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ON EXISTENCE OF STRICT COINCIDENCE AND COMMON STRICT FIXED POINT OF A FAINTLY COMPATIBLE HYBRID PAIR OF MAPS

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Abstract. In this paper, we introduce conditional compatibility, faint compatibility and conditional reciprocal continuity to a hybrid pair of maps involving a single-valued and a multivalued map using δ-distance and establish strict coincidence and common strict fixed point of a faintly compatible hybrid pair without containment requirement of range space of involved maps or completeness of underlying space/subspaces. In the sequel we generalize, extend and improve several results existing in literature, for instance: Bisht and Shahzad [Faintly compatible mappings and common fixed points, Fixed point theory and applications, 2013, 2013:156], Pant and Bisht [Common fixed point theorems under a new continuity condition, Ann. Univ. Ferrara 58(1)(2012), 127-141] and Pant and Bisht [Occasionally weakly compatible mappings and fixed points, Bull. Belg. Math. Soc. Simon Stevin, 19 (2012), 655-661] and references therein. Results obtained are supported by illustrative examples.

1. Introduction

Hybrid fixed point theory, which is the realm of common fixed point theorems for single-valued and multivalued maps has prospective applications in functional inclusions, optimization theory, fractal graphics, oscillator equations, neutral delay differential equations and discrete dynamics for set-valued operators. Recently, Bisht and Shahzad [1] introduced the notion of faint compatibility, as an improvement of conditional compatibility introduced by Pant and Bisht [6], which permitted the existence of a common fixed point or multiple fixed point or coincidence points under both contractive and non-contractive conditions for single valued maps. Further Pant and Bisht [5] introduced the notion of conditional reciprocal continuity, which is weaker than most of the variants of continuity. For a brief development of variants of continuity and the relation between them one may refer to Tomar and Karapinar [9]. In this paper we introduce/extend the notions of conditional compatibility, faint compatibility and conditional reciprocal continuity to a hybrid pair of maps in a metric space and utilize these relatively weaker notions to establish

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strict coincidence and common strict fixed point of a hybrid pair using $\delta-$ distance without using the notion of continuity, containment requirement of range space of involved maps or completeness of underlying space/subspaces.

2. Preliminaries

Throughout this paper, let (X,d) be a metric space and $CB(X)$ be the family of all nonempty closed and bounded subsets of X. For $A,B \in CB(X)$, functions $\delta(A,B)$ and $D(A,B)$ are defined as:
$\delta(A,B) = \sup\{d(a,b); a \in A, b \in B\}$ and
$D(A,B) = \inf\{d(a,b); a \in A, b \in B\}$,
If $A = \{a\}$, then $\delta(A,B) = \delta(a,B)$.
If $A = \{a\}$ and $B = \{b\}$, then $\delta(A,B) = d(a,b)$.
It follows immediately from the definition of δ that

- $\delta(A,B) = \delta(B,A) > 0$,
- $\delta(A,B) \leq \delta(A,C) + \delta(C,B)$,
- $\delta(A,B) = 0 \iff A = B = \{a\}$,
- $\delta(A,A) = \text{diam } A$,

Let H be the Hausdorff metric with respect to d, i.e.,

$$H(A,B) = \max\{\sup_{x\in A} d(x,B), \sup_{x\in B} d(x,A)\}$$

where $d(x,A) = \inf\{d(x,y) : y \in A\}$, for all $A,B \in CB(X)$. Also $H(A,B) = 0$ iff $A = B$.
If $f : X \to X$ is a single valued and $T : X \to CB(X)$ is a multivalued map of metric space (X,d) then a pair (f,T) is known as a hybrid pair.

For a multivalued map $T : X \to CB(X)$, a point $u \in X$ is a

- fixed point if $u \in Tu$;
- strict fixed point (or a stationary point or absolute fixed point) if $Tu = \{u\}$.

For a hybrid pair (f,T), a point $u \in X$ is a

- coincidence point if $fu \in Tu$;
- strict coincidence point if $Tu = \{fu\}$;
- common fixed point if $u = fu \in Tu$;
- common strict fixed point if $fu = Tu = \{u\}$.

Definition 1 A hybrid pair of maps (f,T) of a metric space (X,d) is:

(1) commuting on X [3] if $fTx \in Tfx$ for all $x \in X$.
(2) compatible [4] if $fTx \in CB(X)$ for all $x \in X$ and $\lim_n H(fTx_n, Tfx_n) = 0$ whenever $\{x_n\}$ is a sequence in X such that $\lim_n fx_n = t \in A = \lim_n Tx_n$ for some $t \in X$ and $A \in CB(X)$.
(3) noncompatible [4] if there exists at least one sequence $\{x_n\}$ in X such that $\lim_n fx_n = t \in A = \lim_n Tx_n$ for some $t \in X$ and $A \in CB(X)$ but $\lim_n H(fTx_n, Tfx_n)$ is either non zero or does not exist.
(4) weakly compatible [2] if f and T commute at coincidence points; i.e., for each point u in X such that $Tu = \{fu\}$, we have $Tfu = fTu$.
(5) reciprocally continuous [7] if and only if $fTx \in CB(X)$ for all $x \in X$ and $lim_n fTx_n = fA$ and $\lim_n Tfx_n = Tt$ whenever $\{x_n\}$ is a sequence in X such that $\lim_n fx_n = t \in A = \lim_n Tx_n$ where $t \in X$ and $A \in CB(X)$.

3. Main results

In all that follows $f : X \to X$ is a single valued, $T : X \to CB(X)$ is a multivalued and (f, T) is a hybrid pair of maps of a metric space (X, d) unless otherwise specified. First we introduce conditional compatibility, faint compatibility and conditional reciprocal continuity for a hybrid pair of maps (f, T) using δ- distance. It is worth mentioning here that δ- distance is not a metric like the Hausdorff distance, but shares most of the properties of a metric except self distance for any set need not be equal to zero.

Definition 2 A hybrid pair of maps (f, T) is called conditionally compatible iff whenever the set of sequences $\{x_n\}$ satisfying $\lim_n fx_n = t \in A = \lim_n Tx_n$ for some $t \in X$ and $A \in CB(X)$ is non empty, there exists a sequence $\{y_n\}$ such that $\lim_n fy_n = u \in B = \lim_n Ty_n$ and $\lim_n \delta(fTy_n, Tfy_n) = 0$ for some $u \in X$ and $B \in CB(X)$.

Example 1 Let $X = [0, 2]$, d be the usual metric on X. Let a hybrid pair of map (f, T) on X be defined as follows:

$$fx = \begin{cases} 1, & 0 \le x \le 1 \\ \frac{3}{2}, & 1 < x \le 2, \end{cases} \qquad Tx = \begin{cases} [1, 2 - x], & 0 \le x \le 1 \\ [\frac{3}{2}, 2], & 1 < x \le 2. \end{cases}$$

Consider a sequence $\{x_n\}$ in X satisfying $x_n = 1 + \frac{1}{n}$ and $\lim_n fx_n = \frac{3}{2} \in [\frac{3}{2}, 2] = \lim_n Tx_n$ such that $\lim_n \delta(fTx_n, Tfx_n) = \lim_n \delta(\frac{3}{2}, [\frac{3}{2}, 2]) \neq 0$, i.e, a pair of maps (f, T) is noncompatible.
But it is conditionally compatible as there exists a sequence $\{y_n\}$ in X satisfying $y_n = 1$ and $lim_n fy_n = 1 \in \{1\} = \lim_n Ty_n$ such that $\lim_n \delta(fTy_n, Tfy_n) = 0$.

It is worth mentioning here that at a unique point of strict coincidence, conditional compatibility need not reduce to the class of commutativity. It is well known that most of the weaker forms of commuting maps, though formally distinct from each other, actually coincide when the given maps have a unique point of coincidence. It is well known that weak compatibility is most widely used concept among all weaker forms of commuting maps and remains the minimal condition of commutativity for the existence of common fixed point for a long time. For a brief development of weaker forms of commuting maps and relationship between them one may refer to Singh and Tomar [8].

Definition 3 A hybrid pair of maps (f, T) is called faintly compatible iff f and T are conditionally compatible and f and T commute on a non empty subset of coincidence points whenever the set of coincidences is non empty, i.e., if $C(f, t) \neq \emptyset$ then there exists $x \in M \subseteq C(f, T)$ such that $fx \in Tx$ and $fTx \subseteq Tfx$.

Example 2 Let $X = [0, 12]$, d be the usual metric on X. Let a hybrid pair of map (f, T) on X be defined as follows:

$$fx = \begin{cases} 1, & 0 \le x \le 1 \\ 11, & 1 < x \le 12, \end{cases} \qquad Tx = \begin{cases} \{1\}, & 0 \le x \le 1 \\ [11, 12 - x] & 1 < x \le 12. \end{cases}$$

Consider a sequence $\{x_n\}$ in X satisfying $x_n = 1 + \frac{1}{n}$ and $\lim_n fx_n = 11 \in \{11\} = \lim_n Tx_n$ such that $\lim_n \delta(fTx_n, Tfx_n) = \lim_n \delta(11, [1, 11]) \neq 0$, i.e, pair of maps

(f,T) is noncompatible.
But it is faintly compatible as there exists a sequence $\{y_n\}$ in X satisfying $y_n = 1$ and $\lim_n fy_n = 1 \in \{1\} = \lim_n Ty_n$ such that $\lim_n \delta(fTy_n, Tfy_n) = 0$ and commute on the subset of coincidence points, i.e. $C(f,T) = \{1, 11\}$ then $fx \in Tx$ and $fTx \subseteq Tfx$, $\forall x \in C(f,T)$.

Evidently weakly compatible hybrid pair of maps is also faintly compatible however reverse implication is not in general true. Further faint compatibility and noncompatibility are independent concepts. Also faint compatibility does reduce to the class of commutativity at unique point of strict coincidence.

Definition 4 A hybrid pair of maps (f,T) is called conditionally reciprocally continuous iff the set of sequences $\{x_n\}$ satisfying $\lim_n fx_n = t \in A = \lim_n Tx_n$ where $t \in X$ and $A \in CB(X)$ is non empty, there exists a sequence $\{y_n\}$ satisfying $\lim_n fy_n = u \in B = \lim_n Ty_n$, for some $u \in X$ and $B \in CB(X)$ such that $\lim_n fTy_n = fB$ and $\lim_n Tfy_n = Tu$.

Example 3 Let $X = [0,2]$, d be the usual metric on X. Let a hybrid pair of map (f,T) on X be defined as follows:

$$fx = \begin{cases} 0, & 0 \le x \le 1 \\ 2-x, & 1 < x \le 2, \end{cases} \qquad Tx = \begin{cases} \{0\}, & 0 \le x \le 1 \\ [1,2], & 1 < x \le 2. \end{cases}$$

Consider a sequence $\{x_n\}$ in X satisfying $x_n = 1 + \frac{1}{n}$ and $\lim_n fx_n = 1 \in [1,2] = \lim_n Tx_n$ such that $\lim_n Tfx_n = \{0\} \neq T[1,2]$ and $\lim_n fTx_n = [0,1) \neq f1$ i.e, pair of maps (f,T) is not reciprocally continuous.
But it is conditionally reciprocally continuous as there exists a sequence $\{y_n\}$ in X satisfying $y_n = \frac{1}{n}$ and $\lim_n fy_n = 0 \in \{0\} = \lim_n Ty_n$ such that $\lim_n Tfy_n = \{0\} = T0$ and $\lim_n fTy_n = 0 = f0$. One may verify that maps f and T are discontinuous at $x = 1$.

Clearly continuous or reciprocally continuous hybrid pair of maps is conditionally reciprocally continuous but as shown in Example 3.3 the converse need not be true.

Now as an application of faint compatibility we prove our first main result.
Theorem 1 Let faintly compatible hybrid pair (f,T) of a metric space (X,d) satisfies

$$\delta(Tx,Ty) \le kd(fx,fy), 0 \le k < 1. \tag{1}$$

If f is continuous then f and T have a unique common strict fixed point.

Proof. Faint compatibility of a hybrid pair (f,T) implies that it is conditionally compatible, i.e., there exists a sequence $\{x_n\}$ in X satisfying $lim_n fx_n = t \in A = lim_n Tx_n$. Also there exists a sequence $\{y_n\}$ in X satisfying $\lim_n fy_n = u \in B = \lim_n Tx_n$ such that $\lim_n \delta(fTy_n, Tfy_n) = 0$.
Further, since f is continuous $\lim_n f(fy_n) = fu$ and $\lim_n f(Ty_n) = fB$. Thus $\lim_n Tfy_n = fB$.
By putting $x = u$ and $y = fy_n$ in condition (1), we get $\delta(Tu, Tfy_n) \le kd(fu, ffy_n)$. Taking $\lim n \to \infty$ we get,

$\delta(Tu, fB) \leq kd(fu, fu)$, i.e $\delta(Tu, fB) = 0$ or $Tu = fB$.
Since $u \in B$, $fu \in fB = Tu = \{fu\}$ i.e., fu is a strict coincidence point of f and T.
Further faint compatibility implies $fTu \subseteq Tfu$.
For $x = fu$ and $y = u$ condition (1) gives,
$\delta(Tfu, Tu) \leq kd(ffu, fu)$.
Since $fu \in Tu$, $ffu \in fTu \subseteq Tfu$.
$d(ffu, fu) \leq \delta(Tfu, Tu) \leq kd(ffu, fu)$, a contradiction.
Hence $\{fu\} = \{ffu\} = Tfu$, i.e., fu is a common strict fixed point of f and T.
For uniqueness, suppose that w is also a common strict fixed point other than $fu = z$.
Then by using condition (1), we have $\delta(Tz, Tw) \leq kd(fz, fw)$.
Since $z = fz \in Tz$ and $w = fw \in Tw$.Therefore, $d(fz, fw) \leq \delta(Tz, Tw) \leq kd(fz, fw)$, which is a contradiction.
Hence, $z = w$ i.e., fu is a unique common strict fixed point of f and T.

Example 4 Let $X = [0, 10]$, d be the usual metric on X. Let a hybrid pair of map (f, T) on X be defined as follows:

$$fx = \begin{cases} 1 - x, & 0 \leq x < 1 \\ \frac{2x+1}{3}, & 1 \leq x \leq 10, \end{cases} \qquad Tx = \begin{cases} [\frac{2-x}{2}, \frac{3-x}{3}], & 0 \leq x < 1. \\ \{1\}, & 1 \leq x \leq 10 \end{cases}$$

Then one may verify that f and T satisfy condition (1) of Theorem 1 for $k < 1$.
Let $\{x_n\}$ be a sequence in X where $x_n = \frac{1}{n}$ and $\lim_n fx_n = 1 \in \{1\} = \lim_n Tx_n$ such that $\lim_n \delta(fTx_n, Tfx_n) \neq \lim_n \delta(0, [\frac{1}{2}, \frac{2}{3}]) \neq 0$, i.e., pair of maps (f, T) is noncompatible.
Let $\{y_n\}$ be a sequence in X where $y_n = 1$ and $\lim_n fy_n = 1 \in \{1\} = \lim_n Ty_n$ such that $\lim_n \delta(fTy_n, Tfy_n) = \lim_n \delta(1, \{1\}) = 0$. Also f and T commute on strict coincidence point $1 \in X$. Thus a pair of maps (f, T) is faintly compatible.
Hence f and T satisfy all the conditions of Theorem 1 and have a unique common strict fixed point at $x = 1$. Here one may verify that f is continuous and T is discontinuous at $x = 1$. Moreover $fX \not\subseteq TX$.

One may notice that Theorem 1 is an improved and extended version of Theorem 1 of Bisht and Shahzad [1] without containment requirement of range space of involved hybrid pair of maps. Now we validate the applicability of conditional reciprocal continuity to determine strict coincidence and unique common strict fixed point of a hybrid pair of self-maps, which increase the probability of the study of common strict fixed point from the compatible continuous class of maps to a wider noncompatible and discontinuous class of maps.

Theorem 2 Let faintly compatible hybrid pair (f, T) of a metric space (X, d) be conditionally reciprocally continuous. Then f and T have a coincidence point. Moreover f and T have a unique common strict fixed point provided that the pair satisfies

$$\delta(Tx, Ty) \leq k \max\{d(fx, fy), D(fx, Tx), D(fy, Ty), \frac{1}{2}[D(fx, Ty) + D(fy, Tx)]\}, \tag{2}$$

where, $0 \leq k < 1$.

Proof. Since the hybrid pair (f, T) is conditionally reciprocally continuous, there exists a sequence $\{x_n\}$ such that $\lim_n fx_n = t \in A = \lim_n Tx_n$ where $t \in X$ and $A \in CB(X)$ is non empty, there exists a sequence $\{y_n\}$ satisfying $\lim_n fy_n = u \in B = \lim_n Ty_n$, for some $u \in X$ and $B \in CB(X)$ such that $\lim_n fTy_n = fB$ and $\lim_n Tfy_n = Tu$.
Also, since the pair (f, T) is faintly compatible. It is also conditionally compatible, i.e., $\lim_n \delta(fTy_n, Tfy_n) = 0$ for some $u \in X$ and $B \in CB(X)$. Hence $\delta(fB, Tu) = 0$, i.e., $fB = Tu$.
Now $u \in B$ implies $fu \in fB = Tu = \{fu\}$, i.e. f and T have a strict coincidence. So $C(T, f) \neq 0$.
Hence there exists $u \in M \subseteq C(f, T)$ such that $fTu = Tfu$. Hence $ffu \in fTu \subseteq Tfu$.
Next, we prove that fu is a common strict fixed point of f. Suppose that $ffu \neq fu$. Then by using the condition (2), we have

$$\delta(Tfu, Tu) \leq k \max\{d(ffu, fu), D(ffu, Tfu), D(fu, Tu), \frac{1}{2}[D(ffu, Tu)+D(fu, Tfu)]\},$$
$$\leq k \max d(ffu, fu), 0, 0, \frac{1}{2}[D(ffu, Tu) + D(fu, Tfu)].$$

Since $fu \in Tu$, $D(ffu, Tu) \leq d(ffu, fu)$ and $ffu \in fTu$, $D(fu, Tfu) \leq d(ffu, fu)$.
Therefore, $d(ffu, fu) \leq \delta(Tfu, Tu) \leq kd(ffu, fu)$, which is a contradiction.
Hence, $\{fu\} = \{ffu\} = Tfu$, i.e., fu is a common strict fixed point of f and T.
For uniqueness, suppose that w is also a common strict fixed point other than $fu = z$. Then by using condition (2), we have

$$\delta(Tz, Tw) \leq k \max\{d(fz, fw), D(fz, Tu), D(fw, Tw), \frac{1}{2}[D(fu, Tw) + D(fw, Tu)]\}$$
$$\leq k \max\{d(fz, fw), 0, 0, \frac{1}{2}[D(fz, Tw) + D(fw, Tz)]\}.$$

Since $fz \in Tz$, $D(fw, Tz) \leq d(fw, fz)$ and $w \in fw$, $D(fz, Tw) \leq d(fz, fw)$.
Therefore, $d(fz, fw) \leq \delta(Tz, Tw) \leq kd(fz, fw)$, which is a contradiction.
Hence, $z = w$, i.e., fu is a unique common strict fixed point of f and T.

Example 5 Let $X = [1, 12]$, d be the usual metric on X. Let a hybrid pair of map (f, T) on X be defined as follows:

$$fx = \begin{cases} 2 - x, & 0 \leq x \leq 1 \\ \frac{x+10}{2}, & 1 < x \leq 12, \end{cases} \qquad Tx = \begin{cases} \{1\}, & 0 \leq x \leq 1 \\ [\frac{5}{4}, \frac{3}{2}], & 1 < x \leq 12. \end{cases}$$

Then one may verify that f and T satisfy condition (2) of Theorem 2 for $k < 1$. Consider a sequence $\{x_n\}$ in X satisfying $x_n = 1 - \frac{1}{n}$ and $\lim_n fx_n = 1 \in \{1\} = \lim_n Tx_n$ such that $\lim_n Tfx_n = [\frac{5}{4}, \frac{3}{2}] \neq T\{1\}$ and $\lim_n fTx_n = 1 = f1$ i.e, pair of maps (f, T) is not reciprocally continuous. Also $\lim_n \delta(fTx_n, Tfx_n) \neq 0$ i.e, pair of maps (f, T) is noncompatible.
Let $\{y_n\}$ be a sequence in X where $y_n = 1$ and $\lim_n fy_n = 1 \in \{1\} = \lim_n Ty_n$ such that $\lim_n Tfy_n = \{1\} = T1$ and $\lim_n fTx_n = 1 = f1$. i.e., $\lim_n \delta(fTy_n, Tfy_n) = 0$. Also f and g commute on coincidence point $1 \in X$. Thus pair of maps (f, T) is conditionally reciprocally continuous and faintly compatible.
Hence f and T satisfy all the condition of Theorem 2 and have a unique common strict fixed point at $x = 1$. Here one may verify that f and T are discontinuous at

$x = 1$. Moreover $fX \nsubseteq TX$.

The next theorem illustrates the applicability of faint compatibility and conditional reciprocal continuity by determining unique strict coincidence and unique common strict fixed point of a discontinuous hybrid pair of maps satisfying the strict contractive condition.

Theorem 3 Let faintly compatible hybrid pair (f, T) of a metric space (X, d) be conditionally reciprocally continuous. Then f and T have a coincidence point. Moreover f and T have a unique common strict fixed point provided that the pair satisfies

$$\delta(Tx, Ty) < \max\{d(fx, fy), D(fx, Tx), D(fy, Ty), \frac{1}{2}[D(fx, Ty) + D(fy, Tx)]\}. \tag{3}$$

Proof. Proof of Theorem 3 follows on the similar lines as of Theorem 2.

Theorem 4 Let faintly compatible hybrid pair (f, T) of a metric space (X, d) be conditionally reciprocally continuous. Then f and T have a coincidence point. Moreover f and T have a unique common strict fixed point provided that the pair satisfies

$$\delta(Tx, Ty) \leq kd(fx, fy), 0 \leq k < 1. \tag{4}$$

Proof. Proof of Theorem 4 follows on the similar lines as of Theorem 2.

If T is a single valued mapping in Theorem 2, then we have the following;
Corollary 1 Let faintly compatible pair of single map (f, T) of a metric space (X, d) be conditionally reciprocally continuous. Then f and T have a coincidence point. Moreover f and T have a unique common fixed point provided that the pair satisfies

$$d(Tx, Ty) < k \max\{d(fx, fy), d(fx, Tx), d(fy, Ty), \frac{1}{2}[d(fx, Ty)+d(fy, Tx)]\}, \tag{5}$$

where, $0 \leq k < 1$.

Remark (i) One may notice that in theorems 1, 2 and 3 contractive conditions used to establish strict coincidence and unique common strict fixed point of a hybrid pair of map is more general than used by Bisht and Shahzad [1] and Pant and Bisht [5].
(ii) Theorems 2, 3 and 4 expose the eminence of conditional reciprocal continuity over continuity when the given pair of maps is not even compatible and marks preeminence over all those results wherein the continuity of even single map, containment requirement of range space of involved maps and completeness (or closedness) of the whole space/subspaces are presumed for the existence of coincidence point/strict coincidence point or common fixed point/common strict fixed point.
(iii) Corollary 1 is an improved version of Bisht and Shahzad [1] and Pant and Bisht [5] and [6] for single valued maps without containment requirement of range space of involved pair of maps. Moreover Faint compatibility used to establish common

fixed point is more general then the variants of compatibility.

Conclusion : Our results generalize, extend and improve the results of Bisht and Shahzad [1], Pant and Bisht [5] and [6] and references therein to a faintly compatible hybrid pair of discontinuous maps in non-complete metric space without containment requirement of range space of involved hybrid pair of maps. It is well known that contractivity of maps is not sufficient condition for the existence of fixed point. For instance: If $X = R$ and $fx = x + \frac{1}{x}$; $fx = \sqrt{x^2+1}$ or $fx = \ln(1+e^x)$, then in each case f is contractive but has no fixed points in X. In such cases either the space/subspace is taken to be complete or compact, or some strong conditions are presumed on the maps involved for the existence of fixed point.

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