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ASYMPTOTIC BEHAVIOR MIX AND DIFFUSION PROCESS

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Abstract. We study the asymptotic behavior of a solution of mixed differential equation driven by independent fractional Brownian motion with Hurst index $H \in (0;1)$ and Levy process. This work consists of determining the large deviations principle by means of weak regularity of the coefficients of the stochastic differential equation in temporal distribution space.

1. Introduction

The study of rare events is an important and very active field in a variety of scientific disciplines. Rare event problems arise in the analysis and prediction of major risks, such as earthquakes, floods, air collision risks, nuclear radiation dispersal. Studying major risks can be undertaken by probabilistic modelling of processes such as the stochastic differential equation using certain mathematical tools, or simulation, to obtain an accurate estimate namely the large numbers theory or the large deviations theory. Therefore we study in this paper such a rare event via the large deviations theory.

Let $B^H = \left\{B_t^H, t \in [0,T]\right\}$ be a fractional Brownian motion with Hurst parameter $H \in (0,1)$, $W = \{W_t, t \in [0,T]\}$ be a standard Brownian motion and $\bar{N} = \left\{\bar{N}_t, t \in [0,T]\right\}$ be a compensated Poisson process defined on a white noise probability space $(\mathcal{S}'(\mathbb{R}), \mathcal{B}(\mathcal{S}'(\mathbb{R})), \mathbb{P})$ where $\mathcal{S}'(\mathbb{R})$ is a tempered distribution space called dual of Schwartz space on which $\mathcal{B}(\mathcal{S}'(\mathbb{R}))$ is a Borel algebra. Consider the following time-dependent mixed stochastic differential equation driven by these three

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processes:

$$\begin{aligned} X_t^\varepsilon = x_0 + & \int_0^t b(r, X_r^\varepsilon)dr + \varepsilon \int_0^t \sigma_H(r, X_r^\varepsilon)dB_r^H \\ & + \varepsilon \int_0^t \int_{\mathbb{R}^*} K(r, x, X_r^\varepsilon)\bar{N}(dx, dr) + \varepsilon \int_0^t \sigma_w(r, X_r^\varepsilon)dW_r, \end{aligned} \tag{1}$$

where

- ⋆ $x_0 \in \mathcal{S}'(\mathbb{R})$ and $x \in \mathbb{R}^*$ are measurable random variables;
- ⋆ b, σ_H and $\sigma_w : [0;T] \times \mathcal{S}'(\mathbb{R}) \to \mathcal{S}'(\mathbb{R})$ are measurable functions and are white noise integral (see[3, 15]);
- ⋆ $K : [0;T] \times \mathcal{S}'(\mathbb{R} \times \mathbb{R}^*) \to \mathcal{S}'(\mathbb{R} \times \mathbb{R}^*)$ is measurable function and is white noise integral (see [12]).

These three functions satisfy the following assumption.

Assumption 1. For $x \in \mathbb{R}, z$ and $h \in \mathcal{S}'(\mathbb{R})$, there exist constants M and L such that

$$\begin{aligned} &\bullet |b(h)| \le M \quad , \quad |\sigma(h)| \le M \quad , \quad |K(x,h)| \le M \\ &\bullet |b(h) - b(z)| \le L|h - z| \quad , \quad |\sigma(h) - \sigma(z)| \le L|h - z| \\ &\bullet |K(x,h) - K(x,z)| \le L|h - z|. \end{aligned}$$

The existence and uniqueness of solution for equation of such type have been proved in [12, 13]. The authors consider a class of neutral functional differential equations with finite delay driven simultaneously by a fractional Brownian motion and a Poisson point processes in Hilbert space and prove an existence and uniqueness result. However our goal in this paper is to study the asymptotic behavior of the solution of equation (1) via the Freidlin- Wentzell's large deviation [9] . Moreover in the literature [4, 5, 10, 11, 14, 15] several authors have established the large deviation principle for equations driven by Poisson process and those driven simultaneously by Poisson process and standard Brownian motion. Indeed [15, 14] have established a large deviation principle for general stochastic evolution equation driven by both standard Brownian motion and Poisson jump on a given Hilbert space by using weak convergence method. Concerning the study by the large deviation principle for stochastic differential equations driven by a fractional Brownian motion(fBm) and those driven simultaneously by this process and the standard Brownian motion, we have obtained the results in our papers [7] using the Freidlin-Wentzell [9, 1] methods in $\mathcal{S}'(\mathbb{R})$. To our knowledge, no article presents the study via a large deviation principle of stochastic differential equation controlled simultaneously by fBm and Levy process and therefore we invest in it and consequently what allowed us to write this paper. The approach we have adopted here is different from that used by other authors. As in our paper [7] we proceed by assuming the independence of fBm, standard Brownian motion and Poisson process in the first case where the drift is zero and the diffusion coefficients are equal to one and in the second case where the drift is non-zero. The paper is organized as follows: Section 2, we set up some definitions and theorem of fractional Brownian motion, standard Brownian motion, Poisson jumps process and large deviation principle favourable to our work. Section 3 contains our main results, these results are carried out in two phases. The first is when the drift is 0 and the diffusion coefficients are equal to 1 and the second is when the drift is different from 0.

2. Preliminaries

Consider the white noise space $(\mathcal{S}'(\mathbb{R}), \mathcal{B}(\mathcal{S}'(\mathbb{R})), \mathbb{P})$ and denote $\langle .,. \rangle$ the scalar product and $|.|$ the norm in $\mathcal{S}'(\mathbb{R})$. It well know that $\mathcal{S}(\mathbb{R}) \subset \mathbb{L}^2(\mathbb{R}) \subset \mathcal{S}'(\mathbb{R})$ with $\mathbb{L}^2(\mathbb{R})$ is the Hilbert space.
Let $B_t^H,\ W_t : [0;T] \times \mathcal{S}'(\mathbb{R}) \to \mathcal{S}'(\mathbb{R})$ be respectively a fractional Brownian motion and a standard Brownian with probabilities measures respectively $\mathbb{P}^H$ and $\mathbb{P}^w$ on $\mathcal{B}(\mathcal{S}'(\mathbb{R}))$ and let $\bar{N} : [0;T] \times \mathcal{S}'(\mathbb{R} \times \mathbb{R}) \to \mathcal{S}'(\mathbb{R} \times \mathbb{R})$ be a compensated Poisson process with probability measure ν on $\mathcal{B}(\mathcal{S}'(\mathbb{R}))$. So in $S'(\mathbb{R})$ we recall some definitions and theorems of those three process and large deviation theory.

Definition 2.1. *For η and $\theta \in S'(\mathbb{R})$, the process $\langle \eta, f_{[0,t]} \rangle = \int_0^t f(r) dB_t^H$ and $\langle \theta, \psi_{[0,t]} \rangle = \int_0^t \psi(r) dW_r$ are Gaussian process with respectively covariances*

$$|f|_\phi^2 = \langle f_{[0,t]}, f_{[0,s]} \rangle = \int_0^t \int_0^s f(u) f(r) \phi(u,r) du dr,\ |\psi|^2 = \int_0^t \psi^2(r) dr$$

*for all $f; \psi : [0;T] \to \mathbb{R}$ and $\phi(t,s) = \frac{\partial^2 R_H}{\partial t \partial s} = H(2H-1)|t-s|^{2H-2}$
and $\omega \in \mathcal{S}'(\mathbb{R}), \langle \omega - 1 \circledast, \varphi_t \rangle = \int_0^t \int_{\mathbb{R}} \varphi(x,r) \bar{N}(dx, dr)$
For that , we define the space of continuous square integral functions
$\mathbb{L}_\phi^2 = L_\phi^2(\mathbb{R}) \times L^2(\mathbb{R} \times \mathbb{R}^*) \times L^2(\mathbb{R})$ where*

$$\begin{cases} L_\phi^2(\mathbb{R}) = \{f : [0,T] \to \mathbb{R},\ \int_0^t \int_0^s f(r) f(u) \phi(r,u) du dr < +\infty\} \\ L^2(\mathbb{R}) = \{\psi : [0,T] \to \mathbb{R},\ \int_0^t \psi^2(r) dr < +\infty\} \\ L^2(\mathbb{R}^* \times \mathbb{R}) = \{\varphi : [0,T] \to \mathbb{R},\ I_\nu(\varphi) = \int_0^t \int_0^s \lambda(\varphi(r,u)) \nu(dr) dx < +\infty\}. \end{cases} \tag{2}$$

Definition 2.2. *The family $(X_t^\varepsilon)_{\varepsilon > 0}$ of probability $\mathbb{P}^\varepsilon$ is said to satisfy a large deviation principle (LDP) if there exists a rate function I defined on $\mathbb{L}_\phi^2$ and a speed ε tending to 0 such that:*

i) *$0 \leq I(x) \leq +\infty$, for all $x \in \mathbb{L}_\phi^2$;*
ii) *I is lower semicontinuous on $\mathbb{L}_\phi^2$;*
iii) *for all $a < +\infty$, $\{x : I(x) \leq a\}$ is a compact of $\mathbb{L}_\phi^2$, in which case I is a good rate function;*
iv) *for any closed set $C \subset \mathbb{L}_\phi^2$,*

$$\lim_{\varepsilon \to 0} \sup \varepsilon \log \mathbb{P}^\varepsilon (X_t^\varepsilon \in C) \leq - \inf_{x \in F} I(x) \tag{3}$$

- *for any open set $O \subset \mathbb{L}_\phi^2$,*

$$\lim_{\varepsilon \to 0} \inf \varepsilon \log \mathbb{P}^\varepsilon (X_t^\varepsilon \in O) \geq - \inf_{x \in O} I(x) \tag{4}$$

Theorem 2.1. *(**Contraction principle**, see [8])*
Let E_1 and $E_2 \subset \mathbb{L}_\phi^2$ and $g : E_1 \to E_2$ is a continuous function. If the family $(X_t^\varepsilon)_{\varepsilon > 0}$ satisfies a large deviations principle of a rate function I then the family $g((X_t^\varepsilon)_{\varepsilon > 0})$ satisfies the LDP on E_2 of a rate function J defined by:

$$J(z) = inf\{I(h) : h \in E_1,\ z = g(h)\},$$

for each $z \in E_2$.

3. Main résults

Our results are done in two phases: we show LDP first for a sum of those three independents processes: $B^H,\ \bar{N}$ and W then we show the LDP for the solution of equation (1).

3.1. large deviations for the sum independent processes fBm, Bm and compensated Poisson process.

Consider the process

$$\varepsilon B_t^H + \varepsilon \bar{N}_t + \varepsilon W_t,\ t \in [0;T] \tag{5}$$

with probability measure $\mathbb{P}$ in $\mathbb{L}_\phi^2$ and define $\Phi = (f, \varphi, \psi)$ and

$$I(\Phi) = \begin{cases} \frac{1}{2}|f|_\phi^2 + I_\nu(\varphi) + \frac{1}{2}|\psi|^2 \\ \\ +\infty \quad otherwise \end{cases} \tag{6}$$

Proposition 3.1. *The function $I : \mathbb{L}_\phi^2 \to [0; +\infty]$ defined in (6) is a good rate function and $a \in \mathbb{R}_+^*$, we have:*

1. *I is lower semi-continuous on $\mathbb{L}_\phi^2$;*
2. *$\{\Phi \in \mathbb{L}_\phi^2, I(\Phi) \leq a\}$ is a compact subset of $\mathbb{L}_\phi^2$.*

Proof.
Put $I_1 = \frac{1}{2}|f|_\phi^2 + I_\nu(\varphi)$ and $I_2 = \frac{1}{2}|\psi|^2$.
By [6] I_1 is good rate function in $L_\phi^2(\mathbb{R}) \times L^2(\mathbb{R} \times \mathbb{R}^*)$ and I_2 is too a good rate function in $L^2(\mathbb{R})$, so I been the sum of those functions is a good rate function. □

Theorem 3.2. *The family $(\varepsilon B_t^H + \varepsilon \bar{N}_t + \varepsilon W_t)_{(\varepsilon > 0)}$ satisfies a large deviation principle with a good rate function I. We have:*

1. *for all closed set $C \subset \mathbb{L}_\phi^2$*

$$\lim_{\varepsilon \to 0} \sup \varepsilon^2 \log \mathbb{P}\left(\varepsilon B_t^H + \varepsilon \bar{N}_t + \varepsilon W_t \in C\right) \leq -\left[\frac{1}{2}|f|_\phi^2 + I_\nu(\varphi) + \frac{1}{2}|\psi|^2\right]$$

2. *for an open set $O \subset \mathbb{L}_\phi^2$,*

$$\lim_{\varepsilon \to 0} \inf \varepsilon^2 \log \mathbb{P}\left(\varepsilon B_t^H + \varepsilon \bar{N}_t + \varepsilon W_t \in O\right) \geq -\left[\frac{1}{2}|f|_\phi^2 + I_\nu(\varphi) + \frac{1}{2}|\psi|^2\right].$$

Proof.
Since the families are independent we have:
$\mathbb{P}(\varepsilon B_t^H + \varepsilon \bar{N}_t + \varepsilon W_t) = \mathbb{P}(\varepsilon B_t^H) \times \mathbb{P}(\varepsilon \bar{N}_t) \times \mathbb{P}(\varepsilon W_t)$. In this cases, we obtain:
for any open subset $O \subset \mathbb{L}_\phi^2$

$$\begin{aligned} \lim_{\varepsilon \to 0} \inf \varepsilon^2 \log \mathbb{P}(\varepsilon B_t^H + \varepsilon \bar{N}_t + \varepsilon W_t \in O) &= \lim_{\varepsilon \to 0} \inf \varepsilon^2 \log \mathbb{P}^H(\varepsilon B_t^H \in O) + \lim_{\varepsilon \to 0} \inf \varepsilon^2 \log \nu(\varepsilon \bar{N}_t \in O) \\ &+ \lim_{\varepsilon \to 0} \inf \varepsilon^2 \log \mathbb{P}^w(\varepsilon W_t \in O) \\ &\geq -\left[\frac{1}{2}|f|_\phi^2 + I_\nu(\varphi) + \frac{1}{2}|\psi|^2\right]. \end{aligned}$$

For all closet subset $C \subset \mathbb{L}^2_\phi$

$$\begin{aligned}\lim_{\varepsilon\to 0}\sup \varepsilon^2 \log \mathbb{P}\left(\varepsilon B^H_t + \varepsilon \bar{N}_t + \varepsilon W_t \in C\right) &= \lim_{\varepsilon\to 0}\sup \varepsilon^2 \log \mathbb{P}^H(\varepsilon B^H_t \in C) + \lim_{\varepsilon\to 0}\sup \varepsilon^2 \log \nu(\varepsilon \bar{N}_t \in C)\\ &+ \lim_{\varepsilon\to 0}\sup \varepsilon^2 \log \mathbb{P}^w(\varepsilon W_t \in C)\\ &\le -\left[\frac{1}{2}|f|^2_\phi + I_\nu(\varphi) + \frac{1}{2}|\psi|^2\right].\end{aligned}$$

□

3.2. **Asymptotic behavior of fractional and Levy diffusion process.** In this second section of our work, we will study the diffusion process X^ε_t (1) when the drift is not 0 and ε tends to 0. So we denote the probability law of X^ε_t by $\mu = \mathbb{P}\circ F^{-1}$ where:

- ⋆: $\mathbb{P}$, the probability law of $\varepsilon B^H_t + \varepsilon \bar{N}_t + \varepsilon W_t$;
- ⋆: F is a deterministic function, solution of the following system of ordinary differential equations:

$$\begin{cases} F_t(f_t,\psi_t,\varphi_t) = h(t) = x_0 + \int_0^t b(h(r))dr + \int_0^t \sigma_H(h(r))f_r\phi(r,s)dr + \int_0^t \sigma_w(h(r))\psi_r dr \\ \qquad\qquad + \int_0^t\int_{\mathbb{R}^*} K(x,h(r))(e^{\varphi(x,r)}-1)\nu(dx)dr \\ F_t(f_t,0,0) = z(t) = x_0 + \int_0^t b(z(r))dr + \int_0^t \sigma_H(z(r))f_r\phi(r,s)dr \\ F_t(0,\psi_t,0) = g(t) = x_0 + \int_0^t b(g(r))dr + \int_0^t \sigma_w(g(r))\psi_r dr \\ F_t(0,0,\varphi_t) = p(t) = x_0 + \int_0^t\int_{\mathbb{R}^*} K(x,p(r))(e^{\varphi(x,r)}-1)\nu(dx)dr \\ F_t(0,0,0) = x_0 + \int_0^t b(F_r(0,0,0))dr \end{cases} \tag{7}$$

for which $(f,\varphi,\psi) \in \mathbb{L}^2_\phi$ induced by LDP of the process $\varepsilon B^H_t + \varepsilon \bar{N}_t + \varepsilon W_t$.

Proposition 3.2. *Assume $F(0;0;0)$ defined in (7). Then for $R > 0$ and $\delta > 0$ there exists $\alpha > 0$ such that*

$$\lim_{\varepsilon\to 0}\sup \varepsilon^2 \log \mu\{\|X^\varepsilon_t - F_t(0;0;0)\|_{\mathbb{L}^2_\phi} > \delta, \|B^H_t + \bar{N}_t + W_t\|_{\mathbb{L}^2_\phi} < \alpha\} < -R. \tag{8}$$

Proof.
By assumption (1) and Gronwall's Lemma we have

$$\|X^\varepsilon_t - F_t(0;0;0)\|_{\mathbb{L}^2_\phi} \le \varepsilon M \sup_{0\le t\le T} |B^H_t + \bar{N}_t + W_t| e^{LT}$$

$$\begin{aligned}\mu\{\|X^\varepsilon_t - F_t(0;0;0)\|_{\mathbb{L}^2_\phi} > \delta\} &\le \mu\{\sup_{0\le t\le T} |B^H_t + \bar{N}_t + W_t| > \frac{\delta e^{-LT}}{\varepsilon M}\}\\ &\le 4\exp\{-\frac{\delta^2 e^{-2LT}}{2\varepsilon^2 M^2(t^{2H}+2t)T^2}\}.\end{aligned}$$

$$\begin{aligned}\mu\{\|X^\varepsilon_t - F_t(0;0;0)\|_{\mathbb{L}^2_\phi} &> \delta, \|B^H_t + \bar{N}_t + W_t\|_{\mathbb{L}^2_\phi} < \alpha\}\\ &\le \mu\{\sup_{0\le t\le T} |B^H_t + \bar{N}_t + W_t| > \frac{\delta e^{-LT}}{\varepsilon M}, \|B^H_t + \bar{N}_t + W_t\|_{\mathbb{L}^2_\phi} < \alpha\}\\ &\le 4\exp\{-\frac{\delta^2 e^{-2LT}}{2\varepsilon^2 M^2(t^{2H}+2t)T^2}\}.\end{aligned}$$

Put $R = \dfrac{\delta^2 e^{-2LT}}{2M^2(t^{2H}+2t)T^2}$, thus

$$\lim_{\varepsilon\to 0}\sup \varepsilon^2 \log \mu\{\|X_t^\varepsilon - F_t(0;0;0)\|_{\mathbb{L}^2_\phi} > \delta, \|B_t^H + \bar{N}_t + W_t\|_{\mathbb{L}^2_\phi} < \alpha\} < -R.$$

□

Lemma 3.1. *Let σ be a bounded function and f be a bounded and continuous function. Then there exists $c > 0$ and $N > 0$ such that*
$|f(t)\phi(t,s)| \leq c$ *and* $|\sigma(h(t))\phi(t,s)| \leq N$ *for all* $0 \leq s, t \in [0,T]$.

Proof. f is a bounded function, so there exists δ such that $|f| \leq \delta$. We have for $s, t \in [0;T]$

$$\begin{aligned}|f(t)\phi(s,t)| &= |f(t)||\phi(s,t)| = |f||H(2H-1)|t-s|^{2H-2}| \\ &\leq \delta H|(2H-1)|T^{2H} = c.\end{aligned}$$

σ is bounded, so there exists M such that $|\sigma(h(t))| \leq M \ \forall\ h \in L^2_\phi(\mathbb{R})$, we have for $s, t \in [0;T]$

$$\begin{aligned}|\sigma(h(t))\phi(s,t)| &= |\sigma(h(t))||\phi(s,t)| = |\sigma(h(t))||H(2H-1)|t-s|^{2H-2}| \\ &\leq MH|(2H-1)|T^{2H} = N.\end{aligned}$$

□

Theorem 3.3. *Assume F defined in (7). For $(f,\psi,\varphi) \in \mathbb{L}^2_\phi$ let $\gamma_t = \int_0^r f_r\phi(r,s)dr$, $\Psi_t = \int_0^r \psi_r dr$ and $\Theta_t = \int_0^t \int_{\mathbb{R}^*}(e^{\varphi(x,r)} - 1)\nu(dx)dr$. Then for $R' > 0$ and $\delta > 0$ there exists $\alpha > 0$ such that*

$$\lim_{\varepsilon\to 0}\sup \varepsilon^2 \log \mu\{\|X_t^\varepsilon - F_t(f_t,\varphi_t,\psi_t)\|_{\mathbb{L}^2_\phi} > \delta, \|B_t^H + \bar{N}_t + W_t - \frac{1}{\varepsilon}(\gamma_t + \Theta + \Psi_t)\|_{\mathbb{L}^2_\phi} < \alpha\} < -R'. \tag{9}$$

Proof.
Using again assumption (1); Gronwall's Lemma and the triangular inequality, we have ,

$$\|X_t^\varepsilon - F_t(f_t,\psi_t,\varphi_t)\|_{\mathbb{L}^2_\phi} \leq \varepsilon M \sup_{0\leq t\leq T}[|B_t^H + \frac{1}{\varepsilon}\gamma_t + W_t - \frac{1}{\varepsilon}\Psi_t + \bar{N}_t - \frac{1}{\varepsilon}\Theta_t| + |\frac{2}{\varepsilon}\gamma_t|]e^{LT}.$$

Put $\delta' = \delta + 2McT^{2H}e^{LT}$ *for* $\delta > 0$, $\tilde{B}_t^H = B_t^H + \frac{1}{\varepsilon}\gamma_t$, $\tilde{W}_t = W_t - \frac{1}{\varepsilon}\Psi_t$, $\tilde{\bar{N}}_t = \bar{N}_t - \frac{1}{\varepsilon}\Theta_t$

$$\mu\{\|X_t^\varepsilon - F_t(f_t,\psi_t,\varphi_t)\|_{\mathbb{L}^2_\phi} > \delta'\} \leq \mu\{\sup_{0\leq t\leq T}|\tilde{B}_t^H + \tilde{\bar{N}}_t + \tilde{W}_t| > \frac{\delta e^{-LT}}{\varepsilon M}\}.$$

$$\mu\{\|X_t^\varepsilon - F_t(f_t,\psi_t,\varphi_t)\|_{\mathbb{L}^2_\phi} > \delta', \|B_t^H + \bar{N}_t + W_t - \frac{1}{\varepsilon}(\gamma_t + \Psi_t + \Theta_t)\|_{\mathbb{L}^2_\phi} < \alpha\}$$
$$\leq \mu\{\sup_{0\leq t\leq T} |\tilde{B}_t^H + \tilde{\bar{N}}_t + \tilde{W}_t| > \frac{\delta e^{-LT}}{\varepsilon M}, \|B_t^H + \bar{N}_t + W_t - \frac{1}{\varepsilon}(\gamma_t + \Psi_t + \Theta_t)\|_{\mathbb{L}^2_\phi} < \alpha\}$$
$$\leq \exp\{-\frac{1}{\varepsilon}I(\Phi)\}\tilde{\mu}^\varepsilon\{\sup_{0\leq t\leq T} |\tilde{B}_t^H + \tilde{\bar{N}}_t + \tilde{W}_t| > \frac{\delta e^{-LT}}{\varepsilon M}, \|\tilde{B}_t^H + \tilde{\bar{N}}_t + \tilde{W}_t\|_{\mathbb{L}^2_\phi} < \alpha\}$$
$$\leq \exp\{-\frac{1}{\varepsilon}I(\Phi)\} \times \exp\{-\frac{1}{\varepsilon}R\} = \exp\{-\frac{1}{\varepsilon}(I(\Phi) + R)\}$$
$$\leq \exp\{-\frac{1}{\varepsilon}R\}.$$

$$\lim_{\varepsilon\to 0}\sup \varepsilon^2 \log \mu\{\|X_t^\varepsilon - F_t(f_t,\psi_t,\varphi_t)\|_{\mathbb{L}^2_\phi} > \delta, \|B_t^H + \bar{N}_t + W_t - \frac{1}{\varepsilon}(\Psi_t + \Theta)\|_{\mathbb{L}^2_\phi} < \alpha\} \leq -R'.$$

□

Proposition 3.3. *The function $F : [0,T] \times \mathbb{L}^2_\phi \to \mathbb{L}^2_{\phi^{-1}}$ defined by (7) is a continuous function on the subset of $\mathbb{L}^2_\phi$.*

Proof.
Let's first show $F(f,\varphi,\psi) = h$ is continuous for all $(f,\varphi,\psi) \in \mathbb{L}^2_\phi$.
Let $h_1 = F(f_1,\varphi_1,\psi_1)$ and $h_2 = F(f_2,\varphi_2,\psi_2)$ with

$$h(t) = x_0 + \int_0^t b(h(r))dr + \int_0^t \sigma_H(h(r))f_r\phi(r,s)dr + \int_0^t\int_{\mathbb{R}} K(x,h(r))(e^{\varphi(x,r)} - 1)\nu(dx)dr + \int_0^t \sigma_w(h(r))\psi_r dr$$

Using assumption (1), lemma, Gronwall's lemma, and [6] we have

$$|h_1(t) - h_2(t)| \leq L(1 + c + 2\delta)\int_0^t |h_1(r) - h_2(r)|dr + \alpha(N + 2M)T$$
$$\|F(f_1,\psi_1,\varphi_1) - F(f_2,\psi_2,\varphi_2)\|_{\mathbb{L}^2_\phi} = \|h_1 - h_2\|_{\mathbb{L}^2_\phi} \leq \alpha(N + 2M)Te^{L(1+c+2\delta)T}.$$

Hence F is continuous. □

Theorem 3.4. *The family $(X_t^\varepsilon)_{\varepsilon>0}$ satisfies a large deviation principle with a good rate function $J : \mathbb{L}^2_{\phi^{-1}} \to [0,+\infty]$ given by:*

$$J(z,p,g) = \begin{cases} \frac{1}{2}|\sigma_H^{-1}(z)[\dot{z} - b(z)]|^2_{\phi^{-1}} + \inf_{\psi\in L^2(\mathbb{R}^*\times\mathbb{R})}\{I_\nu(\psi), F_\nu(\psi) = p\} + \frac{1}{2}|\sigma_w^{-1}(g)[\dot{g} - b(g)]|^2 \\ \qquad for\ (z,\varphi,g) \in \mathbb{L}^2_{\phi^{-1}} \\ +\infty\ otherwise \end{cases} \tag{10}$$

In other word:

(1) *J is a good rate function;*
(2) *for all closed set $C \subset \mathbb{L}^2_\phi$ and any open set $O \subset \mathbb{L}^2_\phi$ and for $(z,\varphi,g) \in \mathbb{L}^2_\phi$*

$$\lim_{\varepsilon\to 0}\sup \varepsilon^2 \log \mu(X_t^\varepsilon \in C) \leq -J(z,p,g) \leq \lim_{\varepsilon\to 0}\inf \varepsilon^2 \log \mu(X_t^\varepsilon \in O)$$

Proof.
Put $J_1 = \frac{1}{2}|\sigma_H^{-1}(z)[\dot{z} - b(z)]|^2_{\phi^{-1}} + \inf_{\psi \in L^2(\mathbb{R}^* \times \mathbb{R})} \{I_{\dot{\nu}}(\psi), F_\nu(\psi) = p\}$ and
$J_2 = \frac{1}{2}|\sigma_w^{-1}(g)[\dot{g} - b(g)]|^2$
J_1 is a good rate function and by [9] J_2 is too a good rate function, so $J = J_1 + J_2$ is a good rate function. By the theorem 3.3 and the fact that F is a continuous function, the family $(X_t^\varepsilon)_{\varepsilon>0}$ with probability measure $\mu = \mathbb{P} \circ F^{-1}$ satisfies a LDP with a good rate function J. □

4. Conclusion

In the present paper, we have established a LDP for solution of (1) for any Hurst parameter $H \in (0; 1)$. This construction is carried out in the tempered distribution space $\mathcal{S}'(\mathbb{R})$ using the method of Freidlin-Wentzell [9] or Azencott's method [1]. So it would be very interesting to do this in a space larger than that considered here.

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