COMMON FIXED POINT THEOREM IN INTUITIONISTIC FUZZY METRIC SPACE USING (CLRG) PROPERTY

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ABSTRACT. In this paper, we prove common fixed point theorem for semi-compatible pair of self maps in intuitionistic fuzzy metric space using CLRg Property.

1. INTRODUCTION

The human reasoning involves the use of variable whose values are fuzzy sets. Description of system behavior in the language of fuzzy rules lowers the need for precision in data gathering and data manipulation, and in effect may be viewed as a form of data compression. But there are situations when description by a (fuzzy) linguistic variable given in terms of a membership function only, seems too rough. The use of linguistic variables represents a physical significant paradigm shift in system analysis.

Atanassov [2] introduced the notion of intuitionistic fuzzy sets by generalizing the notion of fuzzy set by treating membership as a fuzzy logical value rather than a single truth value. For an intuitionistic set the logical value has to be consistent (in the sense $\gamma A(x) + \mu A(x) \geq 1$). $\gamma A(x)$ and $\mu A(x)$ denotes degree of membership and degree of non-membership, respectively. All results which hold of fuzzy sets can be transformed Intuitionistic fuzzy sets but converse need not be true. Intuitionistic fuzzy set can be viewed in the context as a proper tool for representing hesitancy concerning both membership and non-membership of an element to a set. To be more precise, a basic assumption of fuzzy set theory that if we specify the degree of membership of an element in a fuzzy set as a real number from [0, 1], say 'a', then the degree of its non-membership is automatically determined as $'(1 - a)'$, need not hold for intuitionistic fuzzy stes. In intuitionistic fuzzy set theory it is assumed that non-membership should not be more than $(1 - a)$. For instant, lack of knowledge (hesitancy concerning both membership and non-membership of an element to a set) and the temperature of a patient changes and other symptoms are not quite clear. The area of intuitionistic fuzzy image processing is just beginning to develop; there are hardly few methods in the literature. Intuitionistic fuzzy

set theory has been used to extract information by reflecting and modeling the hesitancy present in real-life situations. The application of Intuitionistic fuzzy sets instead of fuzzy sets means the introduction of another degree of freedom into a set description. By employing intuitionistic fuzzy sets in databases we can express a hesitation concerning examined objects.

Coker [5] introduced the concept of intuitionistic fuzzy topological spaces. Alaca et al. [1] proved the well-known fixed point theorems of Banach [3] in the setting of intuitionistic fuzzy metric spaces. Later on, Turkoglu et al. [13] proved Jungck's [7] common fixed point theorem in the setting of intuitionistic fuzzy metric space. No wonder that intuitionistic fuzzy fixed point theory has become an area of interest for specialists in fixed point theory as intuitionistic fuzzy mathematics has covered new possibilities for fixed point theorists.

B. Singh et. al. [11] introduced the notion of semi compatible maps in fuzzy metric space. In 2011, Sintunayarat and Kuman [12] introduced the concept of common limit in the range property. Chouhan et. al. [6] utilize the notion of common limit range property to prove fixed point theorems for weakly compatible mapping in fuzzy metric space.

2. Preliminaries

Definition 2.1. Let X be any set. A fuzzy set A in X is a function with domain X and Values in [0,1].

The concepts of triangular norms (t-norms) and triangular conorms (t-conorms) are known as the axiomatic skelton that we use are characterization fuzzy intersections and union respectively. These concepts were originally introduced by Menger [9] in study of statistical metric spaces.

Definition 2.2. [10] A binary operation $* : [0,1] \times [0,1] \to [0,1]$ is continuous t-norm if $*$ satisfies the following conditions for all $a, b, c, d \in [0,1]$,

(i) $*$ is commutative and associative;
(ii) $*$ is continuous;
(iii) $a * 1 = a$;
(iv) $a * b \leq c * d$ whenever $a \leq c$ and $b \leq d$.

Definition 2.3. [10] A binary operation $\lozenge : [0,1] \times [0,1] \to [0,1]$ is continuous t-conorm if $\lozenge$ satisfies the following conditions for all $a, b, c, d \in [0,1]$,

(i) $\lozenge$ is commutative and associative;
(ii) $\lozenge$ is continuous;
(iii) $a \lozenge 0 = a$;
(iv) $a \lozenge b \geq c \lozenge d$ whenever $a \leq c$ and $b \leq d$.

Alaca et al. [1] using the idea of Intuitionistic fuzzy sets, defined the notion of intuitionistic fuzzy metric space with the help of continuous t-norm and continuous t-conorms as a generalization of fuzzy metric space due to Kramosil and Michalek [8] as:

Definition 2.4. [1] A 5-tuple $(X, M, N, *, \Diamond)$ is said to be an intuitionistic fuzzy metric space if X is an arbitrary set, $*$ is a continuous t-norm, $\Diamond$ is a continuous t-conorm and M, N are fuzzy sets on $X^2 \times [0, \infty)$ satisfying the following Conditions:

(i) $M(x, y, t) + N(x, y, t) \leq 1$ for all $x, y \in X$ and $t > 0$;

(ii) $M(x, y, 0) = 0$ for all $x, y \in X$;

(iii) $M(x, y, t) = 1$ for all $x, y \in X$ and $t > 0$ if and only if $x = y$;

(iv) $M(x, y, t) = M(y, x, t)$ for all $x, y \in X$ and $t > 0$;

(v) $M(x, y, t) * M(y, z, s) \leq M(x, z, t + s)$ for all $x, y, z \in X$ and $s, t > 0$;

(vi)   for all $x, y \in X$, $M(x, y, .) : [0, \infty) \to [0, 1]$ is left continuous;

(vii) $\lim_{t \to \infty} M(x, y, t) = 1$ for all $x, y \in X$ and $t > 0$;

(viii) $N(x, y, 0) = 1$ for all $x, y \in X$;

(ix) $N(x, y, t) = 0$ for all $x, y \in X$ and $t > 0$ if and only if $x = y$;

(x) $N(x, y, t) = N(y, x, t)$ for all $x, y \in X$ and $t > 0$;

(xi) $N(x, y, t) \Diamond N(y, z, s) \geq N(x, z, t + s)$ for all $x, y, z \in X$ and $s, t > 0$;

(xii)   for all $x, y \in X$, $N(x, y, .) : [0, \infty) \to [0, 1]$ is right continuous;

(xiii) $\lim_{t \to \infty} N(x, y, t) = 0$ for all $x, y \in X$.

Then (M, N) is called an intuitionistic fuzzy metric space on X. The functions $M(x, y, t)$ and $N(x, y, t)$ denote the degree of nearness and the degree of non-nearness between x and y w.r.t. t respectively.

Remark 2.5. Every fuzzy metric space $(X, M, *)$ is an intuitionistic fuzzy metric space of the form $(X, M, 1 - M, *, \Diamond)$ such that t-norm $*$ and t-conorm $\Diamond$ are associated as $x \Diamond y = 1 - ((1 - x) * (1 - y))$ for all $x, y \in X$.

Remark 2.6. In intuitionistic fuzzy metric space $(X, M, N, *, \Diamond)$, $M(x, y, .)$ is non-decreasing and $N(x, y, .)$ is non-increasing, for all $x, y \in X$.

Alaca et al.[1] introduced the following notions:

Definition 2.7. Let $(X, M, N, *, \Diamond)$ be an intuitionistic fuzzy metric space. Then

(a) a sequence $\{x_n\}$ in X is said to be Cauchy sequence if, for all $t > 0$ and $p > 0$, $\lim_{n \to \infty} M(x_{n+p}, x_n, t) = 1$ and $\lim_{n \to \infty} N(x_{n+p}, x_n, t) = 0$.

(b) a sequence $\{x_n\}$ in X is said to be convergent to a point $x \in X$ if, for all $t > 0$, $\lim_{n \to \infty} M(x_n, x, t) = 1$ and $\lim_{n \to \infty} N(x_n, x, t) = 0$.

Definition 2.8. [1] An intuitionistic fuzzy metric space $(X, M, N, *, \Diamond)$ is said to be complete if and only if every Cauchy sequence in X is convergent.

Example 2.9. Let $X = \left\{ \frac{1}{n} : n = 1, 2, 3, .. \right\} \cup \{0\}$ and let $*$ be the continuous t-norm and $\Diamond$ be the continuous t-conorm defined by $a * b = ab$ and $a \Diamond b = \min\{1, a + b\}$ respectively, for all $a, b \in [0, 1]$. For each $x, y \in X$ and $t > 0$, define (M, N) by $M(x, y, t) = \frac{t}{t + |x - y|}$ if $t > 0$, $M(x, y, 0) = 0$ and $N(x, y, t) = \frac{|x - y|}{t + |x - y|}$ if $t > 0$, $N(x, y, 0) = 1$. Clearly, $(X, M, N, *, \Diamond)$ is complete intuitionistic fuzzy metric space.

Definition 2.10. Two self mappings P and Q of an intuitionistic fuzzy metric space $(X, M, N, *, \Diamond)$ are said to be Compatible, if $\lim_{n \to \infty} M(PQx_n, QPx_n, t) = 1$ and $\lim_{n \to \infty} N(PQx_n, QPx_n, t) = 0$ for all $t > 0$ whenever $\{x_n\}$ is a sequence in

X such that $\lim_{n\to\infty} Px_n = \lim_{n\to\infty} Qx_n = z$, for some z in X.

Definition 2.11. A pair (A, S) of self maps of an intuitionistic fuzzy metric space $(X, M, N, *, \Diamond)$ is said to be Semi compatible if $\lim_{n\to\infty} ASx_n = Sx$, whenever $\{xn\}$ is a sequence such that $\lim_{n\to\infty} Ax_n = \lim_{n\to\infty} Sx_n = x$, for some x in X.

It follows that (A, S) is semi compatible and $Ay = Sy$ then $ASy = SAy$

Definition 2.12. A pair of self mapping P and Q of an intuitionistic fuzzy metric space $(X, M, N, *, \Diamond)$ is said to satisfy the (CLRg) property if there exists a sequence $\{x_n\}$ in X such that $\lim_{n\to\infty} Px_n = \lim_{n\to\infty} Qx_n = Qu$, for some $u \in X$.

Definition 2.13. Two pairs (A, S) and (B, T) of self mappings of an intuitionistic fuzzy metric space $(X, M, N, *, \Diamond)$ are said to share CLRg of S property if there exist two sequence $\{x_n\}$ and $\{y_n\}$ in X such that $\lim_{n\to\infty} Ax_n = \lim_{n\to\infty} Sx_n = \lim_{n\to\infty} By_n = \lim_{n\to\infty} Ty_n = Sz$, for some $z \in X$.

Example 2.14. Let $X = [0, \infty)$ be the usual metric space. Define $g, h : X \to X$ by $gx = x + 3$ and $gx = 4x$, for all $x \in X$. We consider the sequence $\{x_n\} = \{1 + 1/n\}$. Since, $\lim_{n\to\infty} gx_n = \lim_{n\to\infty} hx_n = 4 = h(1) \in X$. Therefore g and h satisfy the (CLRg) property.

Alaca [1] proved the following results:

Lemma 2.15. *Let $(X, M, N, *, \Diamond)$ be intuitionistic fuzzy metric space and for all $x, y \in X, t > 0$ and if for a number $k > 1$ such that $M(x, y, kt) \geq M(x, y, t)$ and $N(x, y, kt) \leq N(x, y, t)$ then $x = y$.*

Lemma 2.16. *Let $(X, M, N, *, \Diamond)$ be intuitionistic fuzzy metric space and for all $x, y \in X, t > 0$ and if for a number $k > 1$ such that $M(y_{n+2}, y_{n+1}, t) \geq M(y_{n+1}, y_n, kt), N(y_{n+2}, y_{n+1}, t) \leq N(y_{n+1}, y_n, kt)$, Then $\{y_n\}$ is a Cauchy sequence in X.*

3. Main Result

Now we prove our main result

Theorem 3.1. Let $(X, M, N, *, \Diamond)$ be a complete intuitionistic fuzzy metric space with $t * t \geq t$ and $(1 - t)\Diamond(1 - t) \leq (1 - t)$. Let A, B, S and T be selfmappings of X such that the following conditions are satisfied :

(i) $A(X) \subseteq T(X), B(X) \subseteq S(X)$,
(ii) (B, T) is semi compatible,
(iii)   There exists $k \in (0, 1)$ such that for every $x, y \in X$ and $t > 0$

$$M(Ax, By, kt) \geq \Big\{ M(Sx, Ty, t) * M(Ax, Sx, t) * M(By, Ty, t) * M(Ax, Ty, t) \Big\}$$
$$(1)$$

$$N(Ax, By, kt) \leq \Big\{ N(Sx, Ty, t)\Diamond N(Ax, Sx, t)\Diamond N(By, Ty, t)\Diamond N(Ax, Ty, t) \Big\} \quad (2)$$

If the pair (A, S) and (B, T) share the common limit in the range of S property, then A, B, S and T have a unique common fixed point.

Proof. Let x_0 be an arbitrary point in X. Since $A(X) \subset T(X)$ and $B(X) \subset S(X)$, there exist $x_1, x_2 \in X$ such that $Ax_0 = Tx_1$ and $Bx_1 = Sx_2$. Inductively, we construct the sequences $\{y_n\}$ and $\{x_n\}$ in X such that

$$y_{2n+1} = Ax_{2n} = Tx_{2n+1}, \ \ y_{2n+2} = Bx_{2n+1} = Sx_{2n+2}$$

for $n = 0, 1, 2, \dots$. Now putting in (1) and (2) $x = x_{2n}, y = x_{2n+1}$, we obtain

$$M(Ax_{2n}, Bx_{2n+1}, kt) \geq \Big\{ M(Sx_{2n}, Tx_{2n+1}, t) * M(Ax_{2n}, Sx_{2n}, t)$$
$$* M(Bx_{2n+1}, Tx_{2n+1}, t) * M(Ax_{2n}, Tx_{2n+1}, t) \Big\}$$

that is

$$M(y_{2n+1}, y_{2n+2}, kt) \geq \Big\{ M(y_{2n}, y_{2n+1}, t) * M(y_{2n+1}, y_{2n}, t)$$
$$* M(y_{2n+2}, y_{2n+1}, t) * M(y_{2n+1}, y_{2n+1}, t) \Big\}$$
$$M(y_{2n+1}, y_{2n+2}, kt) \geq \Big\{ M(y_{2n}, y_{2n+1}, t) * M(y_{2n+1}, y_{2n+2}, t) \Big\}$$
$$M(y_{2n+1}, y_{2n+2}, kt) \geq M(y_{2n}, y_{2n+1}, t)$$

and

$$N(Ax_{2n}, Bx_{2n+1}, kt) \leq \Big\{ N(Sx_{2n}, Tx_{2n+1}, t) \Diamond N(Ax_{2n}, Sx_{2n}, t)$$
$$\Diamond N(Bx_{2n+1}, Tx_{2n+1}, t) \Diamond N(Ax_{2n}, Tx_{2n+1}, t) \Big\}$$

that is

$$N(y_{2n+1}, y_{2n+2}, kt) \leq \Big\{ N(y_{2n}, y_{2n+1}, t) \Diamond N(y_{2n+1}, y_{2n}, t)$$
$$\Diamond N(y_{2n+2}, y_{2n+1}, t) \Diamond N(y_{2n+1}, y_{2n+1}, t) \Big\}$$
$$N(y_{2n+1}, y_{2n+2}, kt) \leq \Big\{ N(y_{2n}, y_{2n+1}, t) \Diamond N(y_{2n+1}, y_{2n+2}, t) \Big\}$$
$$N(y_{2n+1}, y_{2n+2}, kt) \leq N(y_{2n}, y_{2n+1}, t)$$

Similarly,

$$M(y_{2n+2}, y_{2n+3}, kt) \geq M(y_{2n+1}, y_{2n+2}, t)$$

and

$$N(y_{2n+2}, y_{2n+3}, kt) \leq N(y_{2n+1}, y_{2n+2}, t).$$

Thus, we have

$$M(y_{n+1}, y_{n+2}, kt) \geq M(y_n, y_{n+1}, t)$$

and

$$N(y_{n+1}, y_{n+2}, kt) \leq N(y_n, y_{n+1}, t) \ \ \text{for} \ \ n = 1, 2, 3, \dots.$$

Therefore, we have

$$M(y_n, y_{n+1}, t) \geq M\Big(y_n, y_{n+1}, \frac{t}{q}\Big) \geq M\Big(y_{n-1}, y_n, \frac{t}{q^2}\Big)$$
$$\geq \dots \geq M\Big(y_1, y_2, \frac{t}{q^n}\Big) \to 1,$$

and

$$N(y_n, y_{n+1}, t) \leq N\left(y_n, y_{n+1}, \frac{t}{q}\right) \leq N\left(y_{n-1}, y_n, \frac{t}{q^2}\right)$$

$$\leq ... \leq N\left(y_1, y_2, \frac{t}{q^n}\right) \to 0 \text{ when } n \to \infty.$$

For each $\varepsilon > 0$ and $t > 0$, we can choose $n_0 \in \mathrm{N}$ such that $M(y_n, y_{n+1}, t) > 1 - \varepsilon$ and $N(y_n, y_{n+1}, t) < \varepsilon$ for each $n \geq n_0$

For $m, n \in \mathrm{N}$, we suppose $m \geq n$. Then, we have

$$M(y_n, y_m, t) \geq M\left(y_n, y_{n+1}, \frac{t}{m-n}\right) * M\left(y_{n+1}, y_{n+2}, \frac{t}{m-n}\right)$$

$$* ... * M\left(y_{m-1}, y_m, \frac{t}{m-n}\right)$$

$$> \Big((1-\varepsilon) * (1-\varepsilon) * (m-n)times... * (1-\varepsilon)\Big)$$

$$\geq (1-\varepsilon),$$

and

$$N(y_n, y_m, t) \leq N\left(y_n, y_{n+1}, \frac{t}{m-n}\right) \Diamond N\left(y_{n+1}, y_{n+2}, \frac{t}{m-n}\right)$$

$$\Diamond ... \Diamond N\left(y_{m-1}, y_m, \frac{t}{m-n}\right)$$

$$< \Big((\varepsilon)\Diamond(\varepsilon)\Diamond(m-n)times... \Diamond(\varepsilon)\Big)$$

$$\leq (\varepsilon).$$

$$M(y_n, y_m, t) > (1-\varepsilon), \ N(y_n, y_m, t) < \varepsilon.$$

Hence $\{y_n\}$ is a Cauchy sequence in X. As X is complete, $\{y_n\}$ converges to some point $z \in X$. Also, its subsequences converges to this point $z \in X$, i.e. $\{Bx_{2n+1}\} \to z$, $\{Sx_{2n}\} \to z$, $\{Ax_{2n}\} \to z$, $\{Tx_{2n+1}\} \to z$.

Since the pair (A, S) and (B, T) share the common limit in the range of S property, then there exist two sequences $\{x_n\}$ and $\{y_n\}$ in X such that

$$\lim_{n\to\infty} Ax_n = \lim_{n\to\infty} Sx_n = \lim_{n\to\infty} By_n = \lim_{n\to\infty} Ty_n = Sz \text{ for some } z \in X.$$

First we prove that $Az = Sz$

By (1), putting $x = z$ and $y = y_n$, we get

$$M(Az, By_n, kt) \geq \Big\{ M(Sz, Ty_n, t) * M(Az, Sz, t) * M(By_n, Ty_n, t)$$

$$* M(Az, Ty_n, t) \Big\}$$

Taking limit $n \to \infty$, we get

$$M(Az, Sz, kt) \geq \Big\{ M(Sz, Sz, t) * M(Az, Sz, t) * M(Sz, Sz, t) * M(Az, Sz, t) \Big\}$$

$$M(Az, Sz, kt) \geq M(Az, Sz, t) \tag{3}$$

By (2), putting $x = z$ and $y = y_n$, we get

$$N(Az, By_n, kt) \leq \Big\{ N(Sz, Ty_n, t)\Diamond N(Az, Sz, t)\Diamond N(By_n, Ty_n, t)\Diamond N(Az, Ty_n, t) \Big\}$$

Taking limit $n \to \infty$, we get

$$N(Az, Sz, kt) \leq \Big\{ N(Sz, Sz, t) \Diamond N(Az, Sz, t) \Diamond N(Sz, Sz, t) \Diamond N(Az, Sz, t) \Big\}$$
$$N(Az, Sz, kt) \leq N(Az, Sz, t) \tag{4}$$

By lemma 2.15,

$$Az = Sz \tag{5}$$

Since, $A(X) \subseteq T(X)$, therefore there exist $u \in X$, such that

$$Az = Tu \tag{6}$$

Again by inequality (1), putting $x = z$ and $y = u$, we get

$$M(Az, Bu, kt) \geq \Big\{ M(Sz, Tu, t) * M(Az, Sz, t) * M(Bu, Tu, t) * M(Az, Tu, t) \Big\}$$

Using (5) and (6)

$$M(Tu, Bu, kt) \geq \Big\{ M(Tz, Tu, t) * M(Tu, Tu, t) * M(Bu, Tu, t) * M(Tu, Tu, t) \Big\}$$
$$M(Tu, Bu, kt) \geq M(Tu, Bu, t)$$

By (2), putting $x = z$ and $y = u$, we get

$$N(Az, Bu, kt) \leq \Big\{ N(Sz, Tu, t) \Diamond N(Az, Sz, t) \Diamond N(Bu, Tu, t) \Diamond N(Az, Tu, t) \Big\}$$

Using (5) and (6)

$$N(Tu, Bu, kt) \leq \Big\{ N(Tu, Tu, t) \Diamond N(Tu, Tu, t) \Diamond N(Bu, Tu, t) \Diamond N(Tu, Tu, t) \Big\}$$
$$N(Tu, Bu, kt) \leq N(Tu, Bu, t)$$

By lemma 2.15,

$$Tu = Bu \tag{7}$$

Thus from (5), (6), (7), we get

$$Az = Sz = Tu = Bu \tag{8}$$

Now we will prove that $Az = z$

By inequality (1), putting $x = z$ and $y = x_{2n+1}$,

$$M(Az, Bx_{2n+1}, kt) \geq \Big\{ M(Sz, Tx_{2n+1}, t) * M(Az, Sz, t) * M(Bx_{2n+1}, Tx_{2n+1}, t)$$
$$* M(Az, Tx_{2n+1}, t) \Big\}$$

Taking limit $n \to \infty$, we get

$$M(Az, z, kt) \geq \Big\{ M(Sz, z, t) * M(Az, Sz, t) * M(z, z, t) * M(Az, z, t) \Big\}$$
$$M(Az, z, kt) \geq M(Az, z, t)$$

By (2), putting $x = z$ and $y = x_{2n+1}$,

$$N(Az, Bx_{2n+1}, kt) \leq \Big\{ N(Sz, Tx_{2n+1}, t) \Diamond N(Az, Sz, t) \Diamond N(Bx_{2n+1}, Tx_{2n+1}, t)$$
$$\Diamond N(Az, Tx_{2n+1}, t) \Big\}$$

Taking limit $n \to \infty$, we get

$$N(Az, z, kt) \leq \left\{ N(Sz, z, t) \Diamond N(Az, Sz, t) \Diamond N(z, z, t) \Diamond N(Az, z, t) \right\}$$
$$N(Az, z, kt) \leq N(Az, z, t)$$

Using lemma 2.1, $Az = z$

Thus from (8), we get $z = Tu = Bu$

Now Semicompatibility of (B, T) gives $BTy_{2n+1} \to Tz$, i. e. $Bz = Tz$

Now putting $x = z$ and $y = z$ in inequality (1), we get

$$M(Az, Bz, kt) \geq \left\{ M(Sz, Tz, t) * M(Az, Sz, t) * M(Bz, Tz, t) * M(Az, Tz, t) \right\}$$
$$M(Az, Bz, kt) \geq M(Az, Bz, t)$$

By (2), we get

$$N(Az, Bz, kt) \leq \left\{ N(Sz, Tz, t) \Diamond N(Az, Sz, t) \Diamond N(Bz, Tz, t) \Diamond N(Az, Tz, t) \right\}$$
$$N(Az, Bz, kt) \leq N(Az, Bz, t)$$

By lemma 2.15, $Az = Bz$ and hence $Az = Bz = z$

Combining all results, we get $z = Az = Bz = Sz = Tz$.

From this we conclude that z is a common fixed point of A, B, S and T.

Uniqueness: Let z_1 be another common fixed point of A, B, S and T. Then

$$z_1 = Az_1 = Bz_1 = Sz_1 = Tz_1$$

and

$$z = Az = Bz = Sz = Tz$$

then by inequality (1), putting $x = z$ and $y = z_1$, we get

$$M(Az, Bz_1, kt) \geq \left\{ M(Sz, Tz_1, t) * M(Az, Sz, t) * M(Bz_1, Tz_1, t) * M(Az, Tz_1, t) \right\}$$
$$M(z, z_1, kt) \geq M(z, z_1, t)$$

By (2), we get

$$N(Az, Bz_1, kt) \leq \left\{ N(Sz, Tz_1, t) \Diamond N(Az, Sz, t) \Diamond N(Bz_1, Tz_1, t) \Diamond N(Az, Tz_1, t) \right\}$$
$$N(z, z_1, kt) \leq N(z, z_1, t)$$

By lemma 2.15, we get $z = z_1$.

Thus z is the unique common fixed point of A, B, S and T.

If we increase the number of self maps from four to six then we have the following.

Corollary 3.2. Let $(X, M, N, *, \Diamond)$ be a complete intuitionistic fuzzy metric space with $t * t \geq t$ and $(1 - t) \Diamond (1 - t) \leq (1 - t)$. Let A, B, S, T, I and J be self mappings of X such that the following conditions are satisfied :

(i)　　$AB(X) \subseteq J(X)$, $ST(X) \subseteq I(X)$,
(ii)　　(ST, J) is semi compatible,

(iii)   There exists $k \in (0,1)$ such that for every $x, y \in X$, and $t > 0$

$$M(ABx, STy, kt) \geq \Big\{ M(Ix, Jy, t) * M(ABx, Ix, t) * M(STy, Jy, t)$$

$$* M(ABx, Jy, t) \Big\} \tag{1}$$

$$N(ABx, STy, kt) \leq \Big\{ N(Ix, Jy, t) \Diamond N(ABx, Ix, t) \Diamond N(STy, Jy, t) \Diamond N(ABx, Jy, t) \Big\} \tag{2}$$

If the pair (AB, I) and (ST, J) share the common limit in the range of I property, then AB, ST, I and J have a unique common fixed point. Furthermore, if the pairs $(A, B), (A, I), (B, I), (S, T), (S, J)$ and (T, J) are commuting mapping then A, B, S, T, I and J have a unique common fixed point.

Proof. From theorem 3.1, z is the unique common fixed point of AB, ST, I and J.

Finally, we need to show that z is also a common fixed point of A, B, S, T, I and J. For this, let z be the unique common fixed point of both the pairs (AB, I) and (ST, J). Then, by using commutativity of the pair $(A, B), (A, I)$ and (B, I), we obtain

$$Az = A(ABz) = A(BAz) = AB(Az), \; Az = A(Iz) = I(Az), \tag{3}$$

$$Bz = B(ABz) = B\big(A(Bz)\big) = BA(Bz) = AB(Bz), \; Bz = B(Iz) = I(Bz),$$

which shows that Az and Bz are common fixed point of (AB, I), yielding thereby

$$Az = z = Bz = Iz = ABz \tag{4}$$

In the view of uniqueness of the common fixed point of the pair (AB, I). Similarly, using the commutativity of $(S, T), (S, J), (T, J)$, it can be shown that

$$Sz = Tz = Jz = STz = z. \tag{5}$$

Now, we need to show that $Az = Sz(Bz = Tz)$ also remains a common fixed point of both the pairs (AB, I) and (ST, J). For this, put $x = z$ and $y = z$ in (1) and using (4) and (5), we get

$$M(ABz, STz, kt) \geq \Big\{ M(Iz, Jz, t) * M(ABz, Iz, t) * M(STz, Jz, t) * M(ABz, Jz, t) \Big\}$$
$$M(Az, Sz, kt) \geq M(Az, Sz, t)$$

and by (2)

$$N(ABz, STz, kt) \leq \Big\{ N(Iz, Jz, t) \Diamond N(ABz, Iz, t) \Diamond N(STz, Jz, t) \Diamond N(ABz, Jz, t) \Big\}$$
$$N(Az, Sz, kt) \leq N(Az, Sz, t)$$

By lemma 2.15, we get

$$Az = Sz.$$

Similarly, it can be shown that $Bz = Tz$. Thus, z is the unique common fixed point of A, B, S, T, I, and J.

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