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NEW CORONA AND NEW CLUSTER OF GRAPHS AND THEIR WIENER INDEX

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ABSTRACT. The *Wiener index* of G, denoted by $W(G)$, is defined as

$$W(G) = \sum_{u \neq v} d(u, v),$$

where the sum is taken through all unordered pairs of vertices of G. In this paper we introduce new operations namely, new corona and new cluster on graphs and study the Wiener indices of the resulting graphs.

1. Introduction

For a graph $G = (V, E)$ if $u,\ v \in V(G)$ then the *distance* $d(u, v)$ between u and v is defined as the length of a shortest u-v path in G. The *Wiener index* of G, denoted by $W(G)$, is defined as

$$W(G) = \sum_{u \neq v} d(u, v),$$

where the sum is taken through all unordered pairs of vertices of G. Wiener index was introduced by Wiener [14]. It is related to boiling point, heat of evaporation, heat of formation, chromatographic retention times, surface tension, vapour pressure, partition coefficients, total electron energy of polymers, ultrasonic sound velocity, internal energy, etc., *see* [12]. For this reason Wiener index is widely studied by chemists. Mathematical properties and chemical applications of the Wiener index have been intensively studied over the past thirty years. For more information about the Wiener index in chemistry and mathematics *see* [10] and [1, 2, 3, 4, 5, 6, 7, 8, 9, 11, 13], respectively. In this paper we introduce a new corona and new cluster of two graphs and study the Wiener indices of the resulting graphs.

The corona $G_1\ o\ G_2$ is obtained by taking one copy of G_1 and $|G_1|$ copies of G_2, and by joining each vertex of the ith copy of G_2 to the ith vertex of $G_1, i = 1, 2, ..., |G_1|$. We are interested in giving new corona of graphs such that $V(G_1) \cup E(G_1) \cup V(G_2)$ and $V(G_1) \cup V'(G_1) \cup V(G_2)$ are the set of vertices. The cluster $G_1\{G2\}$, is obtained by taking one copy of G_1 and $|G_1|$ copies of a rooted graph G_2, and by identifying the root of the ith copy of G_2 with the ith vertex of $G_1, i = 1, 2, ..., |G_1|$. We are interested in giving new cluster of graphs such that $V(G_1) \cup E(G_1) \cup (V(G_2) - v)$ and $V(G_1) \cup V'(G_1) \cup (V(G_2) - v)$ are the set of vertices, where v is the root vertex of G_2. For this purpose we first recall some operations on graphs.

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The *total graph* $T(G)$ is the graph whose vertex set is $V(G) \cup E(G)$ and two vertices are adjacent if and only if they are adjacent or incident in G. The *middle graph* or *semi-total line graph* $M(G)$ of G is the graph whose vertex set is $V(G) \cup E(G)$. Two vertices in $M(G)$ are adjacent if and only if they are adjacent edges in G, or one is a vertex and another is an edge incident on it in G. The *semi-total point graph* $Q(G)$ of G is the graph whose vertex set is $V(G) \cup E(G)$. Two vertices in $Q(G)$ are adjacent if and only if they are adjacent vertices in G, or one is a vertex and another is an edge incident on it in G. The *subdivision graph* $S(G)$ of G is the graph whose vertex set is $V(G) \cup E(G)$. Two vertices in $S(G)$ are adjacent if and only if one is a vertex and another is an edge incident on it in G. The subdivision graph $S(G)$, the semi-total point graph $Q(G)$, the middle graph $M(G)$ and the total graph $T(G)$ of a graph G are shown in Figure 1.

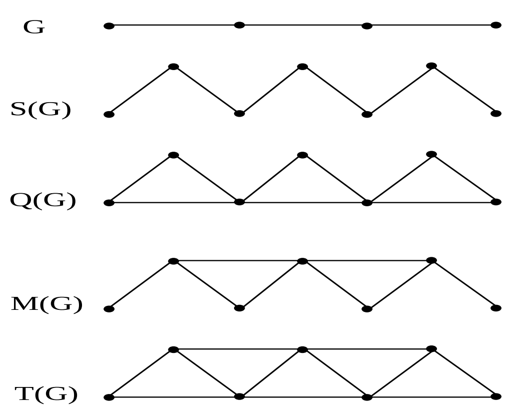

FIGURE 1. The graphs G, $S(G)$, $Q(G)$, $M(G)$, $T(G)$

The *splitting graph* $\Lambda(G)$ of a graph G is the graph whose vertex set is $V(G) \cup V'(G)$, where $V'(G)$ is the copy of $V(G)$ (i.e., $V'(G) = v' : v \in V(G)$) and the edge set $E(G) \cup \{xy' : xy \in E(G)\}$. The vertex v' is called the twin of the vertex v, (and v the twin of v'). The *closed splitting graph* $\overline{\Lambda}(G)$ of a graph G is the graph whose vertex set is $V(G) \cup V'(G)$, where $V'(G)$ is the copy of $V(G)$ (i.e., $V'(G) = v' : v \in V(G)$) and the edge set $E(G) \cup \{xx' : x \in V(G)\} \cup \{xy' : xy \in E(G)\}$. The *shadow graph* of G, denoted by $D_2(G)$, has as the vertex set $V(G) \cup V'(G)$, and the edge set $E(G) \cup \{x'y' : xy \in E(G)\} \cup \{xy' : xy \in E(G)\}$. The *closed shadow graph* of G, denoted by $D_2[G]$, has as the vertex set $V(G) \cup V'(G)$, and the edge set $E(G) \cup \{x'y' : xy \in E(G)\} \cup \{xy' : xy \in E(G)\} \cup \{xx' : x \in V(G)\}$. The vertex v' is called the twin of the vertex v, (and v the twin of v'). We illustrate these definitions in Figure 2.

Let F be one of the symbols $S(G), M(G), Q(G), T(G), \Lambda(G), \overline{\Lambda}(G), D_2(G), D_2[G]$. The F-corona $F(G_1)oG_2$ is obtained by taking one copy of $F(G_1)$ and $|G_l|$ copies of G_2, and by joining each vertex of the ith copy of G_2 to the ith vertex of $G_1, i = 1, 2, ..., |G_1|$. We illustrate these definitions in Figure 3.

Let F be one of the symbols $S(G), M(G), Q(G), T(G), \Lambda(G), \overline{\Lambda}(G), D_2(G), D_2[G]$. The F-cluster $F(G_1)\{G2\}$, is obtained by taking one copy of $F(G_1)$ and $|G_1|$ copies of a rooted graph G_2, and by identifying the root of the ith copy of G_2 with the ith vertex of $G_1, i = 1, 2, ..., |G_1|$. We illustrate these definitions in Figure 4.

2. Wiener index of cluster of two graphs

Lemma 1. [11] *Let G_1 and G_2 be two connected graphs. Then,*

$$W(G_1 \ o \ G_2) = (|G_2| + 1)^2 W(G_1) + |G_1|[|G_2|^2 - |E(G_2)|] + (|G_1|^2 - |G_1|)|G_2|(|G_2| + 1).$$

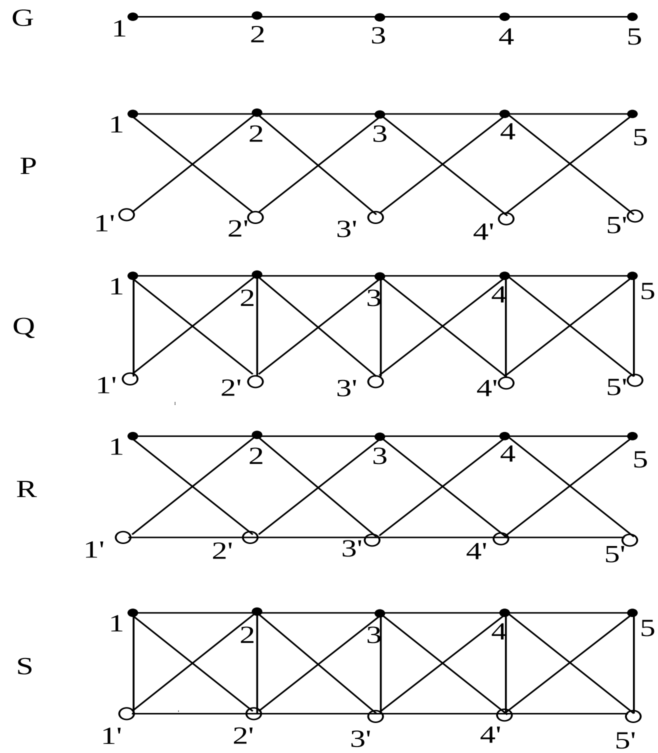

FIGURE 2. The graphs G, $P=\Lambda(G)$, $Q=\overline{\Lambda}(G)$, $R=D_2(G)$ and $S=D_2[G]$

Lemma 2. [11] *Let G_1 and G_2 be two connected graphs. Then,*

$$W(G_1\{G_2\}) = (|G_2|)^2W(G_1)+|G_1|W(G_2)+(|G_1|^2-|G_1|)|G_2|d(r|G_2),$$

where $d(r|G_2)$ is the sum of distances of all the vertices of G_2 from the root vertex of G_2.

Theorem 1. *Let G_1 and G_2 be two connected graphs and $F=S(G),Q(G),M(G),$ or $T(G)$. Then,*

$$\begin{aligned} W(F(G_1)\{G_2\}) &= |G_1|W(G_2)+|G_2|^2\sum_{u,v\in V_1} d(u,v|F(G_1))+(|G_1|^2-|G_1|)|G_2|d(r|G_2) \\ &+ W(F(G_1))-\sum_{u,v\in V_1} d(u,v|F(G_1))+(|G_2|-1)\sum_{u\in V_1,e\in E_1} d(u,e|F(G_1)) \\ &+ |G_1||E_1|d(r|G_2). \end{aligned}$$

Further if $F(G_1)=Q(G_1)$ or $T(G_1)$, then

$$\begin{aligned} W(F(G_1)\{G_2\}) &= |G_1|W(G_2)+|G_2|^2W(G_1)+(|G_1|^2-|G_1|)|G_2|d(r|G_2) \\ &+ W(F(G_1))-W(G_1)+(|G_2|-1)\sum_{u\in V_1,v\in E_1} d(u,v|F(G_1)) \\ &+ |G_1||E_1|d(r|G_2). \end{aligned}$$

Proof. If two vertices u and v belong to the same copy of G_2, then

$$d(u,v|F(G_1)\{G_2\}) = d(u,v|G_2).$$

The respective contribution to $W(F(G_1)\{G_2\})$ is clearly,

$$W_1 = |G_1|W(G_2).$$

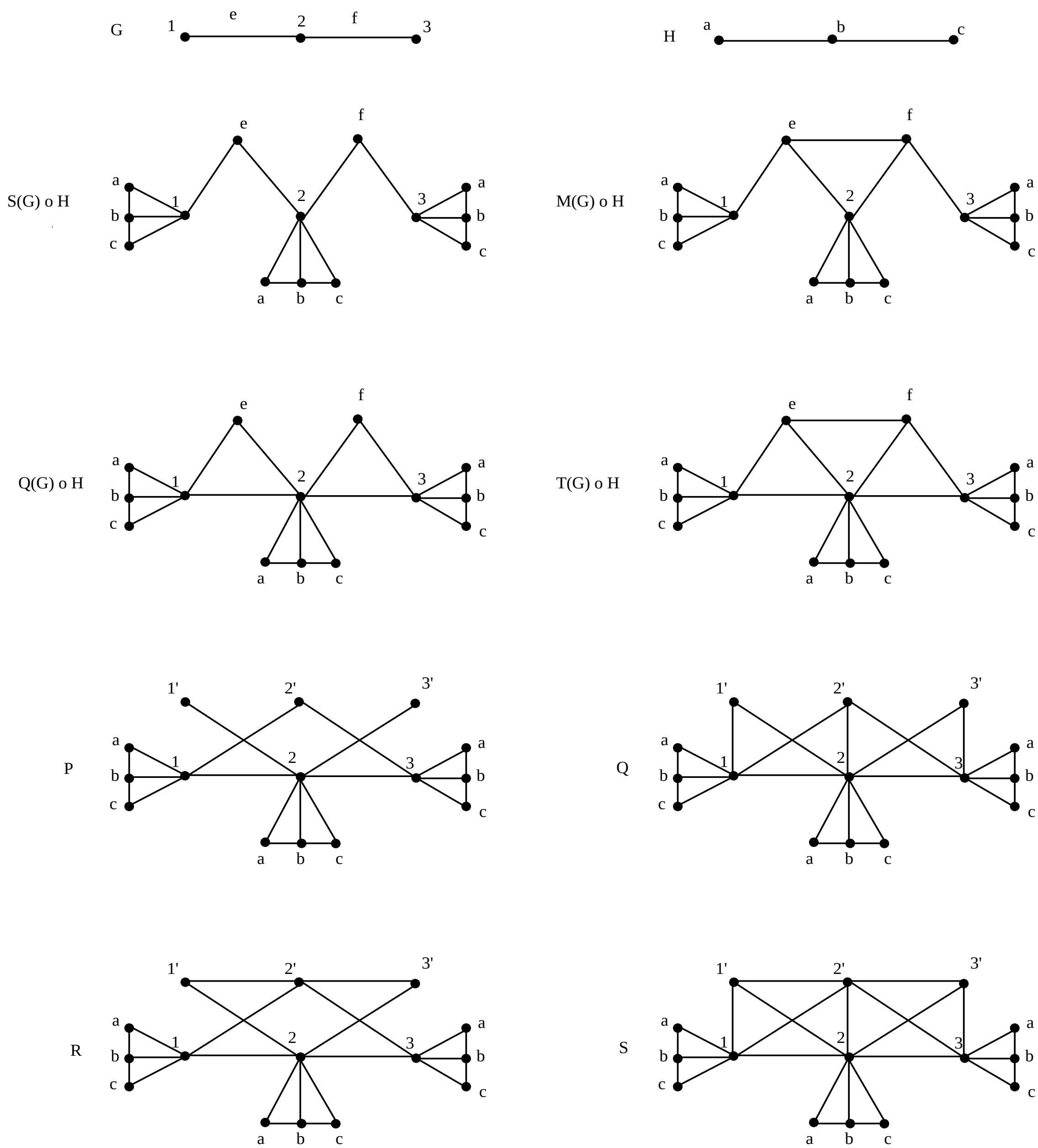

FIGURE 3. The graphs G, H, $S(G)oH$, $Q(G)oH$, $M(G)oH$, $T(G)oH$, $P = \Lambda(G)oH$, $Q = \overline{\Lambda}(G)oH$, $D_2(G)oH$ and $D_2[G]oH$

If, however, the vertices u and v of $F(G_1)\{G_2\}$ belong to different copies of G_2, then

$$d(u, v|F(G_1)\{G_2\}) = d(u, r|G_2) + d(i, j|F(G_1)) + d(v, r|G_2),$$

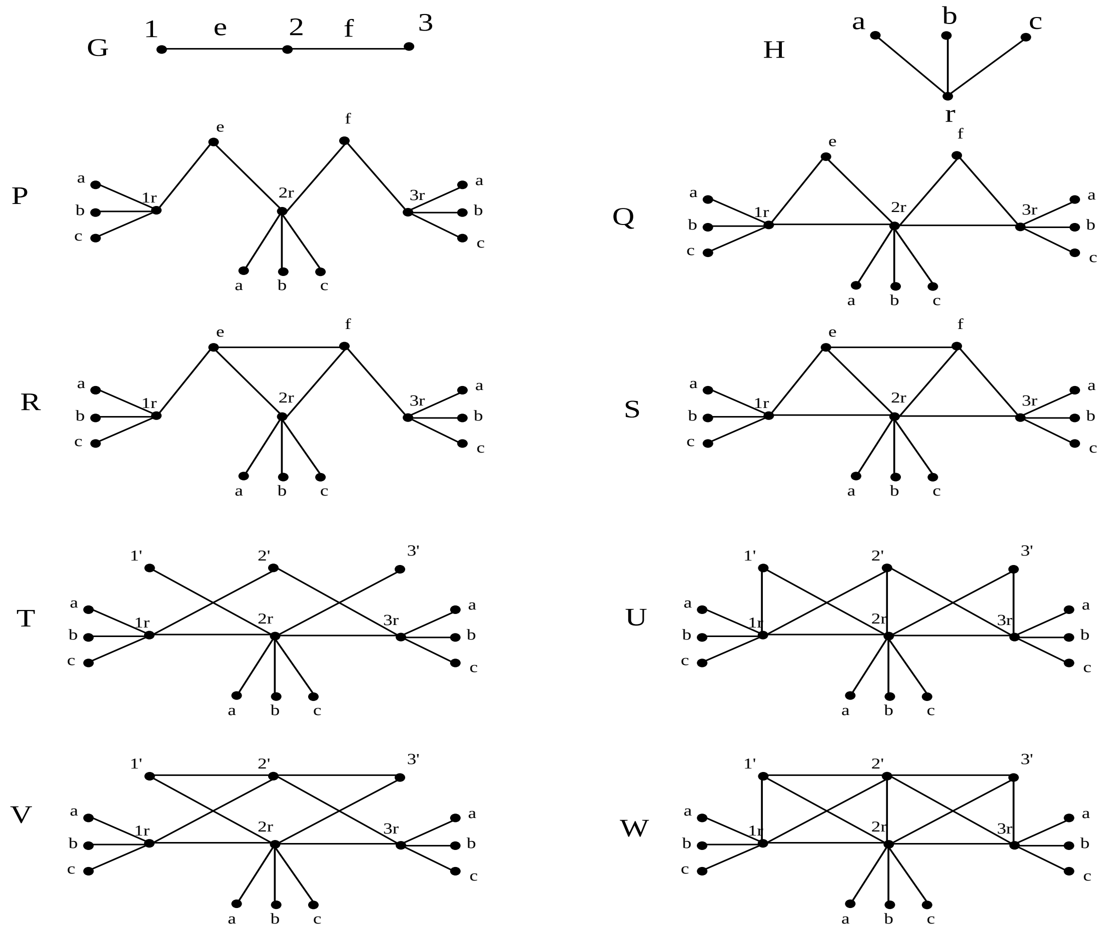

FIGURE 4. The graphs G, H, $P = S(G)\{H\}$, $Q = Q(G)\{H\}$, $R = M(G)\{H\}$, $S = T(G)\{H\}$, $T = \Lambda(G)\{H\}$, $U = \overline{\Lambda}(G)\{H\}$, $V = D_2(G)\{H\}$ and $W = D_2[G]\{H\}$

where i and j denote the vertices of $F(G_1)$ to which the copies of G_2 are attached. For each fixed pair i, j there are $|G_2|^2$ such pairs u, v and their contribution to $W(F(G_1)\{G_2\})$ amounts $2|G_2|d(r|G_2)+|G_2|^2 d(i,j|F(G_1))$. Summing these contribution over all the ${}^{|G_1|}C_2$ distinct pairs i, j, we arrive at

$$W_2 \quad = \quad 2|G_2|^{|G_1|}C_2 d(r|G_2) + |G_2|^2 \sum_{u,v\in V_1} d(u,v|F(G_1)).$$

Now, if the two vertices u and v of $F(G_1)\{G_2\}$ belong to a copy of G_2 and $F(G_1)$ (not an identifying vertex of $F(G_1)$), respectively, then

$$d(u,v|F(G_1)\{G_2\}) = d(u,r|G_2) + d(i,v|F(G_1)),$$

where i denote the vertex of $F(G_1)$ to which the copy of G_2 is attached. For each fixed i, there are $|G_2|$ such pairs u, v and their contribution to $W(F(G_1)\{G_2\})$ is,

$$W_3 \quad = \quad |E_1||G_1|d(r|G_2) + |G_2| \sum_{u\in V_1, v\in E_1} d(u,v|F(G_1)).$$

If the two vertices u and v of $F(G_1)\{G_2\}$ belongs to $F(G_1)$ (corresponding to the edges of G_1), then

$$d(u,v|F(G_1)\{G_2\}) = d(u,v|F(G_1)).$$

So, their contribution is clearly,

$$W_4 = \sum_{u,v\in E_1} d(u,v|F(G_1)).$$

Hence,

$$\begin{aligned}
W(F(G_1)\{G_2\}) &= W_1 + W_2 + W_3 + W_4 \\
&= |G_1|W(G_2) + 2|G_2|^{|G_1|}C_2 d(r|G_2) + |G_2|^2 \sum_{u,v\in V_1} d(u,v|F(G_1)) \\
&+ |E_1||G_1|d(r|G_2) + |G_2| \sum_{u\in V_1, v\in E_1} d(u,v|F(G_1)) + \sum_{u,v\in E_1} d(u,v|F(G_1)) \\
&= |G_1|W(G_2) + |G_2|^2 \sum_{u,v\in V_1} d(u,v|F(G_1)) + (|G_1|^2 - |G_1|)|G_2|d(r|G_2) \\
&+ W(F(G_1)) - \sum_{u,v\in V_1} d(u,v|F(G_1)) + (|G_2|-1) \sum_{u\in V_1, v\in E_1} d(u,v|F(G_1)) \\
&+ |G_1||E_1|d(r|G_2).
\end{aligned}$$

Further if $F(G_1) = Q(G_1)$ or $T(G_1)$, then

$$\sum_{u,v\in V_1} d(u,v|F(G_1)) = \sum_{u,v\in V_1} d(u,v|G_1) = W(G_1),$$

Therefore,

$$\begin{aligned}
W(F(G_1)\{G_2\}) &= |G_1|W(G_2) + |G_2|^2 W(G_1) + (|G_1|^2 - |G_1|)|G_2|d(r|G_2) \\
&+ W(F(G_1)) - W(G_1) + (|G_2|-1) \sum_{u\in V_1, v\in E_1} d(u,v|F(G_1)) \\
&+ |G_1||E_1|d(r|G_2).
\end{aligned}$$

□

Theorem 2. *Let G_1 and G_2 be two connected graphs and $F = \Lambda(G), \overline{\Lambda}(G), D_2(G),$ or $D_2[G]$. Then,*

$$\begin{aligned}
W(F(G_1)\{G_2\}) &= |G_1|W(G_2) + |G_2|^2 W(G_1) + (|G_1|^2 - |G_1|)|G_2|d(r|G_2) \\
&+ W(F(G_1)) - W(G_1) + (|G_2|-1) \sum_{u\in V_1, v\in V_1'} d(u,v|F(G_1)) \\
&+ |G_1|^2 d(r|G_2).
\end{aligned}$$

Further if $F(G_1) = \Lambda(G_1)$ or $D_2(G_1)$, then

$$\begin{aligned}
W(F(G_1)\{G_2\}) &= |G_1|W(G_2) + W(G_1)(|G_2|^2 + 2|G_1| - 3) + ((|G_1|^2 - |G_1|)|G_2| + |G_1|^2)d(r|G_2) \\
&+ W(F(G_1)) + 2|G_1|(|G_2|-1).
\end{aligned}$$

And if $F(G_1) = \overline{\Lambda}(G_1)$ or $D_2[G_1]$, then

$$\begin{aligned}
W(F(G_1)\{G_2\}) &= |G_1|W(G_2) + W(G_1)(|G_2|^2 + 2|G_1| - 3) + ((|G_1|^2 - |G_1|)|G_2| + |G_1|^2)d(r|G_2) \\
&+ W(F(G_1)) + |G_1|(|G_2|-1).
\end{aligned}$$

Proof. If two vertices u and v belong to the same copy of G_2, then

$$d(u,v|F(G_1)\{G_2\}) = d(u,v|G_2).$$

The respective contribution to $W(F(G_1)\{G_2\})$ is clearly,

$$W_1 = |G_1|W(G_2).$$

If, however, the vertices u and v of $F(G_1)\{G_2\}$ belong to different copies of G_2, then

$$d(u,v|F(G_1)\{G_2\}) = d(u,r|G_2) + d(i,j|F(G_1)) + d(v,r|G_2),$$

where i and j denote the vertices of $F(G_1)$ to which the copies of G_2 are attached. For each fixed pair i, j there are $|G_2|^2$ such pairs u, v and their contribution to $W(F(G_1)\{G_2\})$ amounts $2|G_2|d(r|G_2)+|G_2|^2 d(i,j|F(G_1))$. Summing these contribution over all the ${}^{|G_1|}C_2$ distinct pairs i, j, we arrive at

$$W_2 = 2|G_2|^{|G_1|}C_2 d(r|G_2) + |G_2|^2 \sum_{u,v\in V_1} d(u,v|F(G_1)).$$

Now, if the two vertices u and v of $F(G_1)\{G_2\}$ belong to a copy of G_2 and $F(G_1)$ (not an identifying vertex of $F(G_1)$), respectively, then

$$d(u,v|F(G_1)\{G_2\}) = d(u,r|G_2) + d(i,v|F(G_1)),$$

where i denote the vertex of $F(G_1)$ to which the copy of G_2 is attached. For each fixed i, there are $|G_2|$ such pairs u, v and their contribution to $W(F(G_1)\{G_2\})$ is,

$$W_3 = |G_1|^2 d(r|G_2) + |G_2| \sum_{u\in V_1, v\in V_1'} d(u,v|F(G_1)).$$

If the two vertices u and v of $F(G_1)\{G_2\}$ belongs to $F(G_1)$ (corresponding to the twin of the vertices of G_1), then

$$d(u,v|F(G_1)\{G_2\}) = d(u,v|F(G_1)).$$

So, their contribution is clearly,

$$W_4 = \sum_{u,v\in V_1'} d(u,v|F(G_1)).$$

Hence,

$$\begin{aligned}
W(F(G_1)\{G_2\}) &= W_1 + W_2 + W_3 + W_4 \\
&= |G_1|W(G_2) + 2|G_2|^{|G_1|}C_2 d(r|G_2) + |G_2|^2 \sum_{u,v\in V_1} d(u,v|F(G_1)) \\
&+ |G_1|^2 d(r|G_2) + |G_2| \sum_{u\in V_1, v\in V_1'} d(u,v|F(G_1)) + \sum_{u,v\in V_1'} d(u,v|F(G_1)) \\
&= |G_1|W(G_2) + |G_2|^2 \sum_{u,v\in V_1} d(u,v|F(G_1)) + (|G_1|^2 - |G_1|)|G_2|d(r|G_2) \\
&+ W(F(G_1)) - \sum_{u,v\in V_1} d(u,v|F(G_1)) + (|G_2|-1) \sum_{u\in V_1, v\in V_1'} d(u,v|F(G_1)) \\
&+ |G_1|^2 d(r|G_2).
\end{aligned}$$

Also since $F(G) = \Lambda(G), \overline{\Lambda}(G), D_2(G)$ or , $D_2[G]$,

$$\sum_{u,v\in V_1} d(u,v|F(G_1)) = d(u,v|G_1) = W(G_1).$$

Therefore,

$$\begin{aligned}
W(F(G_1)\{G_2\}) &= |G_1|W(G_2) + |G_2|^2 W(G_1) + (|G_1|^2 - |G_1|)|G_2|d(r|G_2) \\
&+ W(F(G_1)) - W(G_1) + (|G_2|-1) \sum_{u\in V_1, v\in V_1'} d(u,v|F(G_1)) \\
&+ |G_1|^2 d(r|G_2).
\end{aligned}$$

Further if $F(G) = \Lambda(G)$ or $D_2(G)$, then

$$\sum_{u\in V_1, v\in V_1'} d(u,v|F(G_1)) = 2W(G_1) + 2|G_1|.$$

Therefore,

$$\begin{aligned} W(F(G_1)\{G_2\}) &= |G_1|W(G_2) + W(G_1)(|G_2|^2 + 2|G_1| - 3) + ((|G_1|^2 - |G_1|)|G_2| + |G_1|^2)d(r|G_2) \\ &+ W(F(G_1)) + 2|G_1|(|G_2| - 1). \end{aligned}$$

Now if $F(G) = \overline{\Lambda}(G)$ or $D_2[G]$, then

$$\sum_{u\in V_1, v\in V_1'} d(u,v|F(G_1)) = 2W(G_1) + |G_1|.$$

Therefore,

$$\begin{aligned} W(F(G_1)\{G_2\}) &= |G_1|W(G_2) + W(G_1)(|G_2|^2 + 2|G_1| - 3) + ((|G_1|^2 - |G_1|)|G_2| + |G_1|^2)d(r|G_2) \\ &+ W(F(G_1)) + |G_1|(|G_2| - 1). \end{aligned}$$

□

3. Wiener index of corona of two graphs

Theorem 3. *Let G_1 and G_2 be two connected graphs and $F = S(G), Q(G), M(G),$ or $T(G)$. Then,*

$$\begin{aligned} W(F(G_1)\ o\ G_2) &= |G_1|(W(G_2) + |G_2|) + (|G_2| + 1)^2 \sum_{u,v\in V_1} d(u,v|F(G_1)) + (|G_1|^2 - |G_1|)(|G_2| \\ &+ 1)|G_2| + W(F(G_1)) - \sum_{u,v\in V_1} d(u,v|F(G_1)) + |G_2| \sum_{u\in V_1, e\in E_1} d(u,e|F(G_1)) \\ &+ |G_1||E_1||G_2|. \end{aligned}$$

Further if $F(G_1) = Q(G_1)$ or $T(G_1)$, then

$$\begin{aligned} W(F(G_1)\ o\ G_2) &= |G_1|(W(G_2) + |G_2|) + (|G_2| + 1)^2 W(G_1) + (|G_1|^2 - |G_1|)(|G_2| + 1)|G_2| \\ &+ W(F(G_1)) - W(G_1) + |G_2| \sum_{u\in V_1, e\in E_1} d(u,e|F(G_1)) + |G_1||E_1||G_2|. \end{aligned}$$

Proof. It is a special case of Theorem 1. We have $F(G_1)\ o\ G_2 \equiv F(G_1)\{G_2 + K_1\}$, where K_1 is a one vertex graph and where the root of $G_2 + K_1$ is chosen to be the vertex belonging to $V(K_1)$. □

Theorem 4. *Let G_1 and G_2 be two connected graphs and $F = \Lambda(G), \overline{\Lambda}(G), D_2(G),$ or $D_2[G]$. Then,*

$$\begin{aligned} W(F(G_1)\ o\ G_2) &= |G_1|(W(G_2) + |G_2|) + (|G_2| + 1)^2 W(G_1) + (|G_1|^2 - |G_1|)(|G_2| + 1)|G_2| \\ &+ W(F(G_1)) - W(G_1) + |G_2| \sum_{u\in V_1, v\in V_1'} d(u,v|F(G_1)) \\ &+ |G_1|^2|G_2|. \end{aligned}$$

Further if $F(G_1) = \Lambda(G_1)$ or $D_2(G_1)$, then

$$\begin{aligned} W(F(G_1)\ o\ G_2) &= |G_1|W(G_2) + W(G_1)(|G_2|^2 + 2|G_1| - 3) + ((|G_1|^2 - |G_1|)|G_2| + |G_1|^2)d(r|G_2) \\ &+ W(F(G_1)) + 2|G_1|(|G_2| - 1). \end{aligned}$$

And if $F(G_1) = \overline{\Lambda}(G_1)$ or $D_2[G_1]$, then

$$\begin{aligned} W(F(G_1)\ o\ G_2) &= |G_1|W(G_2) + W(G_1)(|G_2|^2 + 2|G_1| - 3) + ((|G_1|^2 - |G_1|)|G_2| + |G_1|^2)d(r|G_2) \\ &+ W(F(G_1)) + |G_1|(|G_2| - 1). \end{aligned}$$

Proof. It is a special case of Theorem 2. We have $F(G_1)\ o\ G_2 \equiv F(G_1)\{G_2 + K_1\}$, where K_1 is a one vertex graph and where the root of $G_2 + K_1$ is chosen to be the vertex belonging to $V(K_1)$. □

4. Conclusion

In this paper, we have introduced the new type of corona and cluster operations of two graphs. Further, we have studied the results related to the Wiener index of the said graph operations.

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