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STRONG CONVERGENCE OF THREE STEP ITERATION PROCESS FOR NONEXPANSIVE AND STRONGLY PSEUDOCONTRACTIVE MAPPINGS

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Abstract. In this paper, we introduce a three step iteration process and prove strong convergence theorem for finding the common fixed point associated with nonexpansive and strongly pseudocontractive mappings in real uniformly smooth Banach space. A numerical example is given in support of our result. We remark that the iteration process of Kang et al. [16] can be obtained as a particular case of our iteration process. In our result the necessity of condition (C) is not require to prove strong convergence.

1. Introduction

Let E be a real uniformly smooth Banach space and let K be a nonempty convex subset of E. The mapping $T : K \to K$ is said to be nonexpansive if $\|Tx-Ty\| \leq \|x-y\|$ $\forall x, y \in K$. Let J denote the normalized duality mapping from E to 2^{E^*} defined by

$$J(x) = \{f^* \in E^* : \langle x, f^*\rangle = \|x\|^2, \|f^*\| = \|x\|\},\ \forall x, y \in E \tag{1}$$

where E^* denotes the dual space of E and $\langle ., .\rangle$ denotes the generalized duality pairing. We will denote the single-valued duality map by j.

- The mapping T is said to be pseudocontractive if

$$\|x - y\| \leq \|x - y + t\,((I - T)x - (I - T)y)\,\|\ \forall x, y \in K \text{ and } t > 0 \tag{2}$$

According to Kato [14], T is pseudocontractive if and only if there exists $j(x - y) \in J(x - y)$ such that

$$\langle Tx - Ty, j(x - y)\rangle \leq \|x - y\|^2\ \forall x, y \in K \tag{3}$$

- The mapping T is said to be strongly pseudocontractive if there exists a constant $t > 1$ such that

$$\|x - y\| \leq \|(1 + r)(x - y) - rt(Tx - Ty)\|\ \forall x, y \in K \text{ and } r > 0. \tag{4}$$

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Or equivalently (see [18]) one has for $0 < k < 1$,

$$\langle Tx - Ty, j(x-y)\rangle \leq (1-k)\|x-y\|^2 \ \forall x, y \in K. \tag{5}$$

Whenever the existence of fixed point of a given mapping is established, then an iteration procedure is required to converge the fixed point of the mapping. In 1953, Mann [17] introduced the following iteration process.

Let K be a nonempty convex subset of E and $T : K \to K$ is a mapping. Then the sequence $\{x_n\}$ defined by

$$\begin{cases} x_0 \in K, \\ x_{n+1} = (1-\alpha_n)x_n + \alpha_n T x_n, \ n \geq 0, \end{cases} \tag{6}$$

where $\{\alpha_n\}$ is a sequence in [0,1].

In 1974, S. Ishikawa [12] introduced the following iteration process which is known as Ishikawa iteration process defined by

$$\begin{cases} x_0 \in K, \\ y_n = (1-\beta_n)x_n + \beta_n T x_n, \\ x_{n+1} = (1-\alpha_n)x_n + \alpha_n T y_n, \ n \geq 0, \end{cases} \tag{7}$$

where $\{\alpha_n\}$ and $\{\beta_n\}$ are sequences in $[0, 1]$ and proved the following theorem.

Theorem 1.1. [12] Let K be a compact convex subset of a Hilbert space H and let $T : K \to K$ be a Lipschitzian pseudocontractive mapping. For arbitrary $x_1 \in K$, let $\{x_n\}$ be a sequence defined iteratively by (7) satisfying

(i) $0 < \alpha_n < 1$;

(ii) $\lim_{n\to\infty} \beta_n = 0$;

(iii) $\Sigma_{n=1}^{\infty} \alpha_n \beta_n = \infty$.

Then the sequence $\{x_n\}$ converges strongly to a fixed point of T.

It is well known that the Mann iteration process does not converge always to fixed point of Lipschitz pseudocontractive mapping therefore the Ishikawa iteration process is significantly desirable for these type of mappings. A detail information regarding the convergence of Mann iteration process of such mapping is given in [5]. In a paper Rashwan [20] studied the convergence of Mann iterates to a common fixed point for a pair of mappings in normed space. Many papers have been published on the iterative approximation of fixed points of Lipschitz strongly pseudocontractive mappings using the Ishikawa iteration process (see, e.g., [3, 12, 13, 19, 24, 28, 31] and the references cited therein).

New iterative techniques for approximation of fixed points of Lipschitz pseudocontractive mapping have been studied independently by the authors Bruck [2], Chidume [6], Chidume and Zegeye [7], Schu [23], Zhou [32], Zhange and Su [33] in Hilbert space, uniformly smooth Banach space, refexive real Banach space. Noor et al. [18] obtained strong convergence result for strongly pseudocontractive mapping in real uniformly smooth Banach space using three-step iteration process. In a paper Rhoades and Soltuz [22] studied that Mann and Ishikawa iteration schemes are equivalent to a multi-step iteration scheme for various classes of the operators.

The following S-iteration process is given by Sahu et al. [26, 27]. The sequence $\{x_n\}$ defined by

$$\begin{cases} x_1 \in K, \\ x_{n+1} = T y_n, \\ y_n = (1-\beta_n)x_n + \beta_n T x_n, \ n \geq 1, \end{cases} \tag{8}$$

where $\{\beta_n\}$ is a sequence in $[0,1]$. In 2013, Kang et al. [16] proved the following theorem.

Theorem 1.2. [16] Let K be a nonempty closed convex subset of a real Banach space E, let $S : K \to K$ be a nonexpansive mapping and let $T : K \to K$ be a Lipschitz strongly pseudocontractive mapping such that $p \in F(S) \cap F(T) = \{x \in K : Sx = Tx = x\}$ and

$$\|x - Sy\| \leq \|Sx - Sy\|, \ \|x - Ty\| \leq \|Tx - Ty\|, \ \forall x, y \in K \tag{C}$$

Let $\{\beta_n\}$ be a sequence in [0,1] satisfying
(i) $\Sigma_{n=1}^{\infty}\beta_n = \infty$;
(ii) $\lim_{n\to\infty} \beta_n = 0$.
For arbitrary $x_1 \in K$, let $\{x_n\}$ be a sequence iteratively defined by

$$\begin{cases} x_{n+1} = Sy_n, \\ y_n = (1-\beta_n)x_n + \beta_n Tx_n, \ n \geq 1 \end{cases} \tag{9}$$

Then the sequence $\{x_n\}$ converges strongly to a common fixed point p of S and T.

For approximation of fixed point many iteration process involving multi-step have been discussed by the researchers for various classes of nonlinear operators (see [8, 15, 21, 25, 30]). In many respects it is well known that the approximation of fixed point a three step iteration process is better than a two and single step iteration process under suitable conditions (see [1, 10, 11]). Above facts inspired us to introduce a three step iterative process as follows.

For arbitrary $x_0 \in K$, $\{x_n\}$ be a sequence iteratively defined by

$$\begin{cases} x_0 \in K, \\ x_{n+1} = Sy_n, \\ y_n = (1-\alpha_n)x_n + \alpha_n Tz_n, \\ z_n = (1-\beta_n)x_n + \beta_n Sx_n, \ n \geq 0 \end{cases} \tag{10}$$

and prove strong convergence theorem for finding the common fixed point associated with nonexpansive and strongly pseudocontractive mappings in real uniformly smooth Banach space.

2. Main Results

We will need the following lemmas.

Lemma 2.1. [4, 29] Let $J : E \to 2^{E^*}$ be the normalized duality mapping. Then for any $x, y \in E$,

$$\|x+y\|^2 \leq \|x\|^2 + 2\langle y, j(x+y)\rangle, \ \forall j(x+y) \in J(x+y).$$

Lemma 2.2. [28] Let $\{\rho_n\}$ and $\{\theta_n\}$ be nonnegative sequence satisfying

$$\rho_{n+1} \leq (1-\theta_n)\rho_n + b_n,$$

where $\theta_n \in [0,1)$, $\Sigma_{n=1}^{\infty}\theta_n = \infty$ and $b_n = o(\theta_n)$, Then $\lim_{n\to\infty} \rho_n = 0$.

It is well known that the existence of a fixed point for strongly psedocontractive mapping follows from Deimling [9] and the set of fixed points for strongly pseudocontractive mapping is a singleton (see [31]). Now we will prove our main result.

Theorem 2.3. Let K be a nonempty closed convex subset of a real uniformly smooth Banach space E, let $S : K \to K$ be a nonexpansive mapping and $T : K \to K$ be a strongly pseudocontractive mapping with bounded range such that

$p \in F(S) \cap F(T) = \{x \in K : Sx = Tx = x\}$. Let $\{\alpha_n\}$ and $\{\beta_n\}$ be sequences in [0,1] satisfying
(i) $\lim_{n\to\infty} \alpha_n = 0$,
(ii) $\lim_{n\to\infty} \beta_n = 0$,
(iii) $\sum_{n=0}^{\infty} \alpha_n = \infty$.
For arbitrary $x_0 \in K$, let $\{x_n\}$ be a sequence defined by (10). Then the sequence $\{x_n\}$ converges strongly to a common fixed point p of S and T.
Proof. Let $p \in F(S) \cap F(T)$. Since the mapping T has bounded range, let

$$d_1 = \sup\{\|Tx - Ty\| : x, y \in E\} \tag{11}$$

Then $d_1 < \infty$. We claim that $\|x_n - p\| \leq d_1 + \|x_0 - p\|,\ \forall n \geq 0,\ n \in \mathbb{Z}$.
For $n = 0$, the inequality is true. Suppose the inequality is true for $n = k$, then we will prove for $n = k + 1$.
Using equations (10) and (11), we get

$$\begin{aligned}
\|x_{k+1} - p\| &= \|Sy_k - p\| \\
&= \|Sy_k - Sp\| \\
&\leq \|y_k - p\| \\
&\leq \|(1 - \alpha_k)x_k + \alpha_k Tz_k - p\| \\
&\leq \|(1 - \alpha_k)(x_k - p) + \alpha_n(Tz_k - p)\| \\
&\leq \|(1 - \alpha_k)(x_k - p) + \alpha_n(Tz_k - Tp)\| \\
&\leq (1 - \alpha_k)\|x_k - p\| + \alpha_k\|Tz_k - Tp\| \\
&\leq (1 - \alpha_k)[d_1 + \|x_0 - p\|] + \alpha_k d_1 \\
&\leq d_1 + (1 - \alpha_k)\|x_0 - p\| \\
&\leq d_1 + \|x_0 - p\|
\end{aligned}$$

This implies that $\{\|x_n - p\|\}$ is bounded.
Let $M_1 = d_1 + \|x_0 - p\|$, then $\|x_n - p\| \leq M_1 < \infty\ \forall n \in \mathbb{Z},\ n \geq 0$.
Since,

$$\begin{aligned}
\|x_n - y_n\| &= \|x_n - (1 - \alpha_n)x_n - \alpha_n Tz_n\| \\
&= \alpha_n\|x_n - Tz_n\| \\
&\leq \alpha_n\|x_n - p\| + \alpha_n\|p - Tz_n\| \\
&\leq \alpha_n\|x_n - p\| + \alpha_n\|Tp - Tz_n\| \\
&\leq \alpha_n M_1 + \alpha_n d_1 \\
&\leq \alpha_n(M_1 + d_1) \\
&\to 0 \text{ as } n \to \infty
\end{aligned}$$

This implies that $\{\|x_n - y_n\|\}$ is bounded.
Since,

$$\|y_n - p\| \leq \|x_n - p\| + \|x_n - y_n\|$$

Therefore $\{\|y_n - p\|\}$ is bounded.
Since,

$$\begin{aligned}\|z_n - p\| &= \|(1-\beta_n)x_n + \beta_n S x_n - p\| \\ &\le (1-\beta_n)\|x_n - p\| + \beta_n\|Sx_n - p\| \\ &\le (1-\beta_n)\|x_n - p\| + \beta_n\|Sx_n - Sp\| \\ &\le (1-\beta_n)\|x_n - p\| + \beta_n\|x_n - p\| \\ &\le \|x_n - p\| \\ &\le M_1\end{aligned}$$

This implies that $\{\|z_n - p\|\}$ is bounded.
Let

$$\begin{aligned}d_2 &= \sup\{\|x_n - p\| : n \in \mathbb{Z},\ n \ge 0\} \\ d_3 &= \sup\{\|y_n - p\| : n \in \mathbb{Z},\ n \ge 0\} \\ d_4 &= \sup\{\|z_n - p\| : n \in \mathbb{Z},\ n \ge 0\}\end{aligned}$$

Denote $M = M_1 + d_2 + d_3 + d_4$, then $M < \infty$.
Using equations (10), (5) and Lemma 2.1, we have

$$\begin{aligned}\|y_n - p\|^2 &= \|(1-\alpha_n)x_n + \alpha_n T z_n - p\|^2 \\ &\le \|(1-\alpha_n)(x_n - p) + \alpha_n(Tz_n - p)\|^2 \\ &\le \|(1-\alpha_n)(x_n - p) + \alpha_n(Tz_n - Tp)\|^2 \\ &\le (1-\alpha_n)^2\|x_n - p\|^2 + 2\alpha_n(1-\alpha_n)\langle Tz_n - Tp, j(x_n - p)\rangle \\ &\le (1-\alpha_n)^2\|x_n - p\|^2 + 2\alpha_n(1-\alpha_n)\langle Tz_n - Tp, j(z_n - p)\rangle \\ &\quad + 2\alpha_n(1-\alpha_n)\langle Tz_n - Tp, j(x_n - p) - j(z_n - p)\rangle \\ &\le (1-\alpha_n)^2\|x_n - p\|^2 + 2\alpha_n(1-\alpha_n)(1-k)\|z_n - p\|^2 \\ &\quad + 2\alpha_n(1-\alpha_n)\langle Tz_n - Tp, j(x_n - z_n)\rangle \\ &\le (1-\alpha_n)^2\|x_n - p\|^2 + 2\alpha_n(1-\alpha_n)(1-k)\|z_n - p\|^2 \\ &\quad + 2\alpha_n(1-\alpha_n)\|Tz_n - Tp\|\|x_n - z_n\| \qquad (12)\end{aligned}$$

Also,

$$\begin{aligned}\|x_n - z_n\| &= \|x_n - (1-\beta_n)x_n - \beta_n S x_n\| \\ &= \beta_n\|x_n - Sx_n\| \\ &\le \beta_n\|x_n - p\| + \beta_n\|p - Sx_n\| \\ &\le \beta_n\|x_n - p\| + \beta_n\|Sx_n - Sp\| \\ &\le \beta_n\|x_n - p\| + \beta_n\|x_n - p\| \\ &\le 2\beta_n\|x_n - p\| \\ &\le 2\beta_n M \qquad (13) \\ &\to 0 \text{ as } n \to \infty\end{aligned}$$

Using equation (10) and Lemma 2.1, we have

$$\begin{aligned}
\|z_n - p\|^2 &= \|(1-\beta_n)x_n + \beta_n Sx_n - p\|^2 \\
&= \|(1-\beta_n)(x_n - p) + \beta_n(Sx_n - p)\|^2 \\
&= \|(1-\beta_n)(x_n - p) + \beta_n(Sx_n - Sp)\|^2 \\
&\leq (1-\beta_n)^2\|x_n - p\|^2 + 2\beta_n(1-\beta_n)\langle Sx_n - Sp, j(x_n - p)\rangle \\
&\leq (1-\beta_n)^2\|x_n - p\|^2 + 2\beta_n(1-\beta_n)\|Sx_n - Sp\|\|x_n - p\| \\
&\leq (1-\beta_n)^2\|x_n - p\|^2 + 2\beta_n(1-\beta_n)\|x_n - p\|^2 \\
&\leq (1-\beta_n^2)\|x_n - p\|^2 \qquad (14)
\end{aligned}$$

Using equations (11), (13) and (14) in (12), we get

$$\begin{aligned}
\|y_n - p\|^2 &\leq (1-\alpha_n)^2\|x_n - p\|^2 + 2\alpha_n(1-\alpha_n)(1-k)(1-\beta_n^2)\|x_n - p\|^2 \\
&\quad + 4\alpha_n(1-\alpha_n)d_1\beta_n M \\
&\leq [(1-\alpha_n)^2 + 2\alpha_n(1-\alpha_n)(1-k)(1-\beta_n^2)]\|x_n - p\|^2 \\
&\quad + 4\alpha_n(1-\alpha_n)d_1\beta_n M
\end{aligned} \qquad (15)$$

Using equation (10) and (15), we get

$$\begin{aligned}
\|x_{n+1} - p\|^2 &= \|Sy_n - p\|^2 \\
&= \|Sy_n - Sp\|^2 \\
&\leq \|y_n - p\|^2 \\
&\leq [(1-\alpha_n)^2 + 2\alpha_n(1-\alpha_n)(1-k)(1-\beta_n^2)]\|x_n - p\|^2 \\
&\quad + 4\alpha_n(1-\alpha_n)d_1\beta_n M \qquad (16)
\end{aligned}$$

Since $k \in (0,1)$, $\alpha_n \in [0,1]$ and $\lim_{n\to\infty} \alpha_n = 0$, $\exists$ a natural number $N \in \mathbb{N}$ and a constant C such that for all $n \in \mathbb{N}$, $n \geq N$

$$2k + \alpha_n(1-2k) \geq C > 0$$

Hence,

$$[2k + \alpha_n(1-2k)]\alpha_n \geq C\alpha_n > 0$$

This implies

$$1 - [2k + \alpha_n(1-2k)]\alpha_n \leq 1 - C\alpha_n$$

Thus for all $n \in \mathbb{N}$, $n \geq N$, we have

$$\begin{aligned}
(1-\alpha_n)^2 + 2\alpha_n(1-\alpha_n)(1-k)(1-\beta_n^2) &\leq (1-\alpha_n)^2 + 2\alpha_n(1-\alpha_n)(1-k) \\
&\leq 1 + \alpha_n^2 - 2\alpha_n + 2\alpha_n - 2\alpha_n^2 - (2\alpha_n - 2\alpha_n^2)k \\
&\leq 1 - \alpha_n^2 - 2\alpha_n(1-\alpha_n)k \\
&\leq 1 - \alpha_n[2k + \alpha_n(1-2k)] \\
&\leq 1 - \alpha_n C \qquad (17)
\end{aligned}$$

Thus for all $n \in \mathbb{N},\ n \geq N$, we have

$$\|x_{n+1} - p\|^2 \leq (1-\alpha_n C)\|x_n - p\|^2 + 4\alpha_n(1-\alpha_n)d_1\beta_n M \qquad (18)$$

For all $n \geq 1$, taking

$$\rho_n = \|x_n - p\|$$
$$\theta_n = \alpha_n C$$
$$b_n = 4\alpha_n(1-\alpha_n)d_1\beta_n M$$

Using Lemma 2.2, we obtain from equation (18) that

$$\lim_{n\to\infty} \|x_n - p\| = 0.$$

This completes the proof.
Taking $S = I$ in Theorem 2.3 (I is identity mapping), we obtain the following corollary.
Corollary 2.4. Let K be a nonempty closed convex subset of a real uniformly smooth Banach space E and $T : K \to K$ be a strongly pseudocontractive mapping with bounded range. Let $\{\alpha_n\}$ be a sequence in [0,1] satisfying
(i) $\lim_{n\to\infty} \alpha_n = 0$,
(ii) $\sum_{n=0}^{\infty} \alpha_n = \infty$.
For arbitrary $x_0 \in K$, let $\{x_n\}$ be a sequence defined by

$$x_{n+1} = (1-\alpha_n)x_n + \alpha_n T x_n,\ n \geq 0.$$

Then the sequence $\{x_n\}$ converges strongly to fixed point of T.
Example 2.5. Let $E = \mathbb{R}$, $K = [-2, 2]$ and $S,\ T : K \to K$ be mappings defined by

$$Sx = \begin{cases} x, & x \in [0,2] \\ -x, & x \in [-2,0) \end{cases}$$

and

$$Tx = \begin{cases} 1, & x \in (-2,-1) \\ \sqrt{1-(1+x)^2}, & x \in [-1,0) \\ -\sqrt{1-(x-1)^2}, & x \in [0,1] \\ -1, & x \in (1,2) \end{cases}$$

Clearly S is nonexpansive mapping and 0 is the fixed point of S. Also by a simple calculation one can see that T is strongly pseudocontractive mapping with bounded range and 0 is the fixed point of T. Thus 0 is the common fixed point of mappings S and T.

Let us consider the parameters $\alpha_n = \frac{1}{n+1}$ and $\beta_n = \frac{n}{n^2+1}$ for $n = 0, 1, 2, ...,$ then the conditions (i) to (iii) of Theorem 2.3 are satisfied by the parameters. The following Table shows that the sequence $\{x_n\}$ generated by our three step iteration process converges to the common fixed point 0 of S and T for arbitrary initial values of $x_0 \in K$.

x_0	0.1	-0.1	-1
x_1	0.43588	0.43588	1
x_2	0.43588	0.43588	0
x_3	0.01536	0.01536	0
x_4	0.03212	0.03212	0
x_5	0.02459	0.02459	0
x_6	0.01624	0.01624	0
x_7	0.01171	0.01171	0
x_8	0.00883	0.00883	0
x_9	0.00688	0.00688	0
x_{10}	0.00552	0.00552	0
x_{11}	0.00452	0.00452	0
x_{12}	0.00377	0.00377	0
x_{13}	0.00319	0.00319	0
x_{14}	0.00274	0.00274	0
x_{15}	0.00238	0.00238	0
x_{16}	0.00207	0.00207	0
x_{17}	0.00184	0.00184	0
x_{18}	0.00163	0.00163	0
x_{19}	0.00146	0.00146	0
x_{20}	0.00132	0.00132	0
x_{21}	0.00119	0.00119	0
x_{22}	0.00108	0.00108	0
x_{23}	0.00098	0.00098	0
x_{24}	0.00091	0.00091	0
x_{25}	0.00083	0.00083	0
x_{26}	0.00077	0.00077	0
x_{27}	0.00071	0.00071	0
x_{28}	0.00066	0.00066	0
...	...	...	...
x_{60}	0.00014	0.00014	0

Remark 2.6. Our result improves the result of Kang et al. [16] in the following ways:

(i) If $\beta_n = 0$ in iteration process (10), then it reduces to the iteration process (9).

(ii) The condition (C) is not require to prove the strong convergence.

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