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OPTIMIZED WAVE MODEL DESCRIBING INFECTED CASES AND DEATHS OF COVID-19 AND ITS VARIANTS IN EGYPT

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Abstract. Infections are serious disturbing problem over the time. They are usually showing as a tsunami waves hitting the societies. COVID-19 and its followed variants are the recent pandemic in this decade. In this paper, a mathematical model is proposed to describe the wave propagation of COVID-19 daily cases and deaths. The model is applied to the data of Egyptian society which is representing the confirmed daily cases and deaths of COVID-19 and of its followed variants. The pandemic is studied as a successive flow of waves over the first three years. These waves are described by summation of sech-hyperbolic functions. The parameters of the model are estimated using Newton optimization method. Finally, the relation between cases and deaths is illustrated before and after vaccination to show the vaccination effect on society. Any pandemic data can be inserted to the proposed model, and then optimize the parameters for any country to fit the data inserted.

1. Introduction

In December 2019, a coronavirus (COVID-19) outbreak started from Wuhan, China. During the interval of January 20th to February 5th, 2020 the number of confirmed cases of coronavirus in China reached 28,018 cases with a further increase to 80,880 cases as on the 16th of March [14].

Since the 11th of March, the World Health Organization (WHO) announced that COVID-19 is a pandemic recording more than 118,000 cases worldwide [11]. Then many researches are published, most of them are medical studying symptoms, spread and medication such as [4, 5, 6, 13], while vaccination strategies are reviewed in the article [9]. Many others are presenting global outbreak and analyzing the epidemic such as [8], and others are introducing the usage of artificial intelligence during the pandemic (see [12]).

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On COVID-19 proposed models, there are several published papers. In [2], they used a simple variation on the classic susceptible, latently infected, symptomatic and asymptomatic infectious and removed individuals (SLIAR) epidemic model. While another epidemic model, called SIDARTHE, was proposed to study whether the infected cases in Italy are diagnosed according to their symptoms, or not even if they were COVID cases (see [7]). In Nigeria, two models are proposed in the form of non-linear differential equations based on the cumulative reported cases ([10] and [1]).

At the beginning, the number of accumulated COVID-19 cases were considered and known to be exponentially increasing till it reached a peak and then started to grow slowly. After the repetition of increase and then decrease of the reported cases, the accumulation view wasn't the best to describe the pandemic progress. The confirmed daily cases and deaths are having waveforms, they are showing peaks, repetitive increases and decreases.

In December of 2020, the first mass vaccination programme started. By then several types of vaccines have been developed and safely validated by WHO to be used. Vaccines support the immunity system and increase the antibodies defense against the pandemic. By mid of January 2022, the following vaccines have given EUL (Emergency Use Listing):

- The Pfizer/BioNTech Comirnaty vaccine, 31 December 2020.
- The SII/COVISHIELD and AstraZeneca/AZD1222 vaccines, 16 February 2021.
- The Janssen/Ad26.COV 2.S vaccine developed by Johnson & Johnson, 12 March 2021.
- The Moderna COVID-19 vaccine (mRNA 1273), 30 April 2021.
- The Sinopharm COVID-19 vaccine, 7 May 2021.
- The Sinovac-CoronaVac vaccine, 1 June 2021.
- The Bharat Biotech BBV152 COVAXIN vaccine, 3 November 2021.
- The Covovax (NVX-CoV2373) vaccine, 17 December 2021.
- The Nuvaxovid (NVX-CoV2373) vaccine, 20 December 2021.

In Egypt, the ministry of health hastened to quickly start vaccination to keep our population safe. Vaccination has been started since the early of March, 2021. Most of the medical, government, educational and higher educational sectors had got at least one dose vaccination by October 2022. By the end of 2023, people infected by COVID-19 and its variants, were not experiencing high risk symptoms to be interested in tesing whether it is COVID-19 or not,so that the recorded cases are very limited by who are staying at hospitals only.

In this paper, a proposed mathematical model can be applied to any wave propagated pandemic in any country. The model is applied to the cases and deaths in Egypt, then analyzed to express the progress of COVID-19 after vaccination.

The rest of this paper is organized as follows. In Section 2, we introduce the hyperbolic model. It is applied to the daily infected and death cases, inserting the constants of the model, in Section 3. Sections 4 and 5 present the analytic view of the progress COVID-19 and vaccination effect by using the model in Egypt. The data studied is downloaded from WHO website (https:
covid19.who.int/WHO-COVID-19-globl-data.csv).

2. The Descriptive wave model for Daily Cases

The daily cases and deaths have a consequence of waves. Each wave of infection has its own properties. These properties are considered to be the peak value of the wave, its wideness, and the time at which the peak is reached. Each property depends on how we treat the start of the wave and how people are kept safe from infection. The wider the wave indicates the slower the virus spread and vice versa. For Wave 1, it is clear that the pandemic spread was heavily controlled since it has completed its form. The synchronization of waves indicates that we have uncontrolled infection spread for many reasons such as crowds of people. This crowd is one of the main reasons, which can be caused during the students final examinations or students semester entrance. That specially occurs at schools, where students are able to transmit the infection between themselves and to their families. Children are not careful to keep distance from their classmates, they may not be controlled with wearing masks as adults. This situation could be seen for Wave 2, Wave 3 and Wave 5, as these intervals were the final semester examinations held (in January and May/June). So, quarantines and citizens' obligation level to the control rules are the most effective parameters on the waves occurrence, propagation, and amplitudes.

The wave forms of daily cases and deaths are proposed to be modeled to the wave equation,

$$f(t) = v_p sech(w(t - t_p)), \tag{1}$$

where t is time in days, v_p is the peak value of the wave, w is the wideness of the wave, and t_p is the time of the peak occurrence.

Eq. 1 is set to describe one solitary wave. To cover all waves along the time, the equation will have a submission of each wave in the same equation as follows,

$$F(t) = \sum_i v_p^{(i)} sech(w^{(i)}(t - t_p^{(i)})), i = 1, 2, 3, \ldots. \tag{2}$$

The proposed wave model is finely fitting the pandemic waves and their properties during time. The variation of the wave wideness is an indicator for the spread of infection speed, and the start of occurrence of another wave phase before the present wave is completed refers to the start of a social event without considering the control rules for the pandemic. This causes a sequence of waves with different altitudes. These waves are propagating in the same field (people of one country), so the daily cases and deaths are having the same wave behavior. For each wave, the number of daily deaths are increasing the same as the recorded infected cases and vice versa. This is showing the linear relation between deaths daily recorded number and the infected cases. In the next section, we will introduce how to get the best curves fitting to the waves of the pandemic. The procedures are discussed in detail to be helpful for other researchers to use hyperbolic functions for curve fitting.

3. Sech - Curve Fitting for daily cases and deaths

The nonlinear wave equation is a famous simple application for partial differential equations. Solitary waves solution can be set to be a sech-function. This solution could be including two variable t for time and x for displacement. For our model, we are expressing the pandemic waves in the form of a propagation in time excluding the displacement variable x from the wave solution as mentioned above in Eq. (1).

To get the best waveform fits the data, the model parameters are selected by using optimization. The amplitude of wave v_p, its time occurrence t_p, and the wideness of peak w are estimated and optimized by minimizing the mean square error, using Newton's method. The initial value of each parameter is selected arbitrary as close as it could be fitting the data. The first COVID-19 case was recorded in 2020-02-14, while the first death was in 2020-03-08 due to COVID-19 infection. The days is denoted by t, and first day is considered to be $t = 1$. In Fig. 1, the data of the confirmed cases started from 2020-02-14 to 2020-11-09 is giving a solitary wave form (which is counting 270 days).

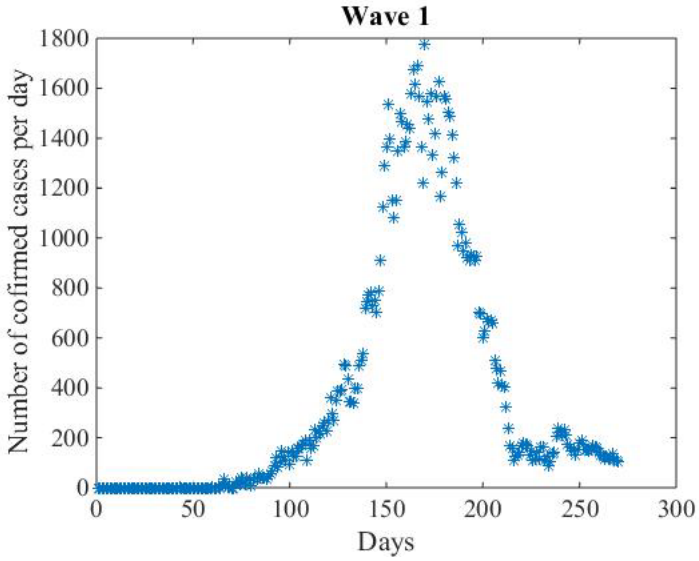

FIGURE 1. Wave 1 data.

The minimization of the mean square error is defined by the following equation

$$minJ = \frac{1}{N}\sum_{i=0}^{N}(y_i - f(t_i))^2 \tag{3}$$

where J is the objective function, y_i is the known data and $f(t_i)$ is given by Eq. (1).

Newton optimization method for Eq.(3) is given by

$$\big(v_p, t_p, w\big)^{(k+1)} = \big(v_p, t_p, w\big)^{(k)} - \nabla J(\nabla^2 J)^{-1} \tag{4}$$

where k id the iteration of the method. Eq. 4 is executed till the tolerance reaches value less than 10^{-5}. The curves are well fitted by the proposed model of sech-function in time for each wave of COVID-19 daily cases and deaths.

Let us discuss the first wave (Wave 1) of the recorded infected cases for the first 270 days in Egypt of COVID-19. The data is divided into 216 trained and 54 tested selected randomly, at which it is divided into 80%and 20% respectively. The three parameters are initialized to have the values $(t_p, v_p, w) = (160, 0.04, 1600)$. Optimizing the parameters after iterating Newton's method, the values are received to be $(t_p, v_p, w) = (169, 0.0497, 1599)$. Therefore our proposed model is ready to fit

the data of Wave 1, which is $f(t) = 1599sech0.0493(t - 131)$. Fig. 2 is showing Wave 1 and the model for the reported daily cases. The correlation between data and sech- model is 0.9835, and root mean square error (RMSE) is 90.4999. By the

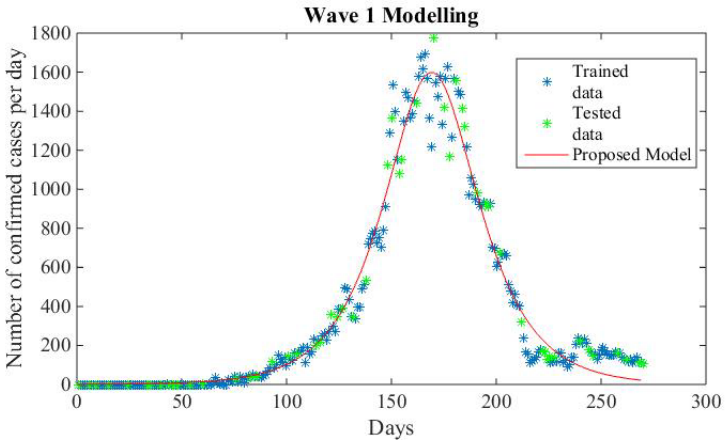

FIGURE 2. Wave 1 modelling

same way, both data, daily reported cases and deaths, is fitted with Eq. (2). Table 1 is presenting the inserted values of t_p, v_p and w for each wave due to the daily cases and daily deaths respectively for the first five waves. The root mean squared error (RMSE) and correlation are 110 and 0.9782 for daily cases respectively, while for daily deaths are 10.1 and 0.8997 during the whole period respectively.

	Daily cases parameters			**Daily deaths parameters**		
Waves	v_p	w	t_p	v_p	w	t_p
Wave 1	1601	0.04966	169	83	0.04837	114
Wave 2	994	0.03816	371	53	0.0252	326
Wave 3	1159	0.03606	493	52	0.03864	434
Wave 4	998	0.02928	675	63	0.03262	618
Wave 5	2200	0.05706	771	54	0.08794	708

TABLE 1. Parameters value of the model.

Fig. 3 and 4 are showing the waves of daily cases and deaths respectively and the corresponding model representing them starting from 03/01/2020 for cases and 08/03/2020 for deaths respectively, up to 12/03/2022 for both cases and deaths.

The data starting from 13/03/2022 up-to-date has missing daily records that was given by zero values. In the next section, the model is used to give these missing data. Also, in Section 5, the relationship between cases and deaths is represented, and how this relation was changed after vaccination.

4. The missing data determination

Since 13/03/2022, the data was not recorded everyday. The model here is used to cover the missing data. The wave appeared during the interval of time, from that day to 05/07/2023. The wave model is then set to be fitting the only recorded

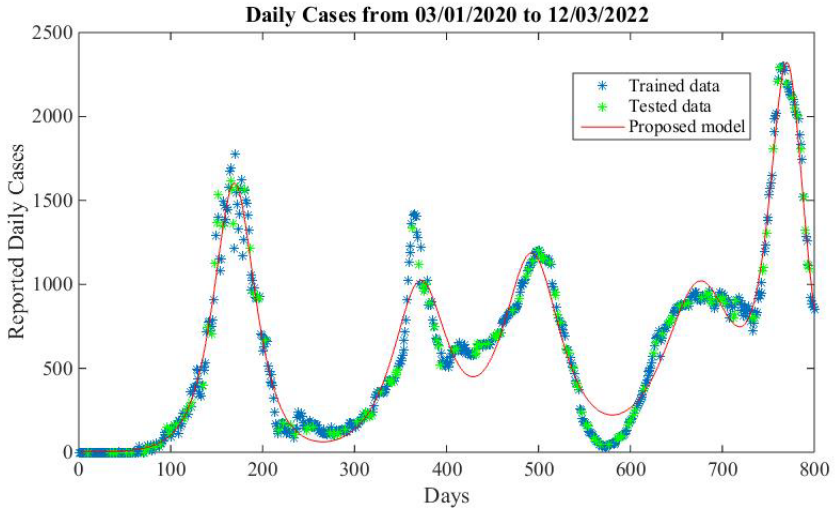

FIGURE 3. Modelling of Daily Cases during 800 days

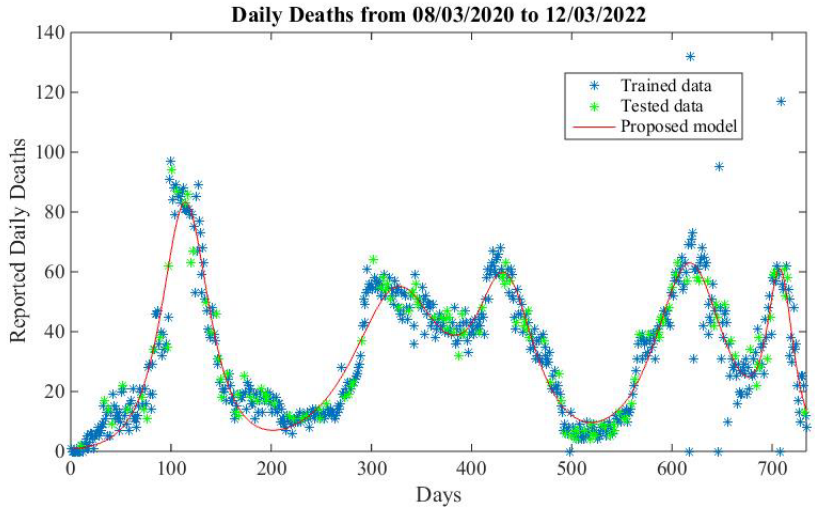

FIGURE 4. Modelling of Daily Deaths during 735 days

data, those are not equal to zero. The curves of data and the model are plotted as shown in Fig. 5 and Fig. 6. They are presenting two successive solitary waves, then the recorded data for cases and deaths have the wave parameters as given in Table 2.

The data is well fitted by sech-model. For daily cases, the correlation coefficient and RMSE are 0.9498 and 327.2076 respectively, while they are 0.8927 and 8.3283 for daily deaths, respectively.

From the model describing less recorded data, we can imagine the progress of COVID-19 and its variants in society. Although there was the highest infection peak of Wave 6 for daily cases since the beginning of the pandemic, the infected cases

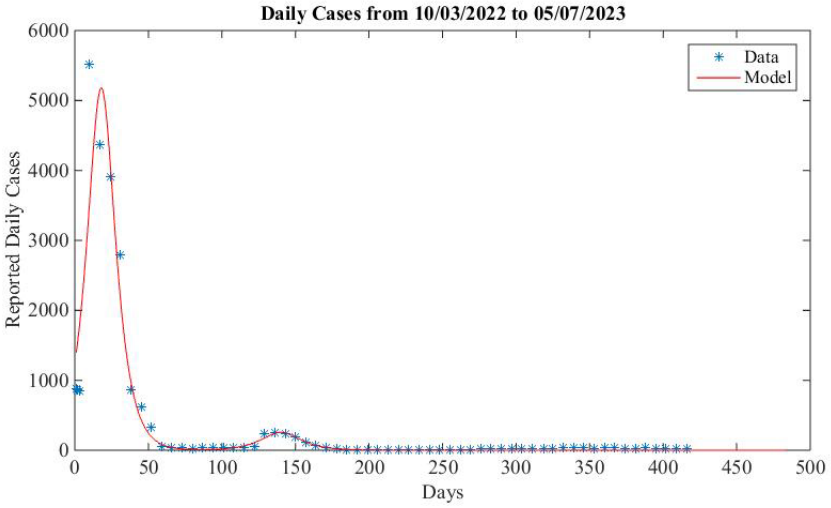

FIGURE 5. Modelling of missed Daily Cases during 483 days

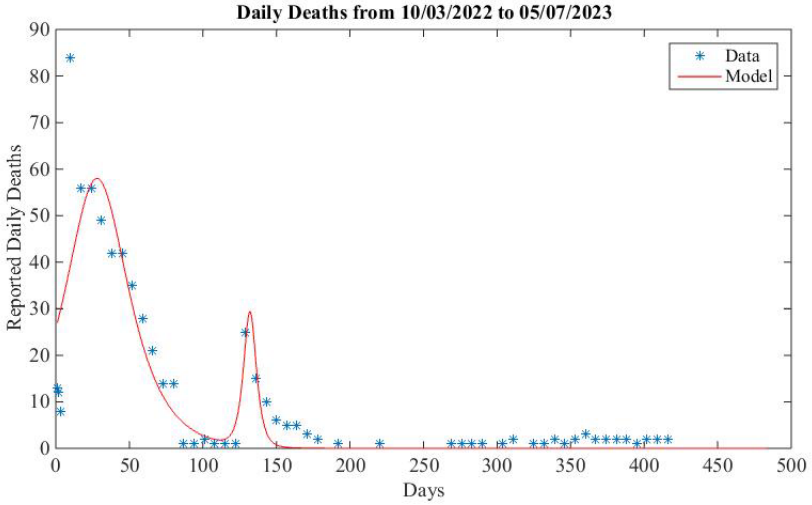

FIGURE 6. Modelling of missed Daily Deaths during 483 days

	Daily cases parameters			**Daily deaths parameters**		
Waves	v_p	w	t_p	v_p	w	t_p
Wave 6	5185	0.11682	18	58	0.051858	28
Wave 7	256	0.083017	140	29	0.25993	132

TABLE 2. Parameters value of the model.

were considering their illness as an usual sickness, and they did not suffer from life risk symptoms to stay at hospitals. On the other hand, the death recorded number of the serious COVID-19 cases was not the highest peak of the waves.

5. Daily Cases and Deaths, and The Effect of Vaccine

For Egyptian society, the proposed model is well describing the daily confirmed COVID-19 cases and reported deaths per day. The propagation of both cases and death waves has a common feature, as the number of waves is the same with some days delay (5-15 days) for death relative to the cases. This delay is normal according to the delayed occurrence of death after being infected by COVID-19 and trying to decrease the illness symptoms. Both cases and deaths have the same criteria to increase, reach a peak value, then decrease. In fact, they are showing a linear relation skipping the delay, so that the ratio between daily cases and deaths is studied for each wave.

The following figures are presenting the linear relation between the data of daily

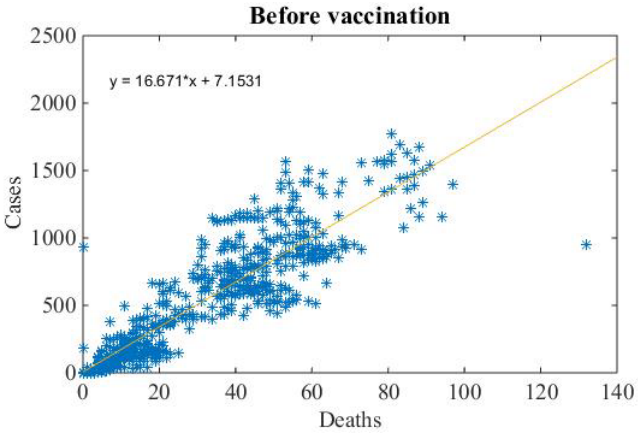

Figure 7. Linear regression between deaths and cases before vaccination widely applied.

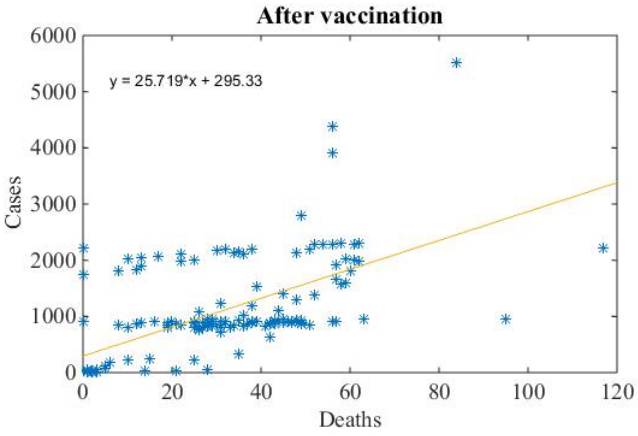

Figure 8. Linear regression between deaths and cases after vaccination.

cases and deaths for each of the first five waves. Also, the linear regression is plotted,

introducing the ratio value between both data (see Fig. 7 and Fig. 8). The last three waves behave differently than the others. Since the vaccination started in January 2022, the ratio between daily cases and deaths has increased. It means that the vaccination didn't decrease the number of daily cases, but it decreases the occurrence of deaths increasing the rate of change between both of them.

Let us check our proposed model to present the linear relation between deaths and cases for the first five waves. The delay in days is here considered by the difference between the peaks of each wave. The average delay between the first five waves (i.e. t_p of cases and deaths) is 56 days As shown in Fig. 9 that the data

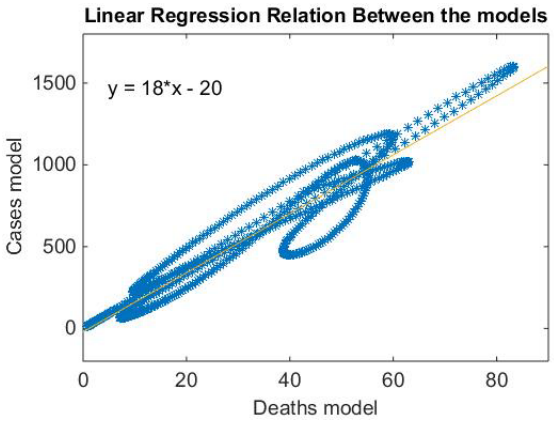

FIGURE 9. Linear regression between deaths and cases models before vaccination.

before vaccination is linearly dependent on each other, the deaths can be linearly related to the recorded cases per day. By then, deaths is not

Vaccination is supporting the immunity system to defend COVID-19 and its variants inside the body, controlling its damage effect on the other body system that may end up with death. Even in Egypt, vaccinated people percent out of the whole population is considered to be very small, but they are the most interacting people with the pandemic through curing, teaching, banking affairs, and so on.

In many other countries, sech-wave model can be applied to COVID-19 daily data. The daily cases and deaths are illustrating the same waves of successive propagation in Russia, Saudi Arabia, Marcoo, Germany, France, Italy, Canada, Australia, Mexico, Brazil and USA.

The values of the parameters are different from one country to another because of different control rules, restrictions,weather, and so on. The proposed model is helpful for detecting the unrecorded data.

6. Conclusion

In Egypt, the daily COVID-19 cases are described by a mathematical model and tested. This model could be applied by changing the values of parameters for any wave propagated pandemic, or for COVID-19 and its variants cases and deaths in any country which is still experiencing its spread in their society. In Egypt, the daily cases are not affected by vaccination, which means the spread of infection is not controlled, while the ratio between infected cases and death cases are less than

that it was before vaccination. These results showed the decrease of death risks due to COVID-19 infection or its variants and vaccine saved many lives of people in Egypt.

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