TRANSLATION-FACTORABLE SURFACES IN THE
3-DIMENSIONAL EUCLIDEAN AND LORENTZIAN SPACES
SATISFYING $\Delta r_i = \lambda_i r_i$

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Abstract. This paper deals with the Translation-Factorable (TF) surfaces in
the 3-dimensional Euclidean space and Lorentzian-Minkowski space with the
condition $\Delta r_i = \lambda_i r_i$ where $\Delta$ denotes the Laplace operator. Our result will
be obtained for the complete classification theorems and give an explicit forms
of these surfaces.

1. Introduction

In 1983 B-Y Chen introduced the notion of Euclidian immersions of finite type.
Basically there are submanifolds whose into $\mathbb{R}^m$ is constructed by making use of
finite number of $\mathbb{R}^m$-valued eigenfunctions of their Lapalacian. Many works were
done to characterize the classification of submanifolds in terms of finite type. Im-
portant results about 2-type spherical closed submanifolds (where spherical means
into a sphere) have been obtained see [9].

A well known are the only surfaces in $\mathbb{R}^3$ satisfying the condition

$$\Delta r = \lambda r \quad \lambda \in \mathbb{R}$$

where $\Delta$ is the Laplace operator associated with the induced metric.

On the other hand Garay [13] determined the complete surfaces of revolution in $\mathbb{R}^3$
whose component functions are eigenfunctions of their Laplace operator i.e.

$$\Delta r^i = \lambda^i r^i \quad \lambda^i \in \mathbb{R}$$

Later Lopez [16] studied the hypersurfaces in $\mathbb{R}^{n+1}$ verifying

$$\Delta r = \lambda r \quad \lambda \in \mathbb{R}^{n+1}$$

Kaimakamis and Papantounion [7] studied surfaces of revolution in the 3-dimensional
Lorentz-Minkowski space satisfying the condition

$$\Delta^{II} r = Ar$$

where $\Delta^{II}$ is the Laplace operator with respect to the second fundamental form
and $A$ is a real $3 \times 3$ array.
Zoubir and Bekkar [8] classified the surfaces of revolution with non zero Gaussian curvature $K_G$ in the 3-dimensional Euclidean space $E^3$ and Lorentzian-Minkowski spaces under the condition

$$\Delta r^i = \lambda^i r^i.$$  

Baba Hamed, Bekkar and Zoubir [4] determined the translation surfaces in the 3-dimensional Lorentz-Minkowski space $E^3_1$, whose component functions are eigenfunctions of their Laplace operator. Baba Hamed, Bekkar [3] studies the helicoidal surfaces without parabolic points in $E^3_1$, which satisfy the condition

$$\Delta II r_i = \lambda_i r.$$  

Bekkar and Senoussi [6] studied the factorable surfaces in the 3-dimensional Minkowski space under the condition

$$\Delta r_i = \lambda_i r.$$  

where $\lambda_i \in \mathbb{R}$ and $dr_i$ are the coordinate of the surface. There has been classification of factorable surface in the 3-dimensional Lorentz-Minkowski Euclidian and pseudo-Galilean space. Lopezand and Moruz [17] studied translation and homothetical non degenerate surfaces in Euclidian in $E^3_1$.

In this paper we classify the factorable surfaces in the 3-dimensional Euclidean space $E^3$ and Lorentzian $E^3_1$ under the condition

$$\Delta r_i = \lambda_i r_i$$  

where $\lambda_i \in \mathbb{R}$.

2. Preliminaries

Let $E^3$ be the 3-dimensional Euclidean space, equipped with the inner product

$$g(X, Y) = x_1 y_1 + x_2 y_2 + x_3 y_3$$

for $X = (x_1, x_2, x_3), Y = (y_1, y_2, y_3) \in E^3$.

Let $E^3_1$ be the 3-dimensional Minkowski space, with the scalar product given by

$$g_L = -dx^2 + dy^2 + dz^2$$

where $(x, y, z)$ is a standard rectangular coordinate system of $E^3_1$.

Let $r : M^2 \to E^3_1$ be an isometric immersion of a surface in the 3-dimensional Lorentzian-Minkowski space.

A surface $M^2$ is said to be of finite type if every component of its position vector field $r$ can be written as a finite sum of eigenfunction of the Laplace $\Delta$ of $M^2$, if

$$r = r_0 + \sum_{i=1}^{k} r_i.$$  

Definition 2.1 (4-15). A surface $M$ is a translation surface if it can be parametrized by

$$x(u, v) = (u, v, f(u) + g(v))$$  

Definition 2.2 (6-18). A surface $M$ is a factorable surface if it can be parameterized by

$$x(u, v) = (u, v, f(u)g(v))$$
Next, we define an extended surface in \(E^3\) using definitions we call it TF-type surface as follows:

**Definition 2.3.** A surface \(M\) is a TF-type surface if it can be parameterized by

\[
x(u, v) = (u, v, A(f(u) + g(v)) + Bf(u)g(v)),
\]

where \(A\) and \(B\) are non-zero real numbers.

**Remark 2.4.** In [4], we have if \(A \neq 0\) and \(B = 0\) in, then surface is a translation surface. In [14], we have if \(A = 0\) and \(B \neq 0\), then surface is a factorable surface.

For vector \(X = (x_1, x_2, x_3)\) and \(Y = (y_1, y_2, y_3)\) in \(E^3\), the Lorentz scalar product and the cross product are defined by:

\[
g_L = -x_1y_1 + x_2y_2 + x_3y_3
\]

The Gauss curvature and the mean curvature are:

\[
K_G = g_L(N, N) \left( \frac{LN - M^2}{EG - F^2} \right), \quad H = \frac{EN + GL - 2FM}{2|EG - F^2|}
\]

Let \(x^i, x^j\) be a local coordinate system of \(M^2\). For the array \((g_{i,j})\) \((i, j = 1, 2)\) consisting of components of the induced metric on \(M^2\), we denote by \((g^{i,j})\) the inverse matrix of the array \((g_{i,j})\). Then the Laplacian operator \(\Delta\) on \(M^2\) is given by:

\[
\Delta = -\frac{1}{\sqrt{|D|}} \sum_{i,j} \frac{\partial}{\partial x^i} \left( \sqrt{|D|} g^{i,j} \frac{\partial}{\partial x^j} \right)
\]

A vector \(V\) of \(E^3\) is said to be timelike if \(g_L(V, V) < 0\), spacelike if \(g_L(V, V) > 0\) or \(V = 0\) and lightlike or null if \(g_L(V, V) = 0\) and \(V \neq 0\). A surface in \(E^3\) is spacelike, timelike or lightlike if the tangent plane at any point is spacelike, timelike or lightlike respectively [19].

3. **Translation-factorable surfaces in \(E^3\)**

In this section, we consider surface in \(E^3\). Assume that \(M^2\) is equivalent to

\[
r(u, v) = (u, v, f(u)g(v) + f(u) + g(v))
\]

the coefficients of the first fundamental form are:

\[
E = (f'g + f')^2 + 1, \quad F = (f'g + f')(fg' + g'), \quad G = (fg' + g')^2 + 1
\]

\[
N = \frac{1}{W}(-f'g - f', -fg' - g', 1)
\]

the coefficients of the second fundamental form are:

\[
L = \frac{f''g + f'}{W}, \quad M = \frac{(f'g + f')(fg' + g')}{W}, \quad N = \frac{f g'' + g''}{W}
\]

where \(W = \sqrt{(f'g + f')^2 + (fg' + g')^2 + 1}\)

The Laplacian \(\Delta\) of \(M^2\) is given by:

\[
\Delta = \frac{1}{W^2} \left( E \frac{\partial^2}{\partial u^2} + G \frac{\partial^2}{\partial v^2} - 2F \frac{\partial^2}{\partial u \partial v} \right) + \frac{2H}{W} \left( (f'g + f') \frac{\partial}{\partial u} + (fg' + g') \frac{\partial}{\partial v} \right)
\]

\[
\Delta u = \lambda_1 u; \quad \Delta v = \lambda_2 v; \quad \Delta (f(u)g(v) + f(u) + g(v)) = \lambda_3 (f(u)g(v) + f(u) + g(v))
\]

By using (1), (7) we get:

\[
2(f'g + f')H = W\lambda_1 u
\]
Next, we explore the classification of the Translation-Factorable surfaces $M^2$ satisfying (1)

Case 1: Let $\lambda_3 \neq 0$.

(i) If $fg + f + g = 0$, then $H = 0$

(ii) If $fg + f + g \neq 0$ we have:

$(k_1)$ If $\lambda_1 = 0$ and $\lambda_2 \neq 0$, equations (10) and (11) imply that:

$$f(u) = a \in \mathbb{R} - \{-1\}, \quad g' \neq 0 \quad \text{and} \quad H = \frac{(a + 1)g''}{2W^3}$$

The system of equations (10), (11) and (12) becomes

$$(1 + a)^2 g'' = \lambda_2 v((a + 1)^2 g'^2 + 1)^2$$

$$(1 + a)g'' = -\lambda_3 (ag + a + g)((a + 1)^2 g'^2 + 1)^2$$

Equation (14) is equivalent to

$$g(v) = \frac{1}{a + 1} \left( -a \pm \sqrt{-\frac{\lambda_2 v^2 + a^2 \lambda_3}{\lambda_3}} \right) \quad (-1 < -\lambda_2 v^2 + a^2 \lambda_3 < 0)$$

Hence, the surface $M^2$ can be expressed by

$$r(u, v, \pm \sqrt{-\frac{\lambda_2 v^2 + a^2 \lambda_3}{\lambda_3}}) \quad (-1 < -\lambda_2 v^2 + a^2 \lambda_3 < 0)$$

$(k_2)$ If $\lambda_2 = 0$ and $\lambda_1 \neq 0$. Equations (10) and (11) imply that:

$$f(u) = c \in \mathbb{R} - \{-1\}, \quad g' \neq 0 \quad \text{and} \quad H = \frac{(c + 1)f''}{2W^3}$$

The system of equations (10), (11) and (12), in this case takes the form

$$(1 + c)^2 f' f'' = \lambda_2 u((c + 1)^2 f'^2 + 1)^2$$

$$(1 + c)f'' = -\lambda_3 (cf + c + f)((c + 1)^2 f'^2 + 1)^2$$

Equation (16) is equivalent to

$$f(u) = \frac{1}{a + 1} \left( -a \pm \sqrt{-\frac{\lambda_2 u^2 + a^2 \lambda_3}{\lambda_3}} \right) \quad (-1 < -\lambda_2 u^2 + a^2 \lambda_3 < 0)$$

Hence, the surface $M^2$ can be expressed by:

$$r(u, v, \pm \sqrt{-\frac{\lambda_2 u^2 + a^2 \lambda_3}{\lambda_3}}) \quad (-1 < -\lambda_2 u^2 + a^2 \lambda_3 < 0)$$

$(k_3)$ If $\lambda_1 \neq 0$ and $\lambda_2 \neq 0$. Equations (10) and (11) imply that:

$$f' \neq 0 \quad \text{and} \quad g' \neq 0$$

We multiply Equation (10) by $fg' + g'$ and Equation (10) by $f'g + f'$, we obtain:

$$\frac{(f + 1)}{f'} \lambda_1 u = \frac{(g + 1)}{g'} \lambda_2 v = e, \quad e \in \mathbb{R}^*$$

$$2(fg' + g')H = W\lambda_2 v$$

$$2H = -W\lambda_3 (fg + f + g)$$
Equations (10) and (12) imply that:

$$\lambda_1 u = -\lambda_3 (fg + f + g)(f'g + f)$$

(18)

equations (17) and (18) imply that:

$$-\lambda_3 (fg + f + g)^2 = e$$

(19)

The functions $f$ and $g$ are constants, hence there are no Translation-Factorable surfaces in this cases satisfying (1)

$k_4$) If $\lambda_1 = 0$ and $\lambda_2 = 0$ equations (10) and (11) imply that:

$$f' = 0 \quad \text{and} \quad g' = 0$$

Hence $\lambda_3 = 0$. Therefore, there are no Translation-Factorable surfaces in this cases satisfying (1)

Case 2: Let $\lambda_3 = 0$. Then, the equation (12) gives rise to $H = 0$ which means that the surfaces are minimal. We get also by the equations (10) and (11): $\lambda_1 = \lambda_2 = 0$

Finally:

**Theorem 3.1.** Let $M^2$ be a Translation-Factorable (TF) given by (6) in $E^3$. Then $M^2$ satisfies $\Delta r_i = \lambda_i r_i, (i=1; 2; 3)$ if and only if the following statements hold

(1) $M^2$ has zero mean curvature

(2) $M^2$ is parameterized as

$$\left(u, v, \pm \sqrt{-\frac{\lambda_2 v^2 + a^2 \lambda_3}{\lambda_3}}\right) \quad (-1 < -\lambda_2 v^2 + a^2 \lambda_3 < 0)$$

(3) $M^2$ is parameterized as

$$\left(u, v, \pm \sqrt{-\frac{\lambda_2 u^2 + a^2 \lambda_3}{\lambda_3}}\right) \quad (-1 < -\lambda_2 u^2 + a^2 \lambda_3 < 0)$$

**Translation-Factorable surfaces in $E^3_1$.** In this section, we consider surfaces in $E^3_1$ and we investigate the classification of the Translation-Factorable satisfying (1). We distinguish $EG - F^2 > 0$ or $EG - F^2 < 0$.

Suppose that $M^2$ is given by (6), the coefficients of the first and second fundamental forms are:

$$E = (f'g + f')^2 - 1; \quad F = (f'g + f')(fg' + g'); \quad G = 1 + (fg' + g)^2$$

(20)

and

$$L = \frac{f''g + f'}{W}; \quad M = \frac{(f'g + f')(fg' + g')}{W}; \quad N = \frac{fg'' + g''}{W}$$

The mean curvature $H$ is

$$H = 1/2W^{-3}H_1$$

Where $H_1 = (1 + (fg' + g')^2)(f''g + f') + (f'g + f')^2 - 1)(fg'' + g'') - 2(f + 1)(g + 1)f^2g'^2$
Spacelike Translation-Factorable surfaces in $E^3_1$. We investigate the spacelike translation and factorable surfaces in $E^3_1$.

If we use (5), the Laplacian $\Delta$ of $M^2$ is given by:

$$\Delta = \frac{1}{W^2} \left( E \frac{\partial^2}{\partial v^2} + G \frac{\partial^2}{\partial u^2} - 2F \frac{\partial^2}{\partial u \partial v} \right) - \frac{2H}{W} \left( (fg' + g') \frac{\partial}{\partial v} - (f'g + f') \frac{\partial}{\partial u} \right)$$  \hspace{1cm} (21)

where $W = \sqrt{EG - F^2}$.

Assume that $EG - F^2 = (f'g + f')^2 - (fg' + g')^2 - 1 > 0$, the metric of $M^2$ is spacelike.

Then using (1), (20) and (21) we have:

$$W^{-4}(f'g + f')H_1 = \lambda_1 u$$ \hspace{1cm} (22)

$$W^{-4}(fg' + g'H_1 = -\lambda_2 v$$ \hspace{1cm} (23)

$$W^{-4}H_1 = \lambda_3(fg + f + g)$$ \hspace{1cm} (24)

First, we examine the classification of the spacelike Translation-Factorable surfaces $M^2$ satisfying (1).

Case 1: Let $\lambda_3 = 0$, then, the equation (24) gives rise to $H_1 = 0$ meaning that the surface are minimal. We get also by the equations (22) and (23) $\lambda_1 = \lambda_2 = 0$.

Case 2: Let $\lambda_3 \neq 0$, then $H_1 \neq 0$ and hence we have necessarily by equation (22) $\lambda_1 \neq 0$.

i) If $\lambda_3 = 0$ we get (23), so $g(v) = a, a \in \mathbb{R} - \{-1\}$.

In this case, the system of equations (22),(23) and (24) takes the form:

$$(a + 1)^2 f' f'' = \lambda_1 u((a + 1)^2 f'^2 - 1)^2$$ \hspace{1cm} (25)

$$(a + 1)f'' = \lambda_3(a f + f + a)((a + 1)^2 f'^2 - 1)^2$$ \hspace{1cm} (26)

Using equation (26)

$$f(u) = \frac{1}{(a + 1)} \left( -a \pm \sqrt{\frac{\lambda_2 u^2 + a^2 \lambda_3}{\lambda_3}} \right)$$ \hspace{1cm} (27)

So the parametrization of the surfaces can be written in the form:

$$r(u, v) = \left( u, v, \pm \sqrt{\frac{\lambda_2 u^2 + a^2 \lambda_3}{\lambda_3}} \right)$$

ii) If $\lambda_2 \neq 0$ we can rewrite the system as follow:

$$\begin{cases} 
\lambda_2 v(fg + f + g) = -a(fg' + g') \\
\lambda_3 u(fg + f + g) = a(f'g + f')
\end{cases} \hspace{1cm} (28)$$

Equation (21) and (22) $(a \neq -1)$ imply that:

$$\lambda_1 u = \lambda_3(fg + f + g)(f'g + f')$$ \hspace{1cm} (29)

From (27) and (28) we obtain:

$$a = \lambda_3(fg + f + g)^2$$

Therefore the functions $f$ and $g$ are constants assuming that there are no Translation-Factorable surfaces in this case satisfying (1). Thus we can give the following result:
Theorem 3.2. Let \( M^2 \) be a spacelike Translation-Factorable (TF) given by (6) in \( E^3_{1} \). Then \( M^2 \) satisfies \( \Delta r_i = \lambda_i r_i \), \((i=1;2;3)\) if and only if the following statements hold

(1) \( M^2 \) has zero mean curvature
(2) \( M^2 \) is parameterized as

\[ r(u,v) = \left( u, v, \pm \sqrt{\frac{\lambda_2 u^2 + a^2 \lambda_3}{\lambda_3}} \right) \quad (0 < \lambda_2 u^2 + a^2 \lambda_3 < 1) \]

Timelike Translation-Factorable surfaces in \( E^3_{1} \). In this section, we deal with the spacelike translation-factorable surfaces in \( E^3_{1} \).

If we use (5), the Laplacian \( \Delta \) of \( M^2 \) is given by:

\[
\Delta = \frac{1}{W^2} \left( E \frac{\partial^2}{\partial v^2} + G \frac{\partial^2}{\partial u^2} - 2F \frac{\partial^2}{\partial u \partial v} - \frac{2H}{W} \left( (fg' + g') \frac{\partial}{\partial v} - (f'g + f') \frac{\partial}{\partial u} \right) \right)
\]

where \( W = \sqrt{F^2 - EG} \).

Assuming that \( EG - F^2 = (f'g + f')^2 - (fg' + g')^2 - 1 < 0 \), the metric of \( M^2 \) is timelike.

Then using (29) and (20) we get

\[
\begin{align*}
\Delta(u) &= -W^{-4}(f'g + f')H_1 \\
\Delta(v) &= W^{-4}(fg' + g')H_1 \\
\Delta(fg + f + g) &= -W^{-4}H_1
\end{align*}
\]

hence

\[
\Delta r = W^{-4}H_1(-f'g + f', fg' + g', -1)
\]

By (1) and (30) we obtain the following system of differential equations

\[
\begin{align*}
W^{-4}(f'g + f')H_1 &= -\lambda_1 u \\
W^{-4}(fg' + g')H_1 &= \lambda_2 v \\
W^{-4}H_1 &= \lambda_3(fg + f + g)
\end{align*}
\]

We explore the classification of the timelike Translation-Factorable surfaces \( M^2 \) satisfying (1.1).

Case 1: Let \( \lambda_3 = 0 \), then, the equation (35) gives rise to \( H_1 = 0 \), which means that the surfaces are minimal. We have also by the equations (33) and (34) \( \lambda_1 = \lambda_2 = 0 \).

Cases 2: Let \( \lambda_3 \neq 0 \).

i) If \( fg + f + g = 0 \), then \( H_1 = 0 \)

ii) If \( fg + f + g \neq 0 \), in this case we have:

a) If \( \lambda_1 = 0 \) and \( \lambda_2 \neq 0 \) equations (33) and (34) imply that:

\[ f' = 0, \quad g' \neq 0, \quad \text{and} \quad g'' \neq 0 \]

It follows that \( f(u) = a, \ a \in \mathbb{R} - \{-1\} \) and \( g'(v) \) is a non constant function.

The system (33), (34) and (35) is reduced to be equivalent to

\[
\begin{align*}
-(a + 1)^2g'g'' &= \lambda_2 v((a + 1)^2g'^2 + 1)^2 \\
(a + 1)g'' &= \lambda_3(AG + g + a)((a + 1)^2f'^2 + 1)^2
\end{align*}
\]
Equation (36) implies
\[ g(v) = \frac{1}{(a + 1)} \left( -a \pm \sqrt{\frac{-\lambda_2 v^2 + a^2 \lambda_3}{\lambda_3}} \right) \] which such that \( 0 < -\lambda_2 v^2 + a^2 \lambda_3 < 1 \)

So the parametrization of the surfaces can be written in the form:
\[ r(u, v) = \left( u, v, \pm \sqrt{\frac{-\lambda_2 u^2 + a^2 \lambda_3}{\lambda_3}} \right) \] which such that \( 0 < -\lambda_2 u^2 + a^2 \lambda_3 < 1 \)

ii) If \( \lambda_2 = 0 \) and \(-\lambda_2 v^2 + a^2 \lambda_3 \neq 0 \) then
\[ g' = 0, \quad f' \neq 0, \quad f'' \neq 0 \]
The system (33), (34) and (35) is reduce equivalently to
\[ - (a + 1)^2 g' g'' = \lambda_1 u ((a + 1)^2 g'^2 + 1)^2 \]
\[ - (a + 1) f'' = \lambda_3 (af + f + a)(1 - (a + 1)^2 f'^2)^2 \]
Hence
\[ f(u) = \frac{1}{(a + 1)} \left( -a \pm \sqrt{\frac{\lambda_2 u^2 + a^2 \lambda_3}{2 \lambda_3}} \right) \] which such that \( 0 < \lambda_2 u^2 + a^2 \lambda_3 < 1 \)

So the parametrization of the surfaces can be written in the form
\[ r(u, v) = \left( u, v, \pm \sqrt{\frac{\lambda_2 u^2 + a^2 \lambda_3}{2 \lambda_3}} \right) \] which such that \( \lambda_2 u^2 + a^2 \lambda_3 < 1 \)

c) If \( \lambda_1 = \lambda_2 = 0 \) we have:
i) If \( f' = g' = 0 \) imply \( H_1 = 0 \). From (35) we obtain \( \lambda_3 = 0 \) which is a contradiction.
ii) If \( f' = 0 \) and \( g' \neq 0 \), then (34) gives \( g = 0 \), which is a contradiction.
iii) If \( f' \neq 0 \) and \( g' = 0 \), then (33) gives \( g = 0 \), which is a contradiction.
3) If \( \lambda_1 \neq 0 \) and \( \lambda_2 \neq 0 \), then
\[ f' \neq 0 \quad g' \neq 0 \]
We multiply Equation (33) by \( g' f + g' \) and (34) by \( f' g + f' \), and we obtain:
\[ \begin{cases} \lambda_2 v(fg + g + f) = a(fg' + g') \\ \lambda_1 u(fg + g + f) = -a(f'g + f') \end{cases} \]
Equation (33) and (33) \( a \neq -1 \) imply
\[ \lambda_1 u = \lambda_3 (fg + f + g)(f'g + f') \]
From (39) and (40) we obtain:
\[ -a = \lambda_3 (fg + f + g)^2 \]
The functions \( f \) and \( g \) are constants and hence there are no Translation-Factorable surfaces in this case satisfying (1). Thus we can give the following result:
Theorem 3.3. Let $M^2$ be a timelike Translation-Factorable (TF) given by (6) in $E^3_1$. Then $M^2$ satisfies $\Delta r_i = \lambda_i r_i$, $(i=1;2;3)$ if and only if the following statements hold

1) $M^2$ has zero mean curvature

2) $M^2$ is parameterized as

$$r(u,v) = \left( u, v, \pm \sqrt{-\lambda_2 v^2 + a^2 \lambda_3 \over \lambda_3} \right) \quad (0 < -\lambda_2 v^2 + a^2 \lambda_3 < 1)$$

3) $M^2$ is parameterized as

$$r(u,v) = \left( u, v, \pm \sqrt{\lambda_2 u^2 + a^2 \lambda_3 \over \lambda_3} \right) \quad (0 < \lambda_2 u^2 + a^2 \lambda_3 < 1)$$

References


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