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TRANSMISSION DYNAMICS OF CHOLERA EPIDEMIC MODEL WITH LATENT AND HYGIENE COMPLIANT CLASS

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Abstract. Cholera is a dreadful disease caused by the intake of food or water infested with bacterium *vibrio cholerae*. This disease remains burdensome for public health practitioners globally to overcome unless drastic measures are taken to stop the disease spread. In this paper, a non - linear, autonomous, system of differential equations incorporating important features of cholera dynamics such as, hygiene, treatment, sanitation, e.t.c. is presented. The model has a unique positive solution, it exists and its behavior relies on the initial data. Also, it is realistic in an epidemic sense. A basic threshold known as the reproduction number is obtained and analyzed. the local and global analysis is investigated at the cholera free and endemic points obtained when the model system is time - independent. However if $R_0 > 1$ cholera invades the population and if $R_0 \leq 1$ cholera as a disease vanishes out of the host population. Numerical simulation is performed on the cholera model system to validate the theoretical results.

1. Introduction

Cholera is a disease which spreads contagiously in an indirect way through reservoir to human and human to human contact. It is a gastrointestinal infection accompanied by clinical manifestations of severe vomiting and diarrhea, loss of body fluid and electrolytes, and death. According to WHO fact sheet (2018) [[1], [2]], between 1.3 to 4 million cases of cholera occurred and between $21,000$ to $143,000$ people died of cholera in the world. Due to the challenge cholera poses to the public health sector, it still remains the leading cause of death rate, most especially in the third world countries [[13], 14], [16], [17]]. Highly unsafe areas which cholera thrives are, slums, refugee camps, people residing on dung hills and riversides. If proper medical attention is made available to these areas such that water sources are being treated, then, the disease would be eradicated drastically [[15], [18], [19]].In order to gain insight and understanding of the epidemiological complexities involved in the transmission dynamics of cholera, several articles have used mathematical analysis and techniques to model the cholera epidemic [[3], [4], [5], [12], [20], [23], [22], [24]]. R_0 is another important threshold in studying the stabilities of epidemic models. It

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is the average number of secondary cases of cholera disease arising when a cholera infected individual is introduced into the population of individuals susceptible to cholera disease during his or her life time of infections [[9], [10], [11]]. [6] proposed a model describing the concentration of *V cholerae* in water and incorporating it into SIR model. Also, [21] in her masters thesis worked on cholera models with focus on south Sudan where he investigated spatially explicit setting of local human communities. Bifurcation analysis is used to investigate qualitatively the existence of endemic equilibrium, and it was shown that the model in this work exhibits backward bifurcation [[7], [8]]. In this work, a cholera model, including a latent and hygiene compliant class is studied and analyzed. The paper is organized as follows; section(2) involves the model formulation, analysis of positivity and boundedness of the model solutions and obtaining the basic reproduction number. Section (3) discusses the local and global analysis is investigated at the cholera free equilibrium point. Section (4) presents the analysis of the local asymptotic stability of the cholera endemic equilibrium point by employing the center manifold theory. In section (5), numerical simulations is carried out and analysed to validate the results. Also, conclusions and recommendations were made.

2. Mathematical Model Formulation

In this section, a six dimensional autonomous first order differential equations is considered. The host population is subdivided into compartmental classes of state variables of susceptible $S(t)$, individuals who are prone to contacting cholera through human or environment. Exposed or Latent $E(t)$, individuals who are infected with cholera but not yet showing clinical manifestation of the disease. Infected $I(t)$, are individuals who have shown symptoms of cholera and are able to transmit the disease. Recovered $R(t)$ are individuals who recovered from cholera through treatment. Hygiene compliant $H(t)$ are individuals who undergo regular hygenic practices. While, $C(t)$ denotes the concentration of vibrios in water reservoir. Thus, the time evolution cholera epidemic model is given by,

$$\begin{aligned}
\dot{S} &= A - \beta_1 S\frac{C}{j+C} - \beta_2 SI - \mu S + \omega R + (1-\rho)R \\
\dot{E} &= \beta_1 S\frac{C}{j+C} + \beta_2 SI - (k+\mu)E \\
\dot{I} &= kE - (\alpha + m + \mu)I \\
\dot{R} &= \alpha I - \mu R - \omega R \\
\dot{H} &= \rho R - \mu H \\
\dot{C} &= \xi I - \delta C - \eta C
\end{aligned} \tag{1}$$

Subject to initial conditions $S(0) = S_0$, $E(0) = E_0$, $I(0) = I_0$, $R(0) = R_0$, $H(0) = H_0$, $C(0) = C_0$.

In (1), A is the per capita rate of recruitment of individuals into the susceptible host population, while β_1 and β_2 are the rates at which vibrios are being ingested through contaminated water and human to human contagious interactions respectively. μ, is the natural death rate of humans, j is denoted as the concentration

of vibrios in contaminated water, while ξ represents the contribution of human to *V.cholerae*, δ is the decay or death rate of vibrios. Also, η represents the rate at which the contaminated water is disinfected with chemicals e.g chlorine. ω represents the loss of immunity of an individual after recovery by becoming susceptible again. k is the progression rate of new clinical manifestations and α is the recovery rate of cholera infected individuals. m is the cholera induced death rate, while ρ represents individuals compliant to hygiene and $(1-\rho)$ also, represents the fraction of individuals that are not compliant to hygiene thus being susceptible to cholera disease. The following assumptions where made in formulating (1)

a. Disinfecting water sources leads to death of *V.cholerae*
b. Per capita birth rate and natural death rate occurs at different states
c. There is no permanent recovery, recovered individuals becomes susceptible to the disease
d. The total host population is not constant
e. fraction of individuals not compliant to hygiene are susceptible to cholera.
f. The population is mixed homogenously

Considering the assumptions listed above coupled with the parameters and state variables involved in the model formulation, (1) is thus established.

2.1. **Positivity of Solution.**

Theorem 1. *Given that* $S(0) \geq 0, E(0) \geq 0, I(0) \geq 0, R(0) \geq 0, H(0) \geq 0, C(0) \geq 0$*, the trajectories of* (1) *is positively invariant.*

Proof. Let $\Omega = Sup\{t > 0 | S > 0, E > 0, I > 0, R > 0, H > 0, C > 0\}$ □

Considering the first state equation,

$$\dot{S} = A - \beta_1 s \frac{C}{j+C} - \beta_2 SI - \mu S + \omega R + (1-\rho)R \tag{2}$$

such that,

$$\dot{S} \geq A - (\beta_1 \frac{C}{j+C} + \beta_2 + \mu)S \tag{3}$$

so that

$$\dot{S} + (\beta_1 \frac{C}{j+C} + \beta_2 + \mu)S = A \tag{4}$$

solving (4), we obtain

$$S(t) = \frac{A}{\beta_1 \frac{C}{j+C} + \beta_2 I + \mu} + Ce^{-(\beta_1 \frac{C}{j+C} + \beta_2 I + \mu)t} \geq 0 \tag{5}$$

Following the same procedure for the remaining five state equations, it can be shown that,

$$E(t) \geq 0, I(t) \geq 0, R(t) \geq 0, H(t) \geq 0, C(t) \geq 0$$

2.2. **Boundedness of** (1)**.**

Theorem 2. *The solutions of* $S(t), E(t), I(t), R(t), H(t), C(t)$ *of* (1) *are bounded*

Proof. Adding up the systems of equations in (1) yields,

$$\dot{N} = A - (S + E + I + R + H + C)\mu - mI - \xi I - \delta C - \eta C \tag{6}$$

at the absence of infections and mortality, we obtain,

$$\dot{N} = A - \mu N \tag{7}$$

integrating both sides of (7) yields,

$$\int \frac{dN}{dt} = \int A - \mu N \tag{8}$$

such that

$$-\frac{1}{\mu} ln(A - \mu N) \leq t + c \tag{9}$$

so that,

$$N = \frac{A}{\mu} + Ce^{-\mu t} \tag{10}$$

where C is a constant. Therefore,

$$\lim_{x \to \infty} (\frac{A}{\mu} + \frac{C}{e^{\mu t}}) = \frac{A}{\mu} \tag{11}$$

this shows that $\frac{A}{\mu}$ is the upper bound while 0 is the lower bound of (1) therefore,

$$\xi = \big[(S, E, I, R, H, C) \in \Re^{+6} | S + E + I + R + H + C \leq \frac{A}{\mu}\big] \tag{12}$$

is positively invariant and the model system (1) is well posed mathematically and realistic in an epidemic sense. □

2.3. **Basic Reproduction Number.** The basic reproduction number is a dimensionless rate [10, 11] defined as the average number of secondary cholera cases generated when a cholera infected individual is introduced into a susceptible host population during his or her life time of infections. However, if $R_0 < 1$, then cholera dies out in the host population. But, if $R_0 > 1$ cholera as a disease invades the population which allows for drastic control measures to be put in place by public health administrators. The R_0 is computed using the next generation matrix operator approach. see[5, 10, 11].

Theorem 3. *The basic reproduction number of* (1) *is given by,*

$$R_0 = \frac{1}{2} \frac{k\beta_2 A + \sqrt{kA\beta_2(k\beta_2 A + 4(k+\mu)(\alpha + m + \mu)\mu)}}{\mu(\alpha + m + \mu)(k + \mu)} \tag{13}$$

Proof. Let $FV^{-1} = \big[\frac{\partial F_i(E^0)}{\partial x_j}\big]\big[\frac{\partial V_1(E^0)}{\partial x_j}\big]$ Where F_i is the clinical manifestations of disease symptoms in the i^{th} compartment
V_i^+ is the rate of transfer of individuals into i by all other means. While V_i^- is the rate of transfer of individuals out of compartment associated with i, such that, $V = V_i^- - V_i^+$; thus, □

$$F = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \beta_2 \frac{A}{\mu} & 0 & 0 & 0 \\ 0 & k & 0 & 0 & 0 & 0 \\ 0 & 0 & \alpha & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \xi & 0 & 0 & 0 \end{pmatrix}$$

and,

$$V = \begin{pmatrix} \mu & 0 & \beta_2 \frac{A}{\mu} & \omega + (1-\rho) & 0 & 0 \\ 0 & k+\mu & 0 & 0 & 0 & 0 \\ 0 & -k & \alpha + m + \mu & 0 & 0 & 0 \\ 0 & 0 & -\alpha & \mu + \omega & 0 & 0 \\ 0 & 0 & 0 & -\rho & \mu & 0 \\ 0 & 0 & -\xi & 0 & 0 & \delta \end{pmatrix}$$

$R_0(FV^{-1})$ *as the spectral radius is the dominant eigen value given by,*

$$R_0 = \frac{1}{2} \frac{k\beta_2 A + \sqrt{kA\beta_2(k\beta_2 A + 4(k+\mu)(\alpha+m+\mu)\mu)}}{\mu(\alpha+m+\mu)(k+\mu)} \tag{14}$$

3. Local and Global Stability Analysis of Cholera Free Equilibrium Point

3.1. **Local Analysis.** In order to analyze the local asymptotic stability of the cholera free equilibrium solutions, we make the model system (1) static by obtaining the time independent solutions of the model, at when $I = E = C = 0$ i.e No cholera disease in the system. Then,

$$E^0 = (S, E, I, R, H, C) = (\frac{A}{\mu}, 0, 0, 0, 0, 0) \tag{15}$$

Theorem 4. *The cholera free equilibrium* E^0 *of* (1) *is locally asymptotically stable if* $R_0 < 1$ *and unstable, if* $R_0 > 1$

Proof. The Jacobian matrix at cholera free equilibrium is given by,

$$J(E^0) = \begin{pmatrix} -\mu & 0 & \beta_2 \frac{A}{\mu} & \omega + (1-\rho) & 0 & 0 \\ 0 & -(k+\mu) & 0 & 0 & 0 & 0 \\ 0 & k & -(\alpha+m+\mu) & 0 & 0 & 0 \\ 0 & 0 & \alpha & -(\mu+\omega) & 0 & 0 \\ 0 & 0 & 0 & \rho & -\mu & 0 \\ 0 & 0 & \xi & 0 & 0 & -(\delta+\eta) \end{pmatrix} \tag{16}$$

The characteristics polynomial of (16) is given as

$$(\mu+\lambda)^2(\mu+\omega+\lambda)(\eta+\delta+\lambda)(m+\mu+\alpha+\lambda)(k+\mu+\lambda)(1-R_0) \tag{17}$$

where,

$$\lambda^6 + d_1\lambda^5 + d_2\lambda^4 + d_3\lambda^3 + d_4\lambda^2 + d_5\lambda + d_6 = 0 \tag{18}$$

employing the Routh - Hurwitz theorem on stability [22], the cholera free equilibrium point associated with (16) is stable if and only if all the determinants of all

the Hurwitz matrices are positive. That is, $Det(Z_j) > 0, \quad j = 1, 2, ...k$ and the trace $-(5\mu + k + \alpha + m + \omega + \delta + \eta) < 0$. The Hurwitz matrix are given by

$$Z_1 = (d_1) > 0 \quad Z_2 = \begin{pmatrix} d_1 & 1 \\ d_3 & d_2 \end{pmatrix} \tag{19}$$

$$\begin{pmatrix} d_1 & 1 & 0 \\ d_3 & d_2 & d_1 \\ d_5 & d_4 & d_3 \end{pmatrix} > 0 \quad \begin{pmatrix} d_1 & 1 & 0 & 0 \\ d_3 & d_2 & d_1 & 1 \\ d_5 & d_4 & d_3 & d_2 \\ 0 & d_6 & d_5 & d_4 \end{pmatrix} > 0 \tag{20}$$

$$\begin{pmatrix} d_1 & 1 & 0 & 0 & 0 & 0 \\ d_3 & d_2 & d_1 & 0 & 0 & \\ d_5 & d_4 & d_3 & d_2 & d_1 & \\ 0 & d_6 & d_5 & d_4 & d_3 & \\ 0 & 0 & 0 & d_6 & d_5 & \end{pmatrix} > 0 \quad \begin{pmatrix} d_1 & 1 & 0 & 0 & 0 & 0 \\ d_3 & d_2 & d_1 & 1 & 0 & 0 \\ d_5 & d_4 & d_3 & d_2 & 0 & 0 \\ 0 & d_6 & d_5 & d_4 & d_3 & 0 \\ 0 & 0 & 0 & d_6 & d_5 & 0 \\ 0 & 0 & 0 & 0 & 0 & d_6 \end{pmatrix} > 0 \tag{21}$$

since all the characteristics polynomials requires that all $d_i > 0$ for $i = 1, 2, ..6$, and all the eigenvalues are negative shows that $R_0 < 1$. Hence, the cholera free equilibrium is locally asymptotically stable. □

3.2. **Global Analysis.**

Theorem 5. *According to Castillo - Chavez et al* [11, 23]*, the following two conditions must be satisfied. given*

$$\frac{dP}{dt} = Q(P, Q)\frac{dQ}{dt} = R(P, Q) \tag{22}$$

such that $R(P, 0) = 0$

Proof. Let $P = (S, R, H)$ and $Q = (E, I, C)$. Where $P \in \Re^3$ represents the number of individuals who are not infected with cholera, and $Q \in \Re^3$ represents the number of individuals who are infected with cholera, such that the cholera free equilibrium can be written as

$$E^0 = (P^*, Q^*) = (P^*, 0)$$

i if for $\frac{dP}{dt} = Q(P, 0)$, P^* is locally asymptotically stable.
ii for $R(P, Q) = GZ - \hat{R}(P, Q)$, $\hat{R}(P, Q) \geq 0$ for $(P, Q) \in \xi$

$G = D_z R(P^*, 0)$ is a Metzler matrix, since all the diagonal elements are non negative and ξ is the domain where the model is realistic in an epidemic sense,

$$Q(P, 0) = \begin{pmatrix} A + \omega R + (1 - \rho)R - \mu S \\ -(\mu + \omega)R \\ \rho R - \mu H \end{pmatrix}, \; R(P, Q) = \begin{pmatrix} \beta_1 S\frac{C}{j+C} + \beta SI - (k + \mu)E \\ kE - (\alpha + m + \mu)I \\ \xi I - \delta C - \eta C \end{pmatrix} \tag{23}$$

Clearly when $E = I = C = 0$, then, $R(P, 0) = 0$. Also,

$$G = \begin{pmatrix} -(k + \mu) & \beta_2 S & \beta_1 S\frac{C}{(j+C)^2} \\ k & -(\alpha + m + \mu) & 0 \\ 0 & \xi & -(\delta + \eta) \end{pmatrix} \tag{24}$$

and,

$$\hat{R}(P,Q) = \begin{pmatrix} \beta_1(S^* - S)\frac{C}{j+C} + \beta(S^* - S)I - (k+\mu)E \\ kE - (\alpha + m + \mu)I \\ \xi I - (\delta + \eta)C \end{pmatrix} \tag{25}$$

since $S^* > S$, it is clear that $\hat{R}(P,Q) \leq 0$, which proves, that the cholera free equilibrium is globally asymptotically stable. □

4. Local Stability Analysis of Cholera Endemic Equilibrium

4.1. **Local Analysis.** In order to analyze the local asymptotic stability of the cholera endemic equilibrium, we make the model system (1) static by obtaining the time independent solutions of the model, at when $I = E = C \neq 0$ i.e cholera disease is present in the system. Then,

$$S^{**} = \frac{R^{**}((\omega - \rho + 1) + A)(j + C^{**})}{C^{**}(I^{**}\beta_2 + \mu + \beta_1) + j(I^{**}\beta_2 + \mu)}$$

$$E^{**} = \frac{S^{**}(I^{**}\beta_2 + I^{**}C^{**}\beta_2 + C^{**}\beta_1)}{(j + C^{**})(k + \mu)}$$

$$I^{**} = \frac{kE^{**}}{\alpha + m + \mu}$$

$$R^{**} = \frac{I^{**}\alpha}{\mu + \omega}$$

$$H^{**} = \frac{\rho R^{**}}{\mu}$$

$$C^{**} = \frac{\xi I^{**}}{\delta + \eta} \tag{26}$$

The asymptotic stability of the cholera endemic equilibrium analyzed at each eigenvalues gotten from the Jacobian matrix of (1) at each equilibrium points (26). The center manifold theory [11, 23] is applied and the following representations where made by changing the variables of (1), such that;

$$S = x_1,\ E = x_2,\ I = x_3,\ R = x_4,\ H = x_5,\ C = x_6 \tag{27}$$

the cholera epidemic model is written as a vector as $\frac{dx}{dt} = U = (u_1, u_2, ..., u_n)$, where,

$$f_1 = \frac{dx_1}{dt} = A - \beta_1 \frac{x_1 x_6}{j + x_6} - \beta_2 x_1 x_3 - \mu x_1 + \omega x_4 + (1 - \rho)x_4$$

$$f_2 = \frac{dx_2}{dt} = \beta_1 \frac{x_1 x_6}{j + x_6} - \beta_2 x_1 x_3 - (k + \mu)x_2$$

$$f_3 = \frac{dx_3}{dt} = kx_2 - (\alpha + m + \mu)x_3$$

$$f_4 = \frac{dx_4}{dt} = \alpha x_3 - (\mu + \omega)x_3$$

$$f_5 = \frac{dx_5}{dt} = \rho x_4 - \mu x_5$$

$$f_6 = \frac{dx_6}{dt} = \xi x_3 - (\delta + \eta)x_6 \tag{28}$$

choosing $\beta_2 = \beta_2^*$ as a bifurcation parameter and $R_0 = 1$ as the bifurcation point, then, the center manifold theory can be applied to study the dynamics of (28) near $\beta_2 = \beta_2^*$. The Jacobian of (1) has a right eigen vector associated with the zero eigen values given by $v = [v_1, v_2, v_3, v_4, v_5, v_6]$ and where,

$$v = \Big[\frac{k\beta_2 A}{\mu(\alpha + m + \mu)}v_2,\ 0,\ \frac{k}{\alpha + m + \mu}v_2 \geq 0,\ \frac{\alpha}{(\mu + \omega)}v_3,\ \frac{\rho}{\mu}v_4,\ \frac{\xi}{\delta + \eta}v_3\Big] \tag{29}$$

and $w = [w_1, w_2, w_3, w_4, w_5, w_6]^T$ is the left eigen vector of the Jacobian of cholera free equilibrium associated with the non zero eigenvalue at $\beta_2 = \beta_2^*$, such that,

$$w = \Big[0, w_2 > 0,\ \frac{(\alpha + m + \mu)}{\alpha}w_4,\ w_4 = \frac{k + \mu}{k}w_5, w_5 > w_4,\ 0\Big] \tag{30}$$

Theorem 6. [8] *Consider the general system of differential equations with a parameter υ such that*

$$\frac{dx}{dt} = g(x, \upsilon), g : \Re^n \times \Re^+ \to \Re^+$$

and $g \in C^2(\Re^n \times \Re)$, and 0 is an equilibrium point of the system for all μ and

- H1 *$B = D_x g(0,0) = \frac{\partial g_i(0,0)}{\partial x_j}$ is the linearization matrix of the system around the equilibrium 0 with θ evaluated at 0*
- H2 *Zero is a simple eigenvalue of B and all other eigenvalues of B have negative real parts*
- H3 *Matrix B has a right eigenvectors y and left eigenvectors z corresponding to zero eigenvalues*

if g_k be the kth component of g then,

- i $a = \sum_{k,i,j=1}^{n} w_k\ v_i\ v_j \frac{\partial^2 g_k(0,0)}{\partial x_i \partial x_j}$
- ii $b = \sum_{k,i,j=1}^{n} w_k\ v_i\ v_j \frac{\partial^2 g_k(0,0)}{\partial x_i \partial \beta_2}$

the local dynamics of the system (1) at the equilibrium solutions is determined by the signs of a and b especially if $a > 0$ and $b > 0$ then a subcritical (or backward) bifurcation occurs at $\mu = 0$.

- i *$a > 0, b > 0$ when $\mu < 0$ with $|\mu| << 1; 0$ is locally asymptotically stable and there exists a positive unstable equilibrium; when $0 < \mu << 1; 0$ is unstable and there exist a negative and locally asymptotically stable equilibrium.*
- ii *$a < 0, b < 0$ when $\mu < 0$ with $|\mu| << 1; 0$ is unstable when $0 < \mu << 1; 0$ is unstable and there exist a positive and unstable equilibrium.*
- iii *$a > 0, b < 0$ when $\mu < 0$ with $|\mu| << 1; 0$ is stable when a locally asymptotically stable negative equilibrium when $0 < \mu << 1; 0$ is unstable and there exist a positive and unstable equilibrium occurs.*
- iv *$a < 0, b > 0$ when $\mu < 0$ if μ changes from negative to positive and 0 changes from stable to unstable there exist a positive and unstable equilibrium. Also, a negative unstable equilibrium becomes a positive and locally asymptotically stable if $a > 0$ and $b > 0$ then backward bifurcation occurs at $\mu = 0$*

4.1.1. *Computations of Bifurcation Signs a and b.* The signs and values of a and b is determined using the center manifold theorem. since $v_1 = v_6 = 0$, for $k = 1, 6$ then, the values for $k = 2, 3, 4, 5$ is obtained. After some simplifications and arrangements, the associated non zero second order partial derivatives at cholera free points are given by;

$$a = v_2(\beta_1 w_1 w_6 + \beta_2 w_1 w_2 + w_6 \beta_1) > 0 \tag{31}$$

and

$$b = v_2(\beta_1 w_1 + \beta_2 w_3) + v_6(\beta_1 w_2 + \xi w_3) + v_3(k w_2) > 0 \tag{32}$$

since $a > 0$ and $b > 0$, going by the statement of theorem (6), [[11], [8]] it guarantees that the local asymptotic stability of the unique cholera endemic equilibrium is established, if $R_0 > 1$. But close to one, the cholera model (1) undergoes backward bifurcation, otherwise it will undergo forward bifurcation.

4.2. **Numerical Simulations. Table 1: Variables in Model** (1) **and their Meanings**

Variable	Descriptions	Values	Source
S(0)	Susceptible individuals	50	Assumed
E(0)	Exposed or Latent individuals	30	Assumed
I(0)	Infected individuals	10	Assumed
R(0)	Recovered individuals	20	Assumed
H(0)	Hygiene Compliant Individuals	15	Assumed
C(0)	Concentration of Vibrios	10	Assumed

Table 1: Parameters in Model (1) **and their Meanings**

Variable	Descriptions	Values	Source
A	Per capita recruitment rate	10 persons/day$^-$1	[24]
μ	Natural death rate	0.15 persons/day$^-$1	Estimated
ω	Loss of immunity	0.22 persons/day$^-$1	[6]
ρ	Rate of compliance to hygiene	0.31 persons/day$^-$1	Estimated
$(1-\rho)$	Fraction of individuals not compliant to hygiene	0.41 persons/day	Estimated
k	Progression rate	0.11 persons/day$^-$1	[4,5]
α	Recovery rate	0.0125 persons/day$^-$1	[6, 4]
m	Cholera induced death rate	0.03 persons/day$^-$1	[6, 20]
ξ	Rate of human contribution to V.cholerae	0.512 persons/day$^-$1	[6, 4]
δ	Death rate of vibrios	0.0012 persons/day$^-$1	Estimated
η	Rate of treatment of water reservoir	0.014/day$^-$1	[6]

Figure (1) describes the disease profile of fraction of individuals that are not compliant to hygiene varied at different parameter values. The sharp rise in the profile depicts that more susceptible individuals will be exposed or infected without proper hygiene thereby leading to hospitalization or death.

FIGURE 1. Graph of $S(t)$ against time t varying $(1-\rho)$

Figure (2) shows the gradual rise of cholera induced death rate when there are no proper control measures to curtail the spread of cholera disease in the susceptible host population which portends danger to the host population

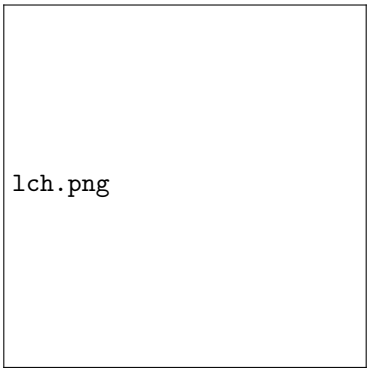

FIGURE 2. Graph of $I(t)$ against time t varying m

Figure (3) describes the decline of the rate at which individuals exposed or infected with cholera recovers from the disease when proper treatment strategies are put in place, which reduces mortality in the host population.

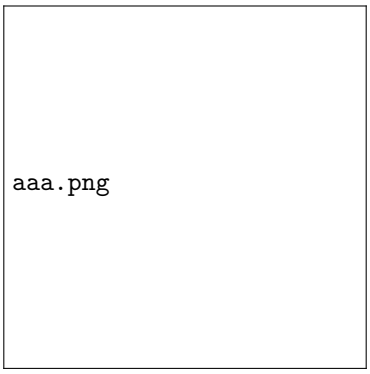

FIGURE 3. Graph of $I(t)$ against time t varying α

Figure (4) depicts the gradual decline of pathogens in infested water reservoir when proper disinfectants e.g. chlorine are applied to treat the water thereby leading to death of vibrios.

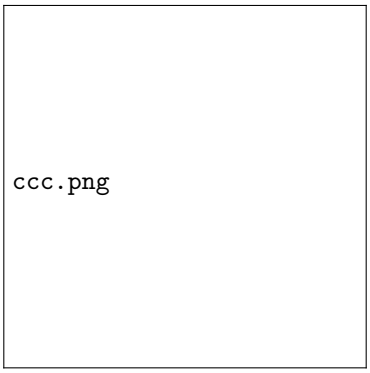

FIGURE 4. Graph of $C(t)$ against time t varying η

Figure (5) shows the class of individuals who are hygiene compliant. The sharp rise and decline in the profile shows that cholera disease infections will be reduced if a consistent hygenic practice is followed.

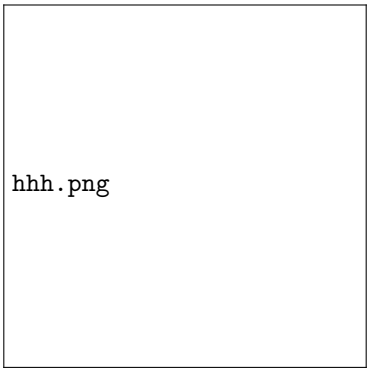

FIGURE 5. Graph of $H(t)$ against time t

Figure(6) describes the interaction between the susceptible and infected class. the disease profile is stable and non periodic.

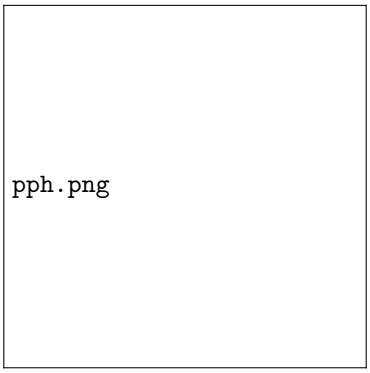

FIGURE 6. Phase diagram of $S(t)$ against $I(t)$

4.3. **Conclusion and Recommendations.** We have presented a mathematical model of cholera by incorporating latency and hygiene compliant class with all other important features describing the dynamics of cholera transmission. The model is analyzed in a region where it is well posed and epidemiologically realistic. We showed that when $R_0 < 1$, the cholera free equilibrium solution is locally and globally asymptotically stable, thereby leading to an extinction of the disease from the system. Also, if $R_0 > 1$, a unique unstable endemic equilibrium is established, leading to the persistence of the disease. The center manifold theory is used to analyze the endemic equilibrium and the model is shown to exhibit backward bifurcation. However, the impact of climatic factors, like, rainfall, temperature, flood, age structure, prevention and intervention strategies as well as bringing about optimal control measures can be used in extending this work further to forestall lasting solutions to curtail cholera spread.

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