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A NOTE ON UNIQUELY REMOTAL SETS IN 2-BANACH SPACES

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Abstract. The study of the continuity of the farthest point mapping for uniquely remotal sets has been used extensively in the literature to prove the singletoness of such sets. In this paper, we introduce the notion of partial statistical continuity of a function which is way weaker than continuity of a function in 2-Banach space. Consequently, we obtain some generalizations of results concerning the singletoness of remotal sets.

1. Introduction

This paper was inspired by [22], where the singletoness of uniquely remotal sets is given. We will often quote some results from [22], that can be easily transferred to the concept of partial statistical continuity is much weaker than continuity and also weaker than partial continuity by means of a sequence in 2-Banach spaces.

The notion of statistical convergence of sequences of real numbers was introduced by H. Fast [5] using the idea of asymptotic density. Statistical convergence had been discussed and developed by many authors including Fridy [6], Salat [23] and Kolk [15].

The concepts of 2-metric spaces and 2-normed spaces were initially introduced by Gähler [7, 8] in 1960's. This notion which is nothing but a two dimensional analogue of a normed space got the attention of a wider audience after the publication of a paper by Albert George White [29]. Siddiqi delivered a series of lectures on this theme in various conferences. His joint paper with Gähler and Gupta [9] also provided valuable results related to the theme of this paper. Results up to 1977 were summarized in the survey paper by Siddiqi [27]. Since then, many researchers have studied these subjects and obtained various results [10, 11, 12, 13, 16, 24, 26, 28].

The study of remotal and uniquely remotal sets has attracted many mathematicians in the last decades, due to its connection with the geometry of Banach spaces. We refer the reader to [1, 3, 18] and [21] for samples of these studies. However, uniquely remotal sets are of special interest. In fact, one of the most interesting and

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hitherto unsolved problems in the theory of farthest points, known as the the farthest point problem, which is stated as: If every point of a normed space E admits a unique farthest point in a given bounded subset T, then must T be a singleton?. This problem gained its importance when Klee [14] proved that: singletoness of uniquely remotal sets is equivalent to convexity of Chybechev sets in Hilbert spaces (which is an open problem too, in the theory of nearest points). Since then, a considerable work has been done to answer this question, and many partial results have been obtained toward solving this problem. We refer the reader to [1, 3, 18] and [21] on uniquely remotal sets.

This paper consists of three sections with the new results in section 3. In Section 3, the concept of the partial statistical continuity of a function is introduced and an example to show that this notion of partial statistical continuity is much weaker than continuity and also weaker than partial continuity introduced by Sababheh et al. [22]. We prove that if T is a non-empty, bounded, uniquely remotal subset in a real 2-Banach space $\mathbb{E}$ such that T has a Chebyshev center c and the farthest point map $F : \mathbb{E} \to T$ restricted to $[c, f(c)]$ is partially statistically continuous at c then T is singleton.

2. Definitions and Notation

In this section we recall some basic definitions and notations which form the background of present work.

Throughout this paper $\mathbb{N}$ will denote the set of positive integers. If K is a subset of $\mathbb{N}$, the set of natural numbers, then K_n denotes the set $\{k \leq n : k \in K\}$. The natural density of K, denoted by $\delta(K)$ is given by

$$\delta(K) = \lim \frac{1}{n} |\{k \leq n : k \in K\}|$$

whenever the limit exists, where $|K_n|$ denotes the cardinality of the set K_n (see [5]).

Let $(X, \|.\|)$ be a normed space. Recall that a sequence $(x_k)_{k\in\mathbb{N}}$ of elements of X is said to be statistically convergent to $x \in X$ if the set $A(\varepsilon) = \{k \in \mathbb{N} : \|x_k - x\| \geq \varepsilon\}$ has natural density zero for each $\varepsilon > 0$.

Now recall the concept of 2-normed space.

Let X be a real vector space of dimension d, where $2 \leq d < \infty$. A 2-norm on X is a function $\|\cdot,\cdot\| : X \times X \to \mathbb{R}$ which satisfies

(i) $\|x, y\| = 0$ if and only if x and y are linearly dependent.

(ii) $\|x, y\| = \|y, x\|$.

(iii) $\|\alpha x, y\| = |\alpha| \|x, y\|$, $\alpha \in \mathbb{R}$.

(iv) $\|x, y + z\| \leq \|x, y\| + \|x, z\|$.

The pair $(X, \|\cdot,\cdot\|)$ is then called a 2-normed space [7]. As an example of a 2-normed space we may take $X = \mathbb{R}^2$ being equipped with the 2-norm $\|x, y\| :=$ the area of the parallelogram spanned by the vectors x and y, which may be given explicitly by the formula

$$\|x, y\| = |x_1 y_2 - x_2 y_1|, \ x = (x_1, x_2), \ y = (y_1, y_2).$$

Observe that in any 2-normed space $(X, \|\cdot,\cdot\|)$ we have $\|x, y\| \geq 0$ and $\|x, y + \alpha x\| = \|x, y\|$ for all $x, y \in X$ and $\alpha \in \mathbb{R}$. Also, if x, y and z are linearly dependent, then $\|x, y + z\| = \|x, y\| + \|x, z\|$ or $\|x, y - z\| = \|x, y\| + \|x, z\|$.

Let $\{x_n\}_{n\in\mathbb{N}}$ be a sequence in 2-normed space $(X, \|\cdot,\cdot\|)$. The sequence $\{x_n\}_{n\in\mathbb{N}}$ is said to be statistically convergent to l, if $\delta(\{n \in \mathbb{N} : \|x_n - L, z\| \geq \varepsilon\}) = 0$ for every $\varepsilon > 0$ and each nonzero z in X. In this case we write $st\text{-}\lim_{n\to\infty} \|x_n, z\| = \|L, z\|$ (see [10]).

Definition 1 ([29]) A linear 2-normed space in which every Cauchy sequence is a convergent sequence is called a 2-Banach space.

Let (E, d) be a metric space and T be a non empty bounded subset of E. The set-valued map defined by

$$Q_T(x) = \left\{ q_T(x) \in T : d(x, q_T(x)) = \sup_{t\in T} d(x,t) \right\}$$

is called the farthest point map of T. If for any $x \in E$, the set $Q_T(x)$ is not empty (resp. is singleton) then T is said to be remotal (resp. uniquely remotal). If T is uniquely remotal set, then $x \to d(x, q_T(x))$ is uniformly continuous. One can easily verify the continuity of the farthest point map for compact uniquely remotal set, and upper (K)-semi continuity of the farthest point map for closed and remotal subset of E [17]. It is easy to see that the farthest point map of a uniquely remotal subset of a metric space is closed, i.e., if $x_n \to x$ and $q_T(x_n) \to y$, then $y = q_T(x)$. It follows that T is singleton iff there is an element x_0 such that $q_T^n(x_0)$ is convergent. One can see that the map $x \to \|q_T(x)\|$ is continuous for any uniquely remotal subset of a normed space. Thus the following problem poses itself: Can we conclude that the map $x \to q_T(x)$ is continuous? This map is continuous if T is compact. If the farthest point map of a uniquely remotal subset of a Banach space is continuous, then T is singleton (see [4]). Panda and Kapoor used the idea of Chebyshev center and obtained the Blatter result for spaces admitting centers (see [20]). A center (or Chebyshev center) of a bounded, non-empty set T in a normed space E is an element c in E for which

$$\sup_{t\in T} \|c - t\| = \inf_{x\in E} \sup_{t\in T} \|x - t\| .$$

Let E be a real Banach space and T be a nonempty bounded subset of E. The farthest distance of t from the set T is denoted by $D(t, T)$ and is defined by

$$D(t, T) = \sup\{\|x - t\| : t \in T\}.$$

The farthest distance from t to T may or may not attained by some elements of T. We say that T is remotal if for every $x \in E$, there exists $t \in T$ such that $D(x, T) = \|x - t\|$. In this case, the collection of all such points of T is denoted by

$$F(x, T) := \{t \in T : D(x, T) = \|x - t\|\}.$$

It is clear that $F(., T) : E \to T$ is a multi-valued function. However, if $F(., T) : E \to T$ is a single-valued function, then T is called uniquely remotal.

Centers of sets have played a major role in the study of uniquely remotal sets, see [1, 2] and [3]. Recall that a center c of a subset E of a normed space X is an element $c \in X$ such that

$$D(c, E) = \inf_{x\in X} D(x, E) .$$

In [19] it was proved that if E is a uniquely remotal subset of a normed space, admitting a center c, and if F, restricted to the line segment $[c, F(c)]$ is continuous at c, then E is a singleton. Then recently, a generalization has been obtained in [22], where the authors proved the singletoness of uniquely remotal sets if the farthest point mapping F restricted to $[c, F(c)]$ is partially continuous at c.

In this note we show that the uniquely remotal sets which the farthest point mapping F restricted to $[c, F(c)]$ is partially statistically continuous at c is singleton.

3. The Main Results

First, we define the notion of partially continuity and partially statistical continuity in 2-Banach space:

Definition 2 Let $\mathbb{E}$ be a real 2-Banach space and T be a nonempty subset of $\mathbb{E}$. The function $F : T \to \mathbb{E}$ is said to be partially continuous at $t \in \mathbb{E}$ if there exists a non constant sequence $\{x_n\}_{n\in\mathbb{N}} \subset T$ in 2-Banach space $(\mathbb{E}, \|\cdot,\cdot\|)$ such that $x_n \to t$ in $\mathbb{E}$ and $F(x_n) \to F(t)$ as $n \to \infty$ in $\mathbb{E}$.

Definition 3 Let $\mathbb{E}$ be a real 2-Banach space and T be a nonempty subset of $\mathbb{E}$. The function $F : T \to \mathbb{E}$ is said to be partially statistically continuous at $t \in \mathbb{X}$ if there exists a non constant sequence $\{x_n\}_{n\in\mathbb{N}} \subset T$ in 2-Banach space $(\mathbb{E}, \|\cdot,\cdot\|)$ such that $\{x_n\}_{n\in\mathbb{N}}$ is statistically convergent to t in $(\mathbb{E}, \|\cdot,\cdot\|)$ and $\{F(x_n)\}_{n\in\mathbb{N}}$ is statistically convergent to $F(t)$ in $(\mathbb{E}, \|\cdot,\cdot\|)$.

Now we give an example to show that this notion of partial statistical continuity is much weaker than partial continuity.

Example 1 Let $\mathbb{E}$ be a real 2-normed space. A function $f : [-1,0]\times[-1,0] \to \mathbb{R}$ be defined by $f(x) = [x]$, $x = (x_1, x_2) \in [-1,0]\times[-1,0]$. It is easy to check that this function is not partially continuous (also not continuous) at the point $x = (0,0)$. Now we show that this function is partially statistically continuous at the point $x = (0,0)$. Let us define the sequence $\{x_n\}_{n\in\mathbb{N}}$ in $[-1,0]\times[-1,0]$ by

$$x_n = \begin{cases} \left(0, -\frac{1}{n}\right), & \text{if } n = m^2 \text{ for some } m \in \mathbb{N} \\ (0,0), & \text{otherwise} \end{cases},$$

and let $x = (0,0)$ and $z = (z_1, z_2)$. Then for every $\varepsilon > 0$ and nonzero $z \in (\mathbb{E}, \|\cdot,\cdot\|)$

$$\{n \in \mathbb{N} : \|x_k - 0, z\| \geq \varepsilon\} \subset \left\{n \in \mathbb{N} : n = m^2 \text{ for some } m \in \mathbb{N}\right\}.$$

We have that $\delta(\{k \leq n : \|x_k - 0, z\| \geq \varepsilon\}) = 0$, for every $\varepsilon > 0$ and $z \in \mathbb{E}$. This implies that the sequence $\{x_n\}_{n\in\mathbb{N}}$ is statistically convergent to 0 in $\mathbb{E}$. Now the sequence $\{f(x_n)\}_{n\in\mathbb{N}}$ is defined by

$$f(x_n) = \begin{cases} (0,-1), & \text{if } n = m^2 \text{ for some } m \in \mathbb{N} \\ (0,0), & \text{otherwise} \end{cases}.$$

Now it is easy to check that the sequence $\{f(x_n)\}_{n\in\mathbb{N}}$ is statistically convergent to $f(0) = (0,0)$ in $\mathbb{E}$. This shows that this function is partially statistically continuous at the point $x = (0,0)$. So the notion of partial statistical continuity in 2-normed space is much weaker than partial continuity in 2-normed space.

We have the statistical analog of the main result related to singletoness in Sababheh et al. [22].

Theorem 1 Let T be a non-empty, bounded subset of a real 2-Banach space $\mathbb{E}$. If T is uniquely remotal such that T has Chebyshev center c and the farthest point map $F : \mathbb{E} \to T$ restricted to $[c, F(c)]$ is partially statistically continuous at c, then T is singleton.

Proof. Since T is uniquely remotal so for each $x \in \mathbb{E}$ there exists unique $t \in T$

such that $\|x-t,z\| = D(x,T;z)$ and the farthest point map $F:\mathbb{E}\to T$ defined by $F(x)=F(x,T,z)$, $\forall x\in\mathbb{E}$ is well defined. We assume that T has Chebyshev center at $c=0$. Suppose to the contrary, that T is not singleton. Hence, we have $F(0)\neq 0$.

Since $F:\mathbb{E}\to T$, restricted to $[0,F(0)]$, is partially statistically continuous at 0, and by Definition 3, there exists a non-constant sequence $\{x_n\}_{n\in\mathbb{N}}$ in $[0,F(0)]$ such that $\{x_n\}_{n\in\mathbb{N}}$ is statistically convergent to 0 and $\{F(x_n)\}_{n\in\mathbb{N}}$ is statistically convergent to $F(0)$.

So, we have $x_n=\lambda_n F(0)$ for some positive sequence (λ_n) with the property λ_n is statistically convergent to 0. Now, for each $n\in\mathbb{N}$, $0\neq z\in\mathbb{E}$ there exists $\psi_n\in\mathbb{E}^*$, the dual of $\mathbb{E}$, such that

$$\varphi_n\left(F\left(x_n\right)-x_n;z\right)=\left\|F\left(x_n\right)-x_n,z\right\| \text{ and } \left\|\varphi_n,z\right\|=1.$$

Further, we have

$$\begin{aligned}
\varphi_n\left(x_n;z\right) &= \varphi_n\left(F\left(x_n\right);z\right)-\varphi_n\left(F\left(x_n\right)-x_n;z\right)\\
&\leq \left\|\varphi_n,z\right\|\left\|F\left(x_n\right),z\right\|-\left\|F\left(x_n\right)-x_n,z\right\|\\
&= \left\|F\left(x_n\right),z\right\|-\left\|F\left(x_n\right)-x_n,z\right\|\\
&= \left\|F\left(x_n\right)-0,z\right\|-\left\|F\left(x_n\right)-x_n,z\right\|\\
&\leq D(0,T;z)-D(x_n,T;z)\\
&\leq 0,
\end{aligned}$$

where we have used the fact that 0 is a center of T in the last inequality. Thus we have shown that $\varphi_n\left(x_n;z\right)\leq 0$ for every $n\in\mathbb{N}$, $0\neq z\in\mathbb{E}$. So, for every $n\in\mathbb{N}$, $0\neq z\in\mathbb{E}$

$$\varphi_n\left(\lambda_n F(0);z\right)\leq 0\Rightarrow \lambda_n\varphi_n\left(F(0);z\right)\leq 0\Rightarrow \varphi_n\left(F(0);z\right)\leq 0.$$

We have

$$st-\lim\left\|F\left(x_n\right)-x,z\right\|=\left\|F\left(0\right),z\right\|=0$$

for every $0\neq z\in\mathbb{E}$. So, for each $\varepsilon>0$ and each nonzero $z\in\mathbb{E}$,

$$\delta\left(\{n\in\mathbb{N}:\|F(x_k)-x_k-F(0),z\|\geq\varepsilon\}\right)=0.$$

Now we have

$$|\|F(x_k)-x_k,z\|-\|F(0),z\||\leq\left\|F(x_k)-x_k-F(0),z\right\|.$$

Let $\varepsilon>0$. So we have

$$\{k\leq n:|\|F(x_k)-x_k,z\|-\|F(0),z\||\geq\varepsilon\}\subset\{k\leq n:\|F(x_k)-x_k-F(0),z\|\geq\varepsilon\}.$$

Therefore

$$\begin{aligned}
&\delta\left(\{k\leq n:|\|F(x_k)-x_k,z\|-\|F(0),z\||\geq\varepsilon\}\right)\\
&\leq\delta\left(\{k\leq n:\|F(x_k)-x_k-F(0),z\|\geq\varepsilon\}\right).
\end{aligned}$$

Since $\delta\left(\{k\leq n:\|F(x_k)-x_k-F(0),z\|\geq\varepsilon\}\right)=0$ for every nonzero $z\in\mathbb{E}$, so

$$\delta\left(\{n\in\mathbb{N}:|\|F(x_k)-x_k,z\|-\|F(0),z\||\geq\varepsilon\}\right)=0.$$

So the sequence $\{\|F(x_k) - x_k, z\|\}_{k\in\mathbb{N}}$ is statistically convergent to $\|F(0), z\|$. Now we have

$$\begin{aligned}\varphi_n(F(x_n) - x_n; z) - \varphi_n(F(0); z) &= \varphi_n(F(x_n) - x_n - F(0); z) \\ &\leq \|\varphi_n, z\| \, \|F(x_n) - x_n - F(0), z\| \\ &= \|F(x_n) - x_n - F(0), z\|\end{aligned}$$

for each nonzero $z \in \mathbb{E}$. The sequence in the right is statistically convergent to 0. So, the sequence $\{\varphi_n(F(0); z)\}_{n\in\mathbb{N}}$ is statistically convergent to $\|F(0), z\|$. But this is possible only when $F(0) = 0$. This is a contradiction. This proves that the uniquely remotal set T is singleton.

As an immediate consequence of Theorem 1, we have the following result.

Corollary 1 Let T be a non-empty, bounded subset of a real 2-Banach space $\mathbb{E}$. If T is remotal such that T has Chebyshev center c and the extracted farthest point map $F : \mathbb{E} \to T$ restricted to $[c, F(c)]$ is partially statistically continuous at c in $(\mathbb{E}, \|\cdot,\cdot\|)$, then T is singleton.

From Theorem 2 of Som and Savaş [25] we have

Theorem 2 Let $\mathbb{E}$ be a real 2-Banach space and T be a non-empty, bounded, uniquely remotal set admitting a Chebyshev center c. If T is not a singleton, then the farthest point map F, restricted to $(c, F(c))$ is not partially statistically continuous at c.

Proof. The uniquely remotal set T has a Chebyshev center c. Let $x \in (c, F(c)]$. Then $x = tc + (1-t)F(c)$ for some $t \in [0, 1)$. Now for each nonzero $z \in \mathbb{E}$,

$$\begin{aligned}\|x - F(x), z\| &= \|tc + (1-t)F(c) - F(x), z\| \\ &= \|tc - tF(x) + (1-t)F(c) - (1-t)F(x), z\| \\ &\leq t\,\|c - F(x), z\| + (1-t)\,\|F(c) - F(x), z\| \\ &\leq t\,\|c - F(c), z\| + (1-t)\,\|F(c) - F(x), z\| \\ &\leq t\,\|x - F(x), z\| + (1-t)\,\|F(c) - F(x), z\|\end{aligned}$$

$$\Rightarrow \|F(x) - F(c), z\| \geq \|x - F(x), z\| \geq \|c - F(c), z\| = r.$$

If we choose $\|c - F(c), z\| = r > 0$ and a sequence $\{x_n\}_{n\in\mathbb{N}}$ in $(c, F(c)]$, statistically convergent to c in $(\mathbb{E}, \|\cdot,\cdot\|)$ but we have

$$\delta(\{k \leq n : \|F(x_k) - F(c), z\| \geq r\}) = 1$$

for all $n \in \mathbb{N}$ and each nonzero $z \in \mathbb{E}$. This shows that the farthest point map F is not statistically continuous at c. This completes the proof.

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