Electronic Journal of Mathematical Analysis and Applications
Vol. 8(1) Jan. 2020, pp. 8-16.
ISSN: 2090-729X(online)
http://math-frac.org/Journals/EJMAA/

COEFFICIENT INEQUALITIES FOR A CLASS OF ANALYTIC FUNCTIONS ASSOCIATED WITH CONIC DOMAINS

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Abstract. Main purpose of this paper is to define and study new class of analytic functions in conic type regions by using concept of Janowski functions. We investigate several interesting properties of newly defined class. Comparison of new results with those that were obtained in earlier investigation are given as Corollaries.

1. Introduction

Let $\mathcal{A}$ denote the class of functions f analytic in the open unit disc

$$\mathbb{U} = \{z : z \in \mathbb{C} \quad \text{and} \quad |z| < 1\}$$

and satisfying the normalization condition $f(0) = f'(0) - 1 = 0$. Thus, the functions in $\mathcal{A}$ are represented by the Taylor-Maclaurin series expansion given by

$$f(z) = z + \sum_{n=2}^{\infty} a_n z^n \qquad (z \in \mathbb{U}). \tag{1}$$

Let $\mathcal{S}$ be the subset of $\mathcal{A}$ consisting of the functions that are univalent in $\mathbb{U}$. The classes $\mathcal{S}^*(\alpha)$ and $\mathcal{C}(\alpha)$ are the well-known classes of starlike and convex univalent functions of order α $(0 \leq \alpha < 1)$, respectively (for details, see [2]).

For two functions f and g, analytic in $\mathbb{U}$, f is said to be subordinate to g in $\mathbb{U}$ and denoted by

$$f(z) \prec g(z) \qquad (z \in \mathbb{U}),$$

if there exists a function $w \in \mathcal{J}_0$ where

$$\mathcal{J}_0 = \{w \in \mathcal{A} : w(0) = 0 \quad \text{and} \quad |w(z)| < 1 \qquad (z \in \mathbb{U})\},$$

such that

$$f(z) = g(w(z)) \qquad (z \in \mathbb{U}).$$

Furthermore, if the function g is univalent in $\mathbb{U}$, then it follows that

$$f(z) \prec g(z) \qquad (z \in \mathbb{U}) \Leftrightarrow f(0) = g(0), \quad f(\mathbb{U}) \subset g(\mathbb{U}).$$

2010 *Mathematics Subject Classification.* 30C45, 30C55.
Key words and phrases. Analytic functions, Subordination, Janowski functions, Conic domain.
Submitted May 10, 2018.

A function h is said to be in the class $\mathcal{P}[A,B]$ if it is analytic in $\mathbb{U}$ with $h(0)=1$ and

$$h(z) \prec \frac{1+Az}{1+Bz} \qquad (-1 \leq B < A \leq 1).$$

Geometrically, a function $h \in \mathcal{P}[A,B]$ maps the open unit disc $\mathbb{U}$ onto the disc $\Omega[A,B]$ defined by

$$\Omega[A,B] = \left\{ w : \left| w - \frac{1-AB}{1-B^2} \right| < \frac{A-B}{1-B^2} \right\}.$$

Let $\mathcal{P}$ be the class of functions with positive real parts. Then we have the following relationship between the classes $\mathcal{P}$ and $\mathcal{P}[A,B]$:

$$h \in \mathcal{P} \Leftrightarrow \frac{(A+1)\,h(z) - (A-1)}{(B+1)\,h(z) - (B-1)} \in \mathcal{P}[A,B].$$

This class was introduced by Janowski [3].

Kanas and Wiśniowska [5, 6] introduced and studied the class $k-\mathcal{UCV}$ of k-uniformly convex functions and the class $k-\mathcal{ST}$ of k-starlike functions. These classes were defined subject to the conic domain Ω_k $(k \geq 0)$ as

$$\Omega_k = \left\{ u+iv : u > k\sqrt{(u-1)^2 + v^2} \right\}.$$

We note that Ω_k is a region in the right half-plane, symmetric with respect to real axis, and contains the point $(1,0)$. More precisely, for $k=0$, Ω_0 is the right half-plane; for $0<k<1$, Ω_k is an unbounded region having boundary $\partial\Omega_k$ a hyperbola; for $k=1$, Ω_1 is still an unbounded region where $\partial\Omega_1$ is a parabola; and for $k>1$, Ω_k is a bounded region enclosed by an ellipse.

The extremal functions for these conic regions are

$$p_k(z) = \begin{cases} \frac{1+z}{1-z} & , \quad k=0 \\ 1+\frac{2}{\pi^2}\left(\log\frac{1+\sqrt{z}}{1-\sqrt{z}}\right)^2 & , \quad k=1 \\ \frac{1}{1-k^2}\cosh\left[\left(\frac{2}{\pi}\arccos k\right)\log\frac{1+\sqrt{z}}{1-\sqrt{z}}\right] - \frac{k^2}{1-k^2} & , \quad 0<k<1 \\ \frac{1}{k^2-1}\sin\left(\frac{\pi}{2K(t)}\int_0^{\frac{u(z)}{\sqrt{t}}} \frac{dx}{\sqrt{1-x^2}\sqrt{1-t^2x^2}}\right) + \frac{k^2}{k^2-1} & , \quad k>1 \end{cases}, \tag{2}$$

where

$$u(z) = \frac{z-\sqrt{t}}{1-\sqrt{t}z} \qquad (z \in \mathbb{U}),$$

and $t \in (0,1)$ is chosen such that $k = \cosh\frac{\pi K'(t)}{4K(t)}$. Here $K(t)$ is Legendre's complete elliptic integral of first kind

$$K(t) = \int_0^1 \frac{dx}{\sqrt{1-x^2}\sqrt{1-t^2x^2}},$$

and

$$K'(t) = K\left(\sqrt{1-t^2}\right)$$

is the complementary integral of $K(t)$; for details, see [1, 5, 6, 9]. If

$$p_k(z) = 1 + T_1(k)z + T_2(k)z^2 + \cdots \qquad (z \in \mathbb{U}),$$

then it was shown in [6] that for (2) one can have

$$T_1(k) = \begin{cases} \frac{2A^2(k)}{1-k^2} & , \quad 0 \le k < 1 \\ \frac{8}{\pi^2} & , \quad k = 1 \\ \frac{\pi^2}{4(k^2-1)K^2(t)\sqrt{t}(1+t)} & , \quad k > 1 \end{cases} , \tag{3}$$

$$T_2(k) = D(k)\, T_1(k),$$

where

$$D(k) = \begin{cases} \frac{A^2(k)+2}{3} & , \quad 0 \le k < 1 \\ \frac{2}{3} & , \quad k = 1 \\ \frac{4K^2(t)\left(t^2+6t+1\right)-\pi^2}{24K^2(t)\sqrt{t}(1+t)} & , \quad k > 1 \end{cases} , \tag{4}$$

with $A(k) = \frac{2}{\pi} \arccos k$.

Recently, Noor and Malik [10] defined the following class:

A function p is said to be in the class $k - \mathcal{P}[A, B]$, if and only if

$$p(z) \prec \frac{(A+1)\, p_k(z) - (A-1)}{(B+1)\, p_k(z) - (B-1)} \qquad (k \ge 0,\ z \in \mathbb{U}),$$

where p_k is defined by (2) and $-1 \le B < A \le 1$.

Geometrically, the function $p \in k - \mathcal{P}[A, B]$ takes all values from the domain $\Omega_k[A, B]$ which is defined as

$$\Omega_k[A, B] = \left\{ w : \Re\left(\frac{(B-1)\, w - (A-1)}{(B+1)\, w - (A+1)} \right) > k \left| \frac{(B-1)\, w - (A-1)}{(B+1)\, w - (A+1)} - 1 \right| \right\}$$

or equivalently $\Omega_k[A, B]$ is a set of numbers $w = u + iv$ such that

$$\begin{aligned} & \left[\left(B^2-1\right)\left(u^2+v^2\right) - 2\,(AB-1)\, u + \left(A^2-1\right)\right]^2 \\ > \; & k^2 \left[\left(-2\,(B+1)\left(u^2+v^2\right) + 2\,(A+B+2)\, u - 2\,(A+1)\right)^2 + 4\,(A-B)^2\, v^2\right]. \end{aligned}$$

This domain represents the conic type regions; for details see [10, 11]. It can be easily seen that $0 - \mathcal{P}[A, B] = \mathcal{P}[A, B]$ introduced by Janowski [3] and $k - \mathcal{P}[1, -1] = \mathcal{P}(p_k)$ introduced by Kanas and Wiśniowska [5].

It was shown by Khan [8] see also [12] that for

$$k \in [0, \infty) \qquad \text{and} \qquad q_k(z) = \frac{(A+1)p_k(z) - (A-1)}{(B+1)p_k(z) - (B-1)}$$

with

$$q_k(z) = 1 + H_1(k)z + H_2(k)z^2 + \cdots \qquad (z \in \mathbb{U}),$$

then

$$\begin{aligned} H_1(k) &= \frac{A-B}{2} T_1(k), \\ H_2(k) &= \frac{A-B}{4} \left[2D(k) - (B+1)H_1(k)\right] T_1(k), \end{aligned}$$

where $T_1(k)$ and $D(k)$ are defined in (3) and (4), respectively.

Throughout this paper, we assume that

$$k \in [0,\infty), \quad 0 \leq \lambda \leq 1 \quad \text{and} \quad -1 \leq B < A \leq 1.$$

The main purpose of this paper is to extend the work of Noor and Malik [10] and define a new class by using the concepts of Janowski functions in conic regions, as follows:

Definition 1. *A function $f \in \mathcal{A}$ is said to be in the class $k-\mathcal{U}(\lambda, A, B)$, if and only if,*

$$\Re\left(\frac{(B-1)F_{\lambda}(z)-(A-1)}{(B+1)F_{\lambda}(z)-(A+1)}\right) > k\left|\frac{(B-1)F_{\lambda}(z)-(A-1)}{(B+1)F_{\lambda}(z)-(A+1)}-1\right| \qquad (z \in \mathbb{U}),$$

or equivalently,

$$F_{\lambda}(z) \in k-\mathcal{P}[A,B], \tag{5}$$

where

$$F_{\lambda}(z) = \frac{zf'(z)+\lambda z^2 f''(z)}{(1-\lambda)f(z)+\lambda z f'(z)}. \tag{6}$$

Special cases:
(i) *$k-\mathcal{U}[0,A,B] = k-\mathcal{ST}[A,B]$ and $k-\mathcal{U}[1,A,B] = k-\mathcal{UCV}[A,B]$, the classes introduced by Noor and Malik* [10].
(ii) *$k-\mathcal{U}[0,1,-1] = k-\mathcal{ST}$ and $k-\mathcal{U}[1,1,-1] = k-\mathcal{UCV}$, the well-known classes of k-uniformly convex and k-starlike functions, respectively, introduced by Kanas and Wiśniowska* [5, 6].
(iii) *$k-\mathcal{U}[0,1-2\alpha,-1] = \mathcal{SD}(k,\alpha)$ and $k-\mathcal{U}[1,1-2\alpha,-1] = \mathcal{KD}(k,\alpha)$, the classes introduced by Shams et al.* [16].
(iv) *$0-\mathcal{U}[0,A,B] = \mathcal{S}^*[A,B]$ and $0-\mathcal{U}[1,A,B] = \mathcal{C}[A,B]$, the well-known classes of Janowski starlike and Janowski convex functions, respectively, introduced by Janowski* [3].
(v) *$0-\mathcal{U}[0,1-2\alpha,-1] = \mathcal{S}^*(\alpha)$ and $0-\mathcal{U}[1,1-2\alpha,-1] = \mathcal{C}(\alpha)$, the well-known classes of starlike and convex functions of order α $(0 \leq \alpha < 1)$, respectively.*

Lemma 1. [14] *Let $h(z) = 1+\sum_{n=1}^{\infty} c_n z^n$ be subordinate to $H(z) = 1+\sum_{n=1}^{\infty} C_n z^n$. If $H(z)$ is univalent in $\mathbb{U}$ and $H(\mathbb{U})$ is convex, then*

$$|c_n| \leq |C_1| \qquad (n \geq 1).$$

2. Main Results

Theorem 1. *A function $f \in \mathcal{A}$ of the form (1) is in the class $k-\mathcal{U}(\lambda, A, B)$, if it satisfies the condition*

$$\sum_{n=2}^{\infty} \{2(k+1)(n-1)+|n(B+1)-(A+1)|\}\,[1+\lambda(n-1)]\,|a_n| < |B-A|. \tag{7}$$

Proof. Suppose that the inequality (7) holds. Then it suffices that

$$k\left|\frac{(B-1)F_{\lambda}(z)-(A-1)}{(B+1)F_{\lambda}(z)-(A+1)}-1\right| - \Re\left(\frac{(B-1)F_{\lambda}(z)-(A-1)}{(B+1)F_{\lambda}(z)-(A+1)}-1\right) < 1,$$

where $F_{\lambda}(z)$ is defined by (6). We get

$$\begin{aligned}
& k\left|\frac{(B-1)F_{\lambda}(z)-(A-1)}{(B+1)F_{\lambda}(z)-(A+1)}-1\right|-\Re\left(\frac{(B-1)F_{\lambda}(z)-(A-1)}{(B+1)F_{\lambda}(z)-(A+1)}-1\right) \\
\leq\ & (k+1)\left|\frac{(B-1)F_{\lambda}(z)-(A-1)}{(B+1)F_{\lambda}(z)-(A+1)}-1\right| \\
=\ & 2(k+1)\left|\frac{F_{\lambda}(z)-1}{(B+1)F_{\lambda}(z)-(A+1)}\right| \\
=\ & 2(k+1)\left|\frac{(1-\lambda)f(z)-(1-\lambda)zf'(z)-\lambda z^2f''(z)}{(1-\lambda)(A+1)f(z)-[(B+1)-\lambda(A+1)]zf'(z)-\lambda(B+1)z^2f''(z)}\right| \\
<\ & 2(k+1)\frac{\sum_{n=2}^{\infty}(n-1)[1+\lambda(n-1)]\,|a_n|}{|B-A|-\sum_{n=2}^{\infty}|n(B+1)-(A+1)|\,[1+\lambda(n-1)]\,|a_n|} \\
<\ & 1 \qquad (\text{by } (7))\,.
\end{aligned}$$

Therefore $f\in k-\mathcal{U}(\lambda,A,B)$. □

Corollary 1. [10] *A function $f\in\mathcal{A}$ of the form (1) is in the class $k-\mathcal{ST}[A,B]$, if it satisfies the condition*

$$\sum_{n=2}^{\infty}\{2(k+1)(n-1)+|n(B+1)-(A+1)|\}\,|a_n|<|B-A|\,.$$

Corollary 2. [10] *A function $f\in\mathcal{A}$ of the form (1) is in the class $k-\mathcal{UCV}[A,B]$, if it satisfies the condition*

$$\sum_{n=2}^{\infty}n\,\{2(k+1)(n-1)+|n(B+1)-(A+1)|\}\,|a_n|<|B-A|\,.$$

Corollary 3. [6] *A function $f\in\mathcal{A}$ of the form (1) is in the class $k-\mathcal{ST}$, if it satisfies the condition*

$$\sum_{n=2}^{\infty}[n+k(n-1)]\,|a_n|<1.$$

Corollary 4. *A function $f\in\mathcal{A}$ of the form (1) is in the class $k-\mathcal{UCV}$, if it satisfies the condition*

$$\sum_{n=2}^{\infty}n\,[n+k(n-1)]\,|a_n|<1.$$

Corollary 5. [16] *A function $f\in\mathcal{A}$ of the form (1) is in the class $\mathcal{SD}(k,\alpha)$, if it satisfies the condition*

$$\sum_{n=2}^{\infty}[n(k+1)-(k+\alpha)]\,|a_n|<1-\alpha.$$

Corollary 6. [16] *A function $f\in\mathcal{A}$ of the form (1) is in the class $\mathcal{KD}(k,\alpha)$, if it satisfies the condition*

$$\sum_{n=2}^{\infty}n\,\{n(k+1)-(k+\alpha)\}\,|a_n|<1-\alpha.$$

Corollary 7. *A function $f \in \mathcal{A}$ of the form (1) is in the class $\mathcal{S}^*[A,B]$, if it satisfies the condition*

$$\sum_{n=2}^{\infty} \{2(n-1) + |n(B+1) - (A+1)|\} |a_n| < |B - A| .$$

Corollary 8. *A function $f \in \mathcal{A}$ of the form (1) is in the class $\mathcal{C}[A,B]$, if it satisfies the condition*

$$\sum_{n=2}^{\infty} n \{2(n-1) + |n(B+1) - (A+1)|\} |a_n| < |B - A| .$$

Corollary 9. [17] *A function $f \in \mathcal{A}$ of the form (1) is in the class $\mathcal{S}^*(\alpha)$, if it satisfies the condition*

$$\sum_{n=2}^{\infty} (n-\alpha) |a_n| < 1 - \alpha.$$

Corollary 10. [17] *A function $f \in \mathcal{A}$ of the form (1) is in the class $\mathcal{C}(\alpha)$, if it satisfies the condition*

$$\sum_{n=2}^{\infty} n (n-\alpha) |a_n| < 1 - \alpha.$$

Theorem 2. *Let $f \in k - \mathcal{U}(\lambda, A, B)$ be of the form (1). Then*

$$|a_n| \leq \frac{1}{1+\lambda(n-1)} \prod_{j=0}^{n-2} \frac{2j + (A-B)|T_1(k)|}{2(j+1)} \qquad (n \geq 2), \tag{8}$$

where $T_1(k)$ is defined by (3).

Proof. Let $f \in k - \mathcal{U}(\lambda, A, B)$. By Definition 1, we have

$$\frac{zf^{'}(z) + \lambda z^2 f^{''}(z)}{(1-\lambda)f(z) + \lambda z f'(z)} = p(z), \tag{9}$$

where

$$\begin{aligned} p(z) &\prec \frac{(A+1)p_k(z) - (A-1)}{(B+1)p_k(z) - (B-1)} \\ &= [(A+1)p_k(z) - (A-1)]\,[(B+1)p_k(z) - (B-1)]^{-1} \\ &= \frac{A-1}{B-1}\left[1 - \frac{A+1}{A-1}p_k(z)\right]\left[1 + \sum_{n=1}^{\infty}\left(\frac{B+1}{B-1}p_k(z)\right)^n\right] \\ &= \frac{A-1}{B-1} + \left(\frac{(A-1)(B+1)}{(B-1)^2} - \frac{A+1}{B-1}\right)p_k(z) \\ &\quad + \left(\frac{(A-1)(B+1)^2}{(B-1)^3} - \frac{(A+1)(B+1)}{(B-1)^2}\right)(p_k(z))^2 + \cdots \end{aligned}$$

If

$$p_k(z) = 1 + T_1(k)z + \cdots \qquad (z \in \mathbb{U}),$$

where $T_1(k)$ is defined by (3), then we obtain

$$p(z) \prec \sum_{n=1}^{\infty} \frac{-2(B+1)^{n-1}}{(B-1)^n} + \left\{\sum_{n=1}^{\infty} \frac{2n(A-B)(B+1)^{n-1}}{(B-1)^{n+1}}\right\} T_1(k)z + \cdots .$$

Since the series

$$\sum_{n=1}^{\infty} \frac{-2(B+1)^{n-1}}{(B-1)^n} \qquad \text{and} \qquad \sum_{n=1}^{\infty} \frac{2n(A-B)(B+1)^{n-1}}{(B-1)^{n+1}}$$

are convergent and converge to 1 and $\frac{A-B}{2}$, respectively, we have

$$p(z) \prec 1 + \frac{A-B}{2} T_1(k) z + \cdots .$$

Now if $p(z) = 1 + \sum_{n=1}^{\infty} c_n z^n$, then by Lemma 1, we get

$$|c_n| \leq \frac{A-B}{2} |T_1(k)| \qquad (n \geq 1). \tag{10}$$

On the other hand, from (9), we have

$$zf^{'}(z) + \lambda z^2 f^{''}(z) = \left[(1-\lambda) f(z) + \lambda z f^{'}(z)\right] p(z),$$

which implies that

$$zf^{'}(z) + \lambda z^2 f^{''}(z) = \left[(1-\lambda) f(z) + \lambda z f^{'}(z)\right] \left(1 + \sum_{n=1}^{\infty} c_n z^n\right),$$

or equivalently,

$$z + \sum_{n=2}^{\infty} n \left[1 + \lambda (n-1)\right] a_n z^n = \left(z + \sum_{n=2}^{\infty} \left[1 + \lambda (n-1)\right] a_n z^n\right) \left(1 + \sum_{n=1}^{\infty} c_n z^n\right).$$

Equating coefficients of z^n on both sides, we find

$$(n-1) \left[1 + \lambda (n-1)\right] a_n = \sum_{j=1}^{n-1} \left[1 + \lambda (j-1)\right] a_j c_{n-j} \qquad (a_1 = 1, \, n \geq 2).$$

This implies that

$$|a_n| \leq \frac{1}{(n-1) \left[1 + \lambda (n-1)\right]} \sum_{j=1}^{n-1} \left[1 + \lambda (j-1)\right] |a_j| \, |c_{n-j}| \qquad (a_1 = 1, \, n \geq 2).$$

Using (10), we have

$$|a_n| \leq \frac{(A-B) |T_1(k)|}{2(n-1) \left[1 + \lambda (n-1)\right]} \sum_{j=1}^{n-1} \left[1 + \lambda (j-1)\right] |a_j| \qquad (a_1 = 1, \, n \geq 2). \tag{11}$$

Now we shall prove that the following inequality holds for all $n \geq 2$:

$$\frac{(A-B) |T_1(k)|}{2(n-1) \left[1 + \lambda (n-1)\right]} \sum_{j=1}^{n-1} \left[1 + \lambda (j-1)\right] |a_j| \leq \frac{1}{1 + \lambda (n-1)} \prod_{j=0}^{n-2} \frac{2j + (A-B) |T_1(k)|}{2(j+1)}. \tag{12}$$

For the proving of this claim, we use the mathematical induction method:

Setting $n = 2$ in (11), we have

$$|a_2| \leq \frac{(A-B) |T_1(k)|}{2(1+\lambda)},$$

which is hold for $n = 2$.

Setting $n = 3$ in (11), we have

$$\begin{aligned} |a_3| &\leq \frac{(A-B)\,|T_1(k)|}{4\,(1+2\lambda)} \left[1 + (1+\lambda)\,|a_2|\right] \\ &\leq \frac{(A-B)\,|T_1(k)|}{4(1+2\lambda)} \left[1 + \frac{(A-B)\,|T_1(k)|}{2}\right], \end{aligned}$$

which is hold for $n = 3$.

Assume the hypothesis (8) be true for $n = m$, that is

$$|a_m| \leq \frac{1}{1+\lambda\,(m-1)} \prod_{j=0}^{m-2} \frac{2j + (A-B)\,|T_1(k)|}{2(j+1)}.$$

We have to show that the result is true for $n = m+1$, that is

$$\begin{aligned} |a_{m+1}| &\leq \frac{(A-B)\,|T_1(k)|}{2m\,(1+m\lambda)} \sum_{j=1}^{m} \left[1 + \lambda\,(j-1)\right]\,|a_j|\,, \\ &= \frac{(A-B)\,|T_1(k)|}{2m\,(1+m\lambda)} \left[1 + (1+\lambda)\,|a_2| + \cdots + \left[1+\lambda\,(m-1)\right]\,|a_m|\right] \\ &\leq \frac{1}{1+m\lambda} \prod_{j=0}^{m-1} \frac{2j + (A-B)\,|T_1(k)|}{2(j+1)}, \end{aligned}$$

which shows that inequality (12) is true for $n = m+1$. Hence by the technic of mathematical induction, inequality (12) holds true for all $n \geq 2$. □

Corollary 11. [13] *Let $f \in \mathcal{S}^*(\alpha)$ be of the form (1). Then*

$$|a_n| \leq \frac{\prod_{j=0}^{n-2} \left[j + 2(1-\alpha)\right]}{(n-1)!} \qquad (n \geq 2)\,.$$

Corollary 12. [13] *Let $f \in \mathcal{C}(\alpha)$ be of the form (1). Then*

$$|a_n| \leq \frac{\prod_{j=0}^{n-2} \left[j + 2(1-\alpha)\right]}{n!} \qquad (n \geq 2)\,.$$

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