C}(X,Y)Z,\xi) \\ &= \left(q-\frac{d}{n-1}\right)[S(Y,Z)\eta(X)-S(X,Z)\eta(Y)] \\ &+ \left[(p+d)+q(n-1)+\frac{r}{n(n-1)}\{p+2(n-1)q\}\right][g(X,Z)\eta(Y) \\ &- g(Y,Z)\eta(X)]. \end{aligned} \tag{28}$$

Taking $Z=\xi$ in (28) yields

$$\eta(\widetilde{C}(X,Y)\xi)=0. \tag{29}$$

Substituting $X=\xi$ in (28), we have

$$\begin{aligned} & \eta(\widetilde{C}(\xi,Y)Z) \\ = & \left[(p+d)+q(n-1)+\frac{r}{n(n-1)}\{p+2(n-1)q\}+(n-1)\left(q-\frac{d}{n-1}\right)\right]\eta(Y)\eta(Z) \\ + & \left(q-\frac{d}{n-1}\right)S(Y,Z) \\ - & \left[(p+d)+q(n-1)+\frac{r}{n(n-1)}\{p+2(n-1)q\}\right]g(Y,Z). \end{aligned} \tag{30}$$

Setting $U = e_i$ in (27) and taking summation over $i$, $1 \leq i \leq n$, we obtain by virtue of (1), (9) and (28)-(29) that

$$\begin{aligned} & (n-1)\eta(\widetilde{C}(\xi, V)W) \\ = & \left[(p+d) + (n-1)\left(q - \frac{d}{n-1}\right) - q\right] S(V,W) + \left[q - \frac{\{p+2(n-1)q\}}{n}\right] rg(V,W) \\ + & \left[\left\{(p+d) + (n-1)q + \frac{r}{n(n-1)}(p+2(n-1)q)\right\}(n-1) - \left\{q - \frac{d}{n-1}\right\}(r+n-1)\right]\eta(V)\eta(W). \end{aligned} \tag{31}$$

Comparing (30) with (31), we obtain

$$\begin{aligned} S(V,W) &= -\frac{1}{(p+d-q)}\left[\left\{r + (n-1)^2\right\} q + (n-1)(p+d)\right] g(V,W) \\ &+ \left(\frac{r+n(n-1)}{p+d-q}\right)\left(q - \frac{d}{n-1}\right)\eta(V)\eta(W), \end{aligned} \tag{32}$$

provided that $\{p + d - q\} \neq 0$.
Setting $V = W = e_i$ in (32) and taking summation over $1 \leq i \leq n$, yields

$$r = -n(n-1), \tag{33}$$

provided that $[(n-1)\{p + (n-2)q\} - nd] \neq 0$. Now, using (32) in (33), we obtain

$$S(V,W) = -(n-1)g(V,W), \tag{34}$$

provided that $\{p + d - q\} \neq 0$ and $[(n-1)\{p + (n-2)q\} - nd] \neq 0$. Thus, $M$ is an Einstein manifold. Hence we state:

**Theorem 5.** *An $n$-dimensional $(n \geq 3)$ pseudo-quasi-conformally semisymmetric P-Sasakian manifold is an Einstein manifold with the scalar curvature tensor $r = -n(n-1)$, provided that $\{p + d - q\} \neq 0$ and $[(n-1)\{p + (n-2)q\} - nd] \neq 0$.*

Furthermore, by taking account of (34) in (28) and (30), respectively we get

$$\eta(\widetilde{C}(X,Y)Z) = 0 \text{ and } \eta(\widetilde{C}(\xi,Y)Z) = 0. \tag{35}$$

Using (29) and (35) in (26), it follows that

$$'\widetilde{C}(U,V,W,Y) = 0, \tag{36}$$

which implies $\widetilde{C}(U,V)W = 0$. Therefore, $M$ is pseudo-quasi-conformally flat. Conversely, (36) trivially implies (24). Hence, we have the following:

**Theorem 6.** *An $n$-dimensional $(n > 3)$ P-Sasakian manifold is pseudo-quasi-conformally semisymmetric if and only if it is pseudo-quasi-conformally flat, provided that $\{p + d - q\} \neq 0$ and $[(n-1)\{p + (n-2)q\} - nd] \neq 0$.*

The above result immediately follows:

**Corollary 1.** *An $n$-dimensional $(n \geq 3)$ pseudo-quasi-conformally semisymmetric P-Sasakian manifold is an SP-Sasakian manifold, provided that $\{p+d-q\} \neq 0$ and $[(n-1)\{p + (n-2)q\} - nd] \neq 0$.*

## 5. $\phi$-pseudo quasi-conformally flat P-Sasakian manifolds

**Definition 3.** *An n-dimensional $(n>3)$ P-Sasakian manifold $M$ is called $\phi$-pseudo quasi-conformally flat if the condition*

$$\phi^2\widetilde{C}(\phi X,\phi Y)\phi Z=0, \tag{37}$$

*holds on $M$.*

Let $M$ be a $\phi$-pseudo-quasi-conformally flat P-Sasakian manifold, then from (37) we have

$$g(\widetilde{C}(\phi X,\phi Y)\phi Z,\phi W)=0,$$

for all vector fields $X,Y,Z,W$ on $\chi(M)$. Making use of (12) in (38) we obtain

$$\begin{aligned}
& (p+d)g(R(\phi X,\phi Y)\phi Z,\phi W) && (38)\\
= & -\left(q-\frac{d}{n-1}\right)[S(\phi Y,\phi Z)g(\phi X,\phi W)-S(\phi X,\phi Z)g(\phi Y,\phi W)]\\
- & q[g(\phi Y,\phi Z)S(\phi X,\phi W)-g(\phi X,\phi Z)S(\phi Y,\phi W)]\\
+ & \frac{r}{n(n-1)}\{p+2(n-1)q\}[g(\phi Y,\phi Z)g(\phi X,\phi W)\\
- & g(\phi X,\phi Z)g(\phi Y,\phi W)]. && (39)
\end{aligned}$$

If $\{e_1,.....,e_{n-1},\xi\}$ be a local orthonormal basis of vector fields in $(M^n,g)$, then $\{\phi e_1,.....,\phi e_{n-1},\xi\}$ is also a local orthonormal basis. Then putting $X=W=e_i$ in (38) and taking the summation over $i$, $0\leq i\leq(n-1)$, we get

$$\begin{aligned}
& (p+d)\sum_{i=1}^{n-1}g(R(\phi e_i,\phi Y)\phi Z,\phi e_i)\\
= & -\left(q-\frac{d}{n-1}\right)\sum_{i=1}^{n-1}[S(\phi Y,\phi Z)g(\phi e_i,\phi e_i)-S(\phi e_i,\phi Z)g(\phi Y,\phi e_i)]\\
- & q\sum_{i=1}^{n-1}[g(\phi Y,\phi Z)S(\phi e_i,\phi e_i)-g(\phi e_i,\phi Z)S(\phi Y,\phi e_i)]\\
+ & \frac{r}{n(n-1)}\{p+2(n-1)q\}\sum_{i=1}^{n-1}[g(\phi Y,\phi Z)g(\phi e_i,\phi e_i)-g(\phi e_i,\phi Z)g(\phi Y,\phi e_i)]. \quad (40)
\end{aligned}$$

In an $n$-dimensional ($n > 3$) P-Sasakian manifold, it can be easily verify that

$$\sum_{i=1}^{n-1} g(R(\phi e_i, \phi Y)\phi Z, \phi e_i) = S(\phi Y, \phi Z) + g(\phi Y, \phi Z), \tag{41}$$

$$\sum_{i=1}^{n-1} S(\phi e_i, \phi e_i) = r + (n-1), \tag{42}$$

$$\sum_{i=1}^{n-1} g(\phi e_i, \phi e_i) = n-1, \tag{43}$$

$$\sum_{i=1}^{n-1} g(\phi Y, \phi e_i) S(\phi e_i, \phi Z) = S(\phi Y, \phi Z), \tag{44}$$

$$\sum_{i=1}^{n-1} g(\phi e_i, \phi Z) g(\phi Y, \phi e_i) = g(\phi Y, \phi Z). \tag{45}$$

So by the use of (41) - (45) the equation (40) turns into

$$\begin{aligned} &\left[(p+d) + (n-2)\left(q - \frac{d}{n-1}\right) - q\right] S(\phi Y, \phi Z) \\ = &\left[\frac{r}{n(n-1)}\{p + 2(n-1)q\}(n-2) - (p+d) - \{r+n-1\}\,q\right] g(\phi Y, \phi Z). \end{aligned} \tag{46}$$

Again putting $Y = Z = e_i$ in (46), taking the summation over $i$, $0 \leq i \leq (n-1)$, we get

$$r = -n(n-1), \tag{47}$$

provided that $[(n-1)\{p + (n-3)q\} + d] \neq 0$. Substituting (47) in (46) and then using (2) and (10), we get

$$S(Y, Z) = -(n-1)g(Y, Z). \tag{48}$$

Hence we have the following:

**Theorem 7.** *An $n$-dimensional ($n > 1$) $\phi$-pseudo quasi-conformally flat P-Sasakian manifold is an Einstein manifold with the scalar curvature $r = -n(n-1)$, provided that $[(n-1)\{p + (n-3)q\} + d] \neq 0$.*

By using the value of $S$ in (38), we get

$$R(\phi X, \phi Y, \phi Z, \phi W) = -\{g(\phi Y, \phi Z)g(\phi X, \phi W) - g(\phi X, \phi Z)g(\phi Y, \phi W)\}. \tag{49}$$

Replacing $X$ by $\phi X$, $Y$ by $\phi Y$, $Z$ by $\phi Z$ and $W$ by $\phi W$, we obtain

$$R(X, Y, Z, W) = -\{g(Y, Z)g(X, W) - g(X, Z)g(Y, W)\}. \tag{50}$$

This implies $M$ is of constant curvature $-1$, That is, $M$ is an SP-Sasakian manifold

**Theorem 8.** *A $\phi$-pseudo quasi-conformally flat P-Sasakian manifold is an SP-Sasakian manifold, provided that $[(n-1)\{p + (n-3)q\} + d] \neq 0$.*

Next, a manifold of constant curvature $-1$ is pseudo-quasi-conformally flat, that is, $\widetilde{C} = 0$.

Conversely, $\widetilde{C}(X, Y)Z = 0$ implies that $g(\widetilde{C}(\phi X, \phi Y)\phi Z, \phi W) = 0$. That is, $\phi$-pseudo-quasi-conformally flat. Hence, we can state the following:

**Theorem 9.** *An n-dimensional* $(n > 3)$ *P-Sasakian manifold is* $\phi$*-pseudo-quasi-conformally flat if and only if it is pseudo-quasi-conformally flat, provided that* $[(n-1)\{p+(n-3)q\}+d] \neq 0$.

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