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ANTI VAGUE SOFT R-SUBGROUP OF NEAR-RING

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ABSTRACT. The concepts of R-subgroup and fuzzy R-subgroup together are used to develop the theory of an anti vague soft R-subgroup. This work mostly focuses on study of R-subgroups of soft sets. In this paper we have defined Anti vague soft R-subgroup of near- ring. We have given its examples and non-examples. Some of its properties are stated. Also, we have defined 0-normal vague soft set and one vague soft set $(\hat{F}, A)^+$. We also have defined 1-normal vague soft set. Properties of 0-normal and 1-normal vague soft sets are also discussed.

1. INTRODUCTION

Fuzzy set along with some of its applications in real life introduced by L. A. Zadeh[7]. Since then fuzzy mathematics is rapidly growing branch of mathematics. According to Zadeh, Fuzzy set assigns a number in $[0, 1]$ to each element of universe set X. By applying concepts of algebra on fuzzy sets, we get most interesting results such as fuzzy group, fuzzy rings and ideals, fuzzy relation and functions, fuzzy cosets and so on. The definition of ring is modified by G. Pilz [11]to invent near-ring and it is found that every ring is a near ring. So, near-rings are more generalized form of a rings. Researchers defined fuzzy near- ring and carried out results parallel to fuzzy ring. Kyung Ho Kim, Young Bae Jun [3]-[6] stated fuzzy R-subgroup of near-ring. Same pair of scientists produce another paper Anti fuzzy R-subgroups of near-ring. They discussed about examples of Anti fuzzy R-subgroups of near ring, fuzzy image and pre-images, sub property and results based on these concepts. Kyung Ho Kim, Young Bae Jun together discussed about normalization of fuzzy R-subgroup. Many results corresponding to concepts stated briefly.

The concept of vague set theory was first initiated by Gau W. L. and Bueher D. J. [10] in 1993. In a vague set V every element of universal set X linked with two membership t_V function and false membership function fV with $t_V + f_V \leq 1$. Vague value of x in X in an interval $[t_V(x), 1 - f_V(x)]$. A fuzzy set contains only one membership function but vague sets contains two membership functions. So vague sets are very useful than fuzzy sets to solve problems on uncertainties. In the world of set theory D. Molodstov [2] introduced soft set theory which is completely new

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approach. Then many scientists contributed to combine fuzzy set with soft set and vague set with soft set. Wei Xu, Jain Ma, Shouyang Wang, Gang Hao [9] developed operations on vague soft set which are very useful for further calculations.

This paper is organized in three sections. In Section 2 we collect some basic definitions as per need of this paper. Section 3 consist of Anti vague soft R-subgroup and its properties. Also, in section 4 we discussed about normality conditions on anti vague soft R-subgroup of near -ring. Section 5 concludes the paper.

2. Preliminaries

In this section we have given few definitions which are useful for further reading.
Definition 1 [11] A non-empty set R is said to be a left near-ring if
i) $(R, +)$ is a group,
ii) $(R,)$ is a semi-group,
iii) $x(y + z) = xy + xz$ for all $x, y, z \in R$.
If condition (iii) is replaced by right distributive law then it is called right near-ring. Through this paper we will use the word near-ring instead of left near-ring. We denote xy for xy.

Definition 2 [11] A two-sided R-subgroup over a near-ring R is a subset H of R such that
i) $(H, +)$ is a subgroup of $(R, +)$,
ii) $RH \subset H$,
iii)$HR \subset H$.
If H satisfies (i) and (ii) then it is called a left R-subgroup over R. If H satisfies (i) and (iii) then it is called a right R-subgroup over R.

Definition 3 Let U be an initial universe set and E be the set of parameters. Let A be a subset of E. Let P(U) denote the power set of U. A pair (F, A) is called a soft set over U, where F is a mapping given by $F \colon A \to P(U)$. We use $F(a)$ to denote the elements of the soft set.

Definition 4 (F, A) is called a soft R-subgroup of R if for each $a \in A$ following conditions holds:
i) $x - y \in F(a)$,
ii) $rx \in F(a)$,
iii)$xr \in F(a)$. for all $x, y \in F(a)$ and $r \in R$.

Definition 5 [1] Let (F, A) be a soft set over a group G. Then (F, A) is called a normal soft group over G if $F(a)$ is a normal subgroup of G for all $a \in A$.

Definition 6 [9] Let U be universal set, $U = \{u_1, u_2, u_3, \cdots u_n, \cdots\}$. A vague set over U is characterized by a truth-membership function t_v and a false-membership function f_v, $t_v : U \to [0, 1]$, $f_v : U \to [0, 1]$, where $t_v(u_i)$ is lower bound on a grade of membership function u_i derived from evidence of u_i, $f_v(u_i)$ is upper bound on the negation of u_i derived from evidence against $u_i$and $t_v(u_i) + f_v(u_i) \leq 1$. The grade membership of u_i in the vague set is bounded to a subinterval $[t_v(u_i), 1 - f_v(u_i)]$ of $[0, 1]$. The vague value $[t_v(u_i), 1 - f_v(u_i)]$ indicates that the exact grade of membership $\mu_v(u_i)$ of u_i may be unknown, but it is bounded by $t_v(u_i) \leq \mu_v(u_i) \leq 1 - f_v(u_i)$.

Definition 7 [9] A pair $(\hat{F}, A)$ is called vague soft set of U where $\hat{F}$ is a mapping given by $\hat{F} : A \to V(U)$ and V(U) is the power set of vague sets. So, vague soft set can be represented as;
$$(\hat{F}, A) = \left\{\hat{F}(a)|for\ each\ a \in A\right\} = \left\{[t_{\hat{F}_a}(u_i), 1 - f_{\hat{F}_a}(u_i)]\ ui; where\ u_i \in U\right\}$$

Definition 8 [9] For each $u_i \in U$ and $a \in A$ if we have $\hat{F}_a(u_i) = [0,0]$ then $(\hat{F}, A)$ is called a zero vague soft set.

Definition 9 [9] For each $u_i \in U$ and $a \in A$ if we have $\hat{F}_a(u_i) = [1,1]$ then $(\hat{F}, A)$ is called a absolute vague soft set.

Definition 10 [8] The (α, β)-cut of a vague soft set $(\hat{F}, A)$ of U is denoted as $(\hat{F}, A)_{(\alpha,\beta)}$ defined as:

$$(\hat{F}, A)_{(\alpha,\beta)} = \Big\{x \in U | t_{\hat{F}_a}(x) \geq \alpha, 1 - f_{\hat{F}_a}(x) \geq \beta, i.e. \hat{F}_a(x) \geq [\alpha, \beta]\Big\}$$

Where $\alpha, \beta \in [0,1]$ and $\alpha \geq \beta$. If $\alpha = \beta$ then it is written as $(\hat{F}, A)_\alpha$.

3. Anti Vague Soft R-Subgroup

In this section, we are defined anti vague soft R-subgroup of near-ring. Its examples and non-examples are provided. Some of its basic properties are investigated.

Definition 1 Anti right vague soft R-subgroup

Let $(R, +)$ be a near-ring, E be the set of parameters and $A \subset E$. let $(\hat{F}, A)$ be a vague soft set over R. Then $(\hat{F}, A)$ is called a Anti right vague soft R-subgroup of R if for each $a \in A$, following condition holds:

i) $\hat{F}_a(x+y) \leq \max\Big(\hat{F}_a(x), \hat{F}_a(y)\Big)$,

i.e $t_{\hat{F}_a}(x+y) \leq \max\Big(t_{\hat{F}_a}(x), t_{\hat{F}_a}(y)\Big)$ and $1 - f_{\hat{F}_a}(x+y) \leq \max\Big(1 - f_{\hat{F}_a}(x), 1 - f_{\hat{F}_a}(y)\Big)$

ii) $\hat{F}_a(xr) \leq \hat{F}_a(x)$,

i.e $t_{\hat{F}_a}(xr) \leq t_{\hat{F}_a}(x)$ and $1 - f_{\hat{F}_a}(xr) \leq 1 - f_{\hat{F}_a}(x)$

Definition 2 Anti left vague soft R-subgroup

Let $(R, +)$ be a near-ring, E be the set of parameters and $A \subset E$. let $(\hat{F}, A)$ be a vague soft set over R. Then $(\hat{F}, A)$ is called a Anti left vague soft R-subgroup of R if for each $a \in A$, following condition holds:

i) $\hat{F}_a(x+y) \leq \max\Big(\hat{F}_a(x), \hat{F}_a(y)\Big)$,

i.e $t_{\hat{F}_a}(x+y) \leq \max\Big(t_{\hat{F}_a}(x), t_{\hat{F}_a}(y)\Big)$ and $1 - f_{\hat{F}_a}(x+y) \leq \max\Big(1 - f_{\hat{F}_a}(x), 1 - f_{\hat{F}_a}(y)\Big)$

ii)$\hat{F}_a(rx) \leq \hat{F}_a(x)$,

i.e $t_{\hat{F}_a}(rx) \leq t_{\hat{F}_a}(x)$ and $1 - f_{\hat{F}_a}(rx) \leq 1 - f_{\hat{F}_a}(x)$.

Definition 3 The Anti left and right vague soft R-subgroup of R together constitute Anti vague soft R-subgroup of R. Thus $(\hat{F}, A)$ is called a Anti vague soft R-subgroup of R if

i) $\hat{F}_a(x+y) \leq \max\Big(\hat{F}_a(x), \hat{F}_a(y)\Big)$,

ii)$\hat{F}_a(rx) \leq \hat{F}_a(x)$ and $\hat{F}_a(xr) \leq \hat{F}_a(x)$ for every $x, y, r \in R$.

Example 1 Consider the near ring $R = \{a, b, c, d\}$ with two binary operations as defined in following table,

+	0	a	b	c
0	0	a	b	c
a	a	0	c	b
b	b	c	0	a
c	c	b	a	0

.	0	a	b	c
0	0	0	0	0
a	a	a	a	a
b	0	0	0	b
c	a	a	a	c

Let$A = e_1, e_2, e_3$ be a subset of set of parameters. Consider a vague soft set $(\hat{F}, A)$ of a near ring R given by, $(\hat{F}, A) = \left\{\hat{F}(e_1), \hat{F}(e_2), \hat{F}(e_3)\right\}$ where,

$$\hat{F}(e_1) = \{[0.3, 0.6]/a, [0.5, 0.6]/b, [0.7, 1]/c, [0.7, 1]/d\}$$

$$\hat{F}(e_2) = \{[0.2, 0.4]/a, [0.5, 0.5]/b, [0.5, 0.5]/c, [0.5, 0.5]/d\}$$

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$$\hat{F}(e_2) = \{[0, 0]/a, [0.3, 0.8]/b, [1, 1]/c, [1, 1]/d\}$$

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Then $(\hat{F}, A)$ is Anti right vague soft R-subgroup but not Anti left vague soft R-subgroup. Because it fails to satisfy $\hat{F}_e(bc) \leq \hat{F}_e(b)$ and $\hat{F}_e(bd) \leq \hat{F}_e(d)$ for $e \in A$.

Example 2

Consider the near ring $K = \{a, b, c, d\}$ with two binary operations as defined in following table,

+	0	a	b	c
0	0	a	b	c
a	a	0	c	b
b	b	c	0	a
c	c	b	a	0

.	0	a	b	c
0	0	0	0	0
a	a	a	a	a
b	0	0	0	b
c	a	a	a	c

Then above vague soft set will form the Anti Vague K-subgroup of near-ring K.

Theorem 1 Let$(\hat{F}, A)$ is a Anti left (right) vague soft R-subgroup of a near-ring R. Then
i) $\hat{F}_a(0) \leq \hat{F}_a(x)$,
ii) $\hat{F}_a(-x) \leq \hat{F}_a(x)$

proof. Let $(\hat{F}, A)$ be a Anti left (right) vague soft R-subgroup of R. By its property,
$\hat{F}_a(x - x) \leq \max\left\{\hat{F}_a(x), \hat{F}_a(x)\right\} = \hat{F}_a(x)$. Hence $\hat{F}_a(0) \leq \hat{F}_a(x)$.
Again, we have,

$$\hat{F}_a(0 - x) \leq \max\left\{\hat{F}_a(0), \hat{F}_a(x)\right\}$$

By (i) we get, $\hat{F}_a(-x) \leq \hat{F}_a(x)$.

Theorem 2 If $(\hat{F}, A)$ is Anti left(right) vague soft R-subgroup of near-ring R. Then $\hat{F}_a(rxr) \leq \hat{F}_a(r)\ \forall r, x \in R$.

proof. Let $(\hat{F}, A)$ is Anti left(right) vague soft R-subgroup of R. Let any $x, r \in R$ and $a \in A$ We have to show that, $\hat{F}_a(rxr) \leq \hat{F}_a(x)$. Consider, $\hat{F}_a(rxr) = \hat{F}_a[(rx)r] \leq \hat{F}_a(rx) \leq \hat{F}_a(r)$. Repeatedly using the property of Anti left(right) vague soft R-subgroup, $\hat{F}_a(rxr) = \hat{F}_a[(rx)r] = \hat{F}_a(rx) \leq \hat{F}_a(r)$. Hence

$$\hat{F}_a(rxr) \leq \hat{F}_a(r).$$

Theorem 3 If $(\hat{F}, A)$ is Anti vague soft R-subgroup of near-ring R. Then $\hat{F}_a(rxr) \leq \hat{F}_a(x)\ \forall r, x \in R$.

Definition Let $(\hat{F}, A)$ be a vague soft set of near ring R. For each $a \in A$, we define the set,

$$S(a) = \Big\{x \in R | \hat{F}_a(0) = \hat{F}_a(x)\Big\}$$

. Clearly $S(a) \subseteq R$. Hence (S, A) is a soft set with universe R.

Theorem 4 If $(\hat{F}, A)$ be a Anti vague soft R-subgroup of R, then soft set (S, A) is R-subgroup of R.

proof. Let $(\hat{F}, A)$ be a Anti vague soft R-subgroup of R. let any $a \in A$ and $x, y \in S(a)$. Therefore $\hat{F}_a(x) = \hat{F}_a(y) = \hat{F}_a(0)$. Now consider,
$\hat{F}_a(x - y) \leq \max\Big\{\hat{F}_a(x),\ \hat{F}_a(y)\Big\} = \max\Big\{\hat{F}_a(0),\ \hat{F}_a(0)\Big\} = \hat{F}_a(0)$.
But $\hat{F}_a(0) \leq \hat{F}_a(x - y)$. Hence $\hat{F}_a(0) = \hat{F}_a(x - y)$.
Thus $x - y \in S(a)$, each $S(a)$ is a subgroup of $(R, +)$.
Let any $r \in R$ and $x \in S(a)$. Then $\hat{F}_a(rx) \leq \hat{F}_a(x) = \hat{F}_a(0)$.
We have $\hat{F}_a(0) \leq \hat{F}_a(rx)$.
Hence $\hat{F}_a(rx) = \hat{F}_a(0)$. Thus $rx \in S(a)$. Similarly, $xr \in S(a)$. This shows that (S, A) is soft R-subgroup of R.

Theorem 5 If R is commutative ring with unity. Let $(\hat{F}, A)$ be a anti vague soft R-subgroup. Then for each $a \in A$, $H(a) = \Big\{x \in R | \hat{F}_a(x) \leq \hat{F}_a(1_R)\Big\}$ is soft R-subgroup of R.
proof. As similar to above result.

Theorem 6 For each soft R-subgroup of R, there exists anti vague soft R-subgroup.

proof. Let (H, A) be a soft R-subgroup of R. Then for each $a \in A$ we can construct a vague soft set of R defined as below,

$$\hat{F}_a(x) = \begin{cases} [\gamma/2, 1 - \omega/2] & \text{where } x \in H(a)\ \gamma, \omega \in [0, 1], \gamma + \omega \leq 1 \\ [1, 1] & \text{where } x \notin H(a) \end{cases} \tag{1}$$

Let any $x, y \in R$.
Case1: If $x, y \in H(a)$ then $x - y \in H(a)$.
Therefore $\hat{F}_a(x - y) = \hat{F}_a(x) = \hat{F}_a(y) = [\gamma/2, 1 - \omega/2]$.
Also, if $x \in H(a)$ *and* $y \in R$ then $xy \in H(a)$ and $yx \in H(a)$.
Hence $\hat{F}_a(xy) = \hat{F}_a(yx) = [\gamma/2, 1 - \omega/2]$.

Case 2: If $x, y \notin H(a)$ *then* $x - y \in H(a)$ *or* $x, y \notin H(a)$.
Therefore, $\hat{F}_a(x - y) \leq \max\Big\{\hat{F}_a(x), \hat{F}_a(y)\Big\}$.
For $x \notin H(a)$ and $y \in R$ *we have* $xy \in H(a)$ *or* $xy \notin H(a)$.
$\hat{F}_a(xy) \leq \hat{F}_a(x)$. Similarly, $\hat{F}_a(yx) \leq \hat{F}_a(x)$.
Hence $(\hat{F}, A)$ is Anti vague soft R-subgroup.
This shows that corresponding to each soft R-subgroup we can construct an anti vague soft R-subgroup of near-ring R.

4. NORMALITY CONDITIONS

A near-ring always contains an additive identity. Applying some condition on membership functions corresponding to additive identity in Anti vague soft R-subgroup we get interesting results.

Definition A Anti vague soft left(right) R-subgroup of R is said to be 0-normal if corresponding to each $a \in A$ there exists $x \in R$ such that $\hat{F}_a(x) = [0, 0]$

$$i.e t_{\hat{F}_a}(x) = 0 \text{ and } 1 - f_{\hat{F}_a}(x) = 0 \tag{2}$$

Every zero vague soft set is o-normal.

In example 3.2 if we define $\hat{F}_{e_1}(a) = \hat{F}_{e_2}(a) = \hat{F}_{e_3}(a) = [0, 0]$ then $(\hat{F}, A)$ is 0-normal vague soft set.

Theorem 7 If $(\hat{F}, A)$ is 0-normal vague soft set of near-ring R.Then $\hat{F}_a(0) = [0, 0]$ for each $a \in A$.

proof. Let $(\hat{F}, A)$ is 0-normal vague soft set of near-ring R. Therefore, corresponding to each $a \in A$ exists $x \in R$ such that

$$\hat{F}_a(x) = [0, 0]$$

But we have, $\hat{F}_a(0) \leq \hat{F}_a(x)$

i.e $t_{\hat{F}_a}(x) = 0$ *and* $1 - f_{\hat{F}_a}(x) = 0$

Hence $0 \leq t_{\hat{F}_a}(0) \leq t_{\hat{F}_a}(x) \leq 0$ and $0 \leq 1 - f_{\hat{F}_a}(0) \leq 1 - f_{\hat{F}_a}(x) \leq 0$

This shows that $t_{\hat{F}_a}(0) = 0$ and $1 - f_{\hat{F}_a}(0) = 0$ Thus, we get $\hat{F}_a(0) = [0, 0]$

Theorem 8 If $(\hat{F}, A)$ is 0-normal Anti vague soft R-subgroup of R then $(\hat{F}, A)_0 = R$.

proof. let $(\hat{F}, A)$ be a 0-normal Anti vague soft R-subgroup of R.

We have, $(\hat{F}, A)_0 \subseteq R$. Let any $x \in R$.

Thus $[0, 0] = \hat{F}_a(0) \leq \hat{F}_a(x)$. This implies $x \in (\hat{F}, A)_0$ This gives $R \subseteq (\hat{F}, A)_0$

Hence $(\hat{F}, A)_0 = R$.

Theorem 9 Let $(\hat{F}, A)$ is 0-normal vague soft set of near-ring R. Define the set $S(a) = \Big\{x \in R | \hat{F}_a(x) = [0, 0] \ for\ a \in A\Big\}$. Then (S, A) is normal soft set of $(R, +)$.

proof. Let $(\hat{F}, A)$ is 0-normal vague soft set of near-ring R. Then $\hat{F}_a(0) = [0, 0]$.

To show that (S, A) is normal soft set of $(R, +)$.

Let $a \in A$ and any $x, y \in S(a)$. Hence $\hat{F}_a(x) = \hat{F}_a(y) = [0, 0]$.

$$\begin{aligned} Consider \hat{F}_a(x + y - x) &\leq max \Big\{\hat{F}_a(x + y), \hat{F}_a(x)\Big\} \\ &\leq \max \Big\{\max[\hat{F}_a(x) + \hat{F}_a(y)], \hat{F}_a(x)\Big\} \\ &= \max \Big\{\hat{F}_a(x), \hat{F}_a(y)\Big\} \\ &= [0, 0] \end{aligned}$$

But $[0, 0] \leq \hat{F}_a(x + y - x) \leq [0, 0]$ $\hat{F}_a(x + y - x) = [0, 0]$ This implies $x + yx?S(a)$.

For every $a \in A$, $S(a)$ is normal soft set of $(R, +)$.

Defination Let $(\hat{F}, A)$ be a vague soft set of near- ring R. We define new vague soft set $(\hat{F}, A)^+$ as fallows:

$$t^+_{\hat{F}_a} + (x) = t_{\hat{F}_a}(x) - t_{\hat{F}_a}(0) \textit{ and } f^+_{\hat{F}_a}(x) = 1 + f_{\hat{F}_a}(x) - f_{\hat{F}_a}(0)$$

Clearly,$t^+_{\hat{F}_a}(x) + f^+_{\hat{F}_a}(x) \leq 1$.

Theorem 10 If $(\hat{F}, A)$ is 0-normal vague soft set of near-ring R then $(\hat{F}, A)^+$is also 0-normal vague soft set.

Proof. Let $(\hat{F}, A)$ be a 0-normal vague soft set of near-ring R. Then $(\hat{F}, A)$ is a left (right) R-subgroup of R. let $a \in A$ and $x, y \in R$.

$$\begin{aligned} t^+_{\hat{F}_a}(x-y) &= t_{\hat{F}_a}(x-y) - t_{\hat{F}_a}(0) \\ &\leq \max\Big\{t_{\hat{F}_a}(x), t_{\hat{F}_a}(y)\Big\} - t_{\hat{F}_a}(0) \\ &\leq \max\Big\{t_{\hat{F}_a}(x) - t_{\hat{F}_a}(0), t_{\hat{F}_a}(y) - t_{\hat{F}_a}(0)\Big\} \\ &= \max\Big\{t^+_{\hat{F}_a}(x), t^+_{\hat{F}_a}(y)\Big\} \end{aligned}$$

$t^+_{\hat{F}_a}(xr) = t_{\hat{F}_a}(xr) - t_{\hat{F}_a}(0) \leq t_{\hat{F}_a}(x) - t_{\hat{F}_a}(0) = t^+_{\hat{F}_a}(x)$.
On the similar way we can show that $1 - f^+_{\hat{F}_a}(x-y) \leq \max\Big\{1 - f^+_{\hat{F}_a}(x), 1 - f^+_{\hat{F}_a}(y)\Big\}$ $\textit{and } 1 - f^+_{\hat{F}_a}(xr) \leq 1 - f^+_{\hat{F}_a}(x)$. This shows that $(\hat{F}, A)^+$ is also left(right) R-subgroup of R.
As $(\hat{F}, A)$ is 0- normal,$\hat{F}_a(0) = [0, 0]$ and for $a \in A$ there exist $x \in R$ such that $\hat{F}_a(x) = [0, 0]$.
Now consider $t^+_{\hat{F}_a}(x) = t_{\hat{F}_a}(x) - t_{\hat{F}_a}(0) = t_{\hat{F}_a}(x) - 0 = t_{\hat{F}_a}(x)$ and $f^+_{\hat{F}_a}(x) = 1 + f_{\hat{F}_a}(x) - f_{\hat{F}_a}(0) = f_{\hat{F}_a}(x) + 0 = f_{\hat{F}_a}(x)$. As we can see that for $a \in A$ the same x will also work for $\hat{F}^+_a$.

Defination Let $(\hat{F}, A)$ be a vague soft set of near- ring R. We define new vague soft set $(\hat{F}, A)^*$ as fallows:

$$t^*_{\hat{F}_a}(x) = 1 - t_{\hat{F}_a}(x) \textit{ and } f^*_{\hat{F}_a}(x) = 1 - f_{\hat{F}_a}(x)$$

Clearly $t^*_{\hat{F}_a}(x) + f^*_{\hat{F}_a}(x) \leq 1$.

Theorem 11 If $(\hat{F}, A)$ is 0-normal vague soft set of near-ring R then $\hat{F}^*_a(x) = [1, 1]$ for each $a \in A$ and some $x \in R$.

Proof. Let $(\hat{F}, A)$ be a 0-normal vague soft set of near-ring R and let $a \in A$. So there exist $x?R$ such that, $\hat{F}_a(x) = [0, 0]$ Consider, $t^*_{\hat{F}_a}(x) = 1 - t_{\hat{F}_a}(x) = 1$ And $f^*_{\hat{F}_a}(x) = 1 - f_{\hat{F}_a}(x) = 0$ Hence $1 - f^*_{\hat{F}_a}(x) = 1$.
Therefore $\hat{F}^*_a(x) = [1, 1]$

5. Conclusion

In this paper we have defined Anti vague soft R-subgroup of near-ring. We have given its examples. Some basic theorems are obtained by using definition of Anti vague soft R-subgroup. Also, we have worked to define 0-normal vague soft set.

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